Polyvision: 4D Space Manipulation through Multiple Projections

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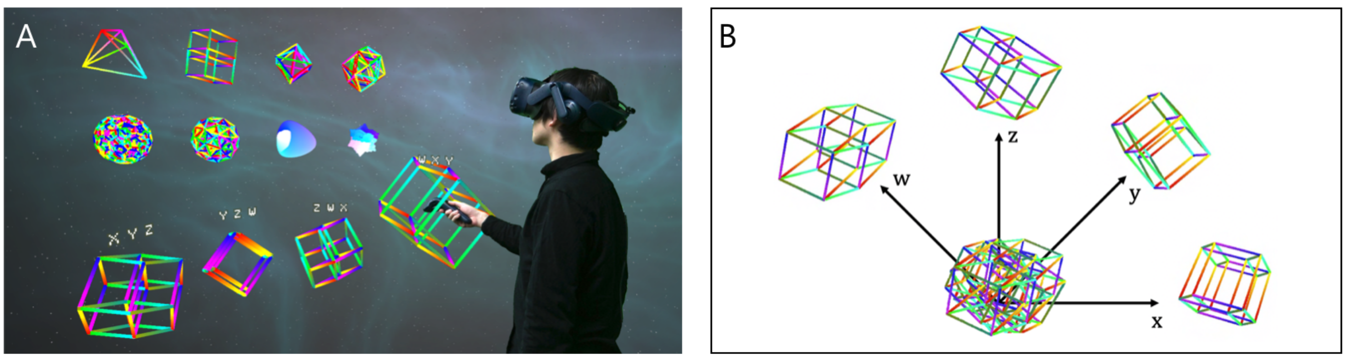

Figure 1: (A) Overview of *Polyvision*: Visualization and manipulation of four-dimensional (4D) objects. (B) Illustration of four projections of a 4D.

ABSTRACT

Seeing is believing. Our novel virtual reality system, *Polyvision*, applies this old saying to the fourth dimension. Various shadows of an object in a four-dimensional (4D) space are simultaneously projected onto multiple three-dimensional (3D) screens created in a virtual environment to reveal its intricate shape. The understanding of high-dimensional shapes and data can essentially be enhanced when good visualization is complemented by interactive functionality. However, a method to implement an interface for handling complex 4D transformations in a user-friendly manner must be developed. Using our Polyvision system, the user can manipulate each shadow as if it were a 3D object in their hand. The user's action on each projection is reflected to the original 4D object, and in turn its projections, in real time. While controlling the object's orientation minutely on one shadow, the user can grasp its global structure from multiple changing projections. Our system has a wide variety of applications in visualization, education, mathematical research, and entertainment, as we demonstrate with a variety of 4D objects that appear in mathematics and data sciences.

*Both authors contributed equally to this work and are listed in alphabetical order.

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CCS CONCEPTS

• **Human-centered computing** → **Scientific visualization**.

KEYWORDS

four-dimensional space, visualization, education, virtual reality

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1 INTRODUCTION

What does a four-dimensional (4D) space look like? 4D space has always been a focus of imagination and intellectual activities for human beings [Abbott 1884; Banchoff 1990; Jos Leys and Alvarez 2010]. Simultaneously, there are practical demands for understanding 4D spaces as they appear in mathematics and physics, such as manifolds and fractal sets, as well as in data analysis. This has led to numerous attempts to visualize 4D spaces ([Kageyama 2016] presents a good survey of the literature).

Herein, we present a novel virtual reality (VR) system, *Polyvision*, that allows the user to not only view, but also manipulate, objects in a 4D space (see Figure 1). The key idea is to utilize multiple projections of the 4D space onto a three-dimensional (3D) environment constructed in VR. Each projected 3D image can be manipulated using controllers, and the user's manipulation is amalgamated into a four-dimensional transformation in real time. Any 4D object provided in the standard PLY format, which is a format for storing graphical objects that are described as collections of polygons, can be imported into the system easily. The system also offers multi-user functionality, which is useful for collaboration and education.

To compare Polyvision with its predecessors, we briefly describe some of the recent attempts to interactively visualize objects in 4D space. *4Dice* [Hanson 2012] realizes 4D rotations of a hypercube through multi-touch controls on a 2D screen, *4D Maze* and *4D Draw* [Weeks 2016] utilize color gradation to represent the fourth dimension, and *4D Toys* [ten Bosch 2017] presents a single 3D slice of a 4D object interactively in a virtual environment. These systems employ either 3D slices or 2D projections to visualize 4D objects, and the user can change the view interactively. However, only a single slice or projection can be viewed at a time. The novelty of our system is the use of multiple 3D projections simultaneously. Different 3D projections substantially enrich the information that a user can gain regarding a 4D object while simultaneously making 4D object manipulation more accessible.

2 CONCEPT

When we examine the shape of a 3D object, we move it, rotate it, and zoom into it. Similarly, to investigate a 4D object we want to apply *similarity transformations*, which are combinations of translations, rotations, and scaling. Translations and scaling in 4D space are as simple to handle as in 3D space. However, 4D rotation is intrinsically more complex than its 3D counterpart, primarily owing to its high degree of freedom (six as opposed to three) and non-commutativity. In a 3D space, a rotation has three degrees of freedom; any rotation can be decomposed into three simple rotations around the x, y, and z axes, respectively. One way to see that a 4D rotation has six degrees of freedom is to decompose it into six simple rotations, preserving the xy-, xz-, xw-, yz-, yw-, and zw-planes (see, e.g., [Noll 1968]). It is difficult to control the six parameters simultaneously at will, and it is non-trivial to define a user interface to specify them.

The approach we adopt in Polyvision is to decompose a 4D space into a number of 3D spaces by using projections. Each 3D projected image can be manipulated using the controllers, as if it were a genuine 3D object. Each manipulation is integrated into a 4D one in real time. In this manner, we can interactively explore a 4D object. In theory, any two independent projections suffice to achieve full control of 4D transformations. However, we choose to have four projections along the x-, y-, z-, and w-axes onto the yzw-, zwx-, wxy-, and xyz-spaces, respectively (see Figure 2).

3 SYSTEM DESIGN

Polyvision consists of three components: 4D object data representation, multi-view visualization, and a user interface for 4D transformations. We use the standard PLY format for simple data exchange. 4D objects are represented as point clouds, specified by their $xyzw$ coordinates and RGB color. To render a 4D point cloud, we developed a special shader Pcx4D[1] based on its 3D counterpart Pcx [Takahashi 2017]. The shader works directly with 4D coordinates; 4D points are applied geometric transformations and projected to the 3D space before rendered as a point cloud in 3D. Rendering time is a small multiple of the usual 3D point cloud rendering. Several 4D objects can be imported into the system (Figure 1 (B)). Each 4D object is projected orthogonally onto four 3D objects (*shadows*); then, the shadows are mapped onto the 3D virtual environment side-by-side (Figure 1 (C)). To manipulate the 4D object, the user selects any of the four shadows to move, rotate, and scale by using the controller (Figure 1 (A)). The user observes that the selected object behaves in exactly the same manner as a 3D object. On the other hand, the other three shadows appear to move through complicated transformations. The user can switch to other shadows at any time. By iteratively applying 3D transformations to different shadows, the user can specify 4D transformations.

Multiple Polyvision systems can be connected through the Internet, and multiple users can share a single environment in which to work together. This feature would be useful for collaboration, education, and entertainment.

4 CONCLUSION AND FUTURE WORK

We have proposed a VR system for the interactive visualization of 4D objects. This has versatile potential applications, especially in visualizing abstract ideas and data, which should be further investigated through concrete examples. Its multi-user functionality can be exploited for a new style of collaboration, education, and entertainment. There are various approaches to visualizing higher-dimensional objects, which poses interesting questions concerning human perception. For example, we can ask which arrangement of multiple projections provides a better grasp of objects, or whether two people (hence with four eyes) can understand 4D objects better than a single person. In the future, we will conduct a user study to improve the interface of our system and understand high-dimensional perception.

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[1] https://github.com/romanesco/Pcx4D