Sinha's spectral sequence and model category of operads

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Sinha constructed a Bousfield-Kan type homology spectral sequence using the *d*-dimensional Kontsevich operad for each $d \geq 3$. This spectral sequence is closely related to the homology of long knots as follows. Let Emb_d be the space of long knots in \mathbb{R}^d , i.e., smooth embeddings $\mathbb{R} \to \mathbb{R}^d$ which coincide with a fixed embedding outside of a (fixed) compact subset, and Imm_d be the space of immersions $\mathbb{R} \to \mathbb{R}^d$ which coincide with a fixed embedding outside of a (fixed) compact subset. When $d \geq 4$, Sinha's spectral sequence converges to the homology of the homotopy fiber $\overline{Emb_d}$ of the inclusion $Emb_d \to Imm_d$.

Lambrechts, Turchin and Volic proved this spectral sequence collapses at E^2 -page if $d \ge 4$ and the coefficient is \mathbb{Q} . As the E^2 -page is isomorphic to the Hochschild cohomology of the Poisson operad, and there is a homotopy equivalence $\overline{Emb}_d \simeq Emb_d \times \Omega^2(S^{d-1})$, we get a good algebraic presentation of the homology of Emb_d by this collapse. The key of their proof is formality of the little disks operad proved by Tamarkin and Kontsevich.

In this talk, we simplify the arguments of Lambrechts-Turchin-Volic using model category of operads, and reprove the result of them. As byproducts, we obtain two new consequences. One is collapse of Sinha's spectral sequence in the case of d = 3 (and the coefficient being \mathbb{Q}). The other says there is no non-trivial extension for the Gerstenhaber algebra structure on the spectral sequence.