

# Conference on K-theory and index theory

Gennadi Kasparov (Vanderbilt, USA)

Other participants:

Yoshiyasu Fukumoto (ECNU, China)

Peter Hochs (Adelaide, Australia)

Hideki Inoue (Nagoya, Japan)

Tsuyoshi Kato (Kyoto, Japan)

Yosuke Kubota (Riken, Japan)

Yanli Song (Washington, USA)

Doman Takata (Tokyo, Japan)

Dates: 2018 August 8-10

Place: Department of Mathematics, room no 127, Kyoto University

Program:

	8	9	10
10:00-11:00		Hochs	Kato
11:20-12:20		Fukumoto	Song
14:30-15:30	Inoue	Kubota	
15:50-16:50	Kasparov	Takata	

G. Kasparov

Title: Spectral K-theory.

Abstract: We introduce a new operator K-theory group  $SK(X;B)$  on the category of  $C_0(X)$ -algebras which behaves like a K-homology group with respect to the locally compact space  $X$  and as a K-theory group with respect to its second argument, a  $C_0(X)$ -algebra  $B$ . This group is closely related with the KK-bifunctor and has good properties with respect to the tensor product of  $C_0(X)$ -algebras over  $C_0(X)$ . Applications to the index theory, especially for transversally elliptic operators, will be discussed.

Y. Fukumoto (ECNU)

Title: Project on twisted  $L^2$  index formula for proper actions

Abstract: When a locally compact group  $G$  acts on a manifold  $X$  properly and a  $U(1)$ -valued 2-cocycle  $\forall\alpha$  of  $G$  has vanishing Dixmier Douady class, then the  $\forall\alpha$ -twisted  $G$ -index of a  $G$ -invariant Dirac operator is realized using the usual  $G_{\forall\alpha}$ -index, here,  $G_{\forall\alpha}$  denotes the  $U(1)$ -central extension of  $G$ . More precisely,  $\forall\alpha$ -twisted  $G$ -index and  $G_{\forall\alpha}$ -index have coincident trace and moreover, its trace can be represented some cohomology classes of  $X$  thanks to the index formula by H. Wang. We want to do this without the condition that Dixmier Douady class vanishes, and we can expect more delicate information will be extracted from the operator via the twisted  $G$ -index.

P. Hochs

Title: Orbital integrals in K-theory of group  $C^*$ -algebras

Abstract: K-theory of group  $C^*$ -algebras is a useful tool in representation theory, geometry and topology. These K-theory groups are not well-understood in general, as illustrated by the unresolved Baum-Connes conjecture. One way to study the K-theory of a group  $C^*$ -algebra and its elements is via maps into the complex numbers induced by trace maps on dense, holomorphically closed subalgebras. In recent years, it has turned out that traces defined by orbital

integrals yield useful information in this context. An orbital integral is the integral of a function on the group over a conjugacy class. For the conjugacy class of the identity element, this is the classical von Neumann trace. For semisimple Lie groups, we use a fixed point theorem for equivariant indices to show what information can be recovered via these traces. This includes Harish-Chandra's character formula for the discrete series, and Shelstad's character identities, in the case of the discrete series. This is joint work with Hang Wang.

H. Inoue

Title: Index theorems for Fredholm, semi-Fredholm and almost-periodic operators; all in one example

Abstract: Based on operators borrowed from scattering theory, we propose several concrete realisations of index theorems. The corresponding operators belong to some  $C^*$ -algebras generated by bounded pseudo-differential operators with coefficients, which either have limits at  $\pm \infty$ , or which are periodic, or which are asymptotically periodic, or which are uniform almost-periodic. These various situations can be deduced from a single partial isometry which depends on several parameters. The resulting relations correspond to the topological version of Levinson's theorem for a family of Schrödinger operators with inverse square potentials on the half-line. This talk is based on a joint work with S. Richard.

T. Kato

Title:  $L^2$  harmonic forms and a reduced Fredholm map

Abstract: In this talk I will explain a new phenomenon on non compact complete Riemannian four manifolds, where the image of one forms does not exhaust densely on  $L^2$  self dual forms on each compact subset if a certain  $L^2$  self dual harmonic form exists. This leads us to a new construction of some reduced Fredholm map.

Y. Kubota

Title: The relative higher index and geometry

Abstract: In this talk I will give a geometric perspective on the invariant of manifolds with boundary called the relative higher index, which is introduced by Chang-Weinberger-Yu in 2015. I will introduce some geometric reflections of this invariant concerning with K-area, enlargeability and positive scalar curvature metrics.

D. Takata

Title: Towards an infinite-dimensional index theorem

Abstract: The overall goal of my research is to formulate and prove an infinite-dimensional version of the equivariant index theorem for locally compact manifold with a proper and cocompact group action. More precisely, I want to replace "locally compact manifold and locally compact group" with "infinite-dimensional manifold and loop group", keeping the action "properly and cocompactly". Although this project has not been completed, I have constructed several substitutes for several core objects for the analytic side:  $L^2$ -space, Dirac operator, group  $C^*$ -algebra, K-group valued analytic index, a part of assembly map and several  $C^*$ -algebras related to this map. In this talk, I will explain the progress so far.

Organized by Tsuyoshi Kato

Cosponsored by Daisuke Kishimoto