

On a property of homology equivalences and its application to the decomposition of special polyhedral products

Kouyemon Iriye

It is well-known that the Whitehead theorem plays a very important role in homotopy theory. But so far the study of maps which induce isomorphisms of fundamental groups and all integral homology groups does not seem to appear in the literature. In this talk we show the following theorem and give its application to the decomposition of special polyhedral products.

Theorem. *Let X be a connected finite CW-complex, Y be a simply connected finite CW-complex which admits a co-H structure, and S be a (finite) bouquet of circles. Let $f : S \vee Y \rightarrow S \vee Y$ be a map such that f induces an isomorphism of fundamental groups and isomorphisms of integral homology groups. If $f \circ g : X \rightarrow S \vee Y$ is null-homotopic for a map $g : X \rightarrow S \vee Y$, then g is also null-homotopic.*

Remark. Let $f : S \vee Y \rightarrow S \vee Y$ be a map as in the theorem above and F_f be its homotopy fiber. The theorem says that the canonical map $F_f \rightarrow S \vee Y$ is a phantom map. It is easy to construct a map f such that this phantom map is essential.