Mirror Symmetry and Related Topics, Kyoto 2023

Duration: 18-22 December 2023
Venue: Room 127 (Graduate School of Science Bldg no.3), Department of Mathematics, Kyoto University

The workshop website:
`https://www.math.kyoto-u.ac.jp/~iritani/mirrorsymmetry2023/Workshop2023.html`.

	9:00-10:00	10:15-11:15	11:30-12:30	14:00-15:00	15:30-16:30	17:00-18:00
Mon	Ikeda1	Ikeda2	Shamoto1	Shamoto2	Yamamoto	Tsutsui
Tue	Cho1	Wang	Coman	Inoue	Matsumoto	Ohta
Wed	Cho2	Ueda1	Tanaka	$\star$	$\star$	$\star$
Thu	Karabas	Ueda2	Morimura	$\star$	$\star$	$\star$
Fri	Kinjo	Liu	Hara	$\star$	$\star$	$\star$

$\star$: free discussion

Speakers and Titles:

Cheol-Hyun Cho	*Floer theory for the variation operator of an isolated singularity 1, 2*
Ioana Coman	*Mock 3-manifold invariants*
Wahei Hara	*Spherical objects in dimension two and three*
Akishi Ikeda	*Flat structures from generalized root systems of type A in genus zero*
Rei Inoue	*Cluster realization of Weyl group and its applications to representation theory*
Dogancan Karabas	*Wrapped Fukaya category of plumbings*
Tasuki Kinjo	*Mirror symmetry for the moduli space of Higgs bundles and cohomological Donaldson-Thomas theory*
Henry Liu	*The 3-fold K-theoretic DT/PT vertex correspondence holds*
Keiho Matsumoto	*On a smooth proper family of dg-categories and $\mathbb{Z}_p$-local system*
Hayato Morimura	*Homological mirror symmetry for complete intersections in algebraic tori*
Hiroshi Ohta	*Note on Stokes curves, spectral curves and holomorphic curves*
Yota Shamoto	*Mirror symmetry and Stokes structure of difference modules*
Hiro Lee Tanaka	*Toward a higher algebra of symplectic geometry*
Yuki Tsutsui	*Graded modules associated with permissible C^∞-divisors on tropical manifolds*
Kazushi Ueda	1: *Homological mirror symmetry for Milnor fibers of simple elliptic hypersurface singularities* 2: *Homological mirror symmetry for maximally degenerate Calabi-Yau manifolds*
Zhe Wang	*Linear reciprocal transformation of bihamiltonian integrable hierarchies*
Yuto Yamamoto	*The Gross–Siebert program and non-archimedean SYZ fibrations*

Organizers: Hiroshi Iritani, Yukiko Konishi, Atsushi Takahashi

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Cheol-Hyun Cho

Floer theory for the variation operator of an isolated singularity 1, 2

The variation operator in singularity theory maps relative homology cycles to compact cycles in the Milnor fiber using the monodromy. We construct its symplectic analogue for an isolated singularity. We define a new Floer cohomology, called monodromy Lagrangian Floer cohomology, which provides categorifications of the standard theorems on the variation operator and the Seifert form. The key ingredients are a special class Γ in the symplectic cohomology of the inverse of the monodromy and its closed-open images. For isolated plane curve singularities whose A'Campo divide has depth zero, we find an exceptional collection consisting of non-compact Lagrangians in the Milnor fiber corresponding to a distinguished collection of vanishing cycles under the variation operator. This is a joint work with Hanwool Bae, Dongwook Choa and Wonbo Jeong.

Ioana Coman

Mock 3-manifold invariants

A new family of topological 3-manifold invariants has been proposed recently, with the property that they are q-series with integrality properties that allow categorification. They have a mathematical definition based on the data which specifies the associated 3-manifold, though this is of limited applicability and restricted to cases which satisfy a certain negativity condition. Aside from their relevance in topology, these invariants have proven to be of broad interest through a web of relations. Physically, they capture the partition functions of certain 3-dimensional SQFTs, while from a number theory perspective they provide examples of holomorphic quantum modular forms. Here I will discuss an underlying hidden symmetry of these invariants and how considerations of modularity can be leveraged to predict what these should be for manifolds not covered by their original definition, as well as interesting wider implications of these results.

Wahei Hara

Spherical objects in dimension two and three

During this talk we discuss the classification problem of spherical objects in various geometric settings including the minimal resolution of Kleinian singularities and threefold flopping contractions with Gorenstein terminal singularities. Spherical objects in the derived category of coherent sheaves provide algebro-geometric substitute of Lagrangian submanifolds in symplectic geometry, and their classification is important from the viewpoints of autoequivalence groups and Bridgeland stability conditions. However algebraic geometry asks us to classify something more general than spherical objects. This talk will start from this background with some more details, and then the idea of the proof will be explained. If the homological mirror exists, the classification also proves results in symplectic geometry. This is all joint work with Michael Wemyss.

Akishi Ikeda

Flat structures from generalized root systems of type A in genus zero

In this talk, we construct flat structures from generalized root systems of type A coming from marked bordered surfaces of genus zero. We also see that where this construction is in the context of mirror symmetry. More explicitly, we explain that

(1) the derived category of a gentle algebra is a homological mirror symmetry counterpart of the construction,

(2) the B-model counterpart of the construction is Dubrovin's Hurwitz-Frobenius manifold.

This is a joint work with T. Otani, Y. Shiraishi and A. Takahashi.

Rei Inoue

Cluster realization of Weyl group and its applications to representation theory

We introduce a realization of Weyl groups in terms of cluster mutations, for finite dimensional semisimple Lie algebras. We briefly explain the idea of its applications to

(1) the higher Teichmuller theory introduced by Fock and Goncharov, and

(2) the q-characters of quantum non-twisted affine algebras introduced by Frenkel and Reshetikhin.

This talk is based on joint works with Thomas Lam, Pavlo Pylyavskyy, Tsukasa Ishibashi, Hironori Oya, and Takao Yamazaki.

Dogancan Karabas

Wrapped Fukaya category of plumbings

Let Q be any quiver whose vertices are equipped with cotangent bundles and whose arrows are valued in $\{+1, -1\}$. I will define a Weinstein manifold from this data, called the plumbing $P(Q)$ along Q. Then, I will describe a combinatorial way to express the wrapped Fukaya category of plumbings using Ganatra-Pardon-Shende and Karabas-Lee results on computing wrapped Fukaya categories in a local-to-global way. As an application, I will explicitly describe the wrapped Fukaya category of plumbings along any quiver equipped with cotangent bundles of oriented surfaces or n-spheres, and show that the latter is equivalent to Ginzburg dg categories with zero potential when n is greater than 2. This is a joint work with Sangjin Lee.

Tasuki Kinjo

Mirror symmetry for the moduli space of Higgs bundles and cohomological Donaldson-Thomas theory

The mirror symmetry for the moduli spaces of G-Higgs bundles on Riemann surfaces has close connection with the geometric Langlands conjecture. More precisely, the moduli space of G-Higgs bundles is expected to be the mirror dual of the moduli space for the Langlands dual of G, and the homological mirror symmetry for them can be thought of as the Dolbeault version of the geometric Langlands conjecture.

In this talk, we focus on the topological mirror symmetry (i.e., the symmetry of the Hodge number) for the moduli space of G-Higgs bundles. Since these moduli spaces are singular, the ordinary Hodge number does not satisfy the expected symmetry in general. We will propose a definition of the "correct" Hodge number for these moduli spaces using an idea from cohomological Donaldson-Thomas theory and explain some speculations and known results for it. In particular, we will see that the modified Hodge number has the symmetry expected from the mirror conjecture in the case of $G = GL_n$. This talk is partly based on joint work with Naoki Koseki.

Henry Liu

The 3-fold K-theoretic DT/PT vertex correspondence holds

On smooth quasi-projective toric 3-folds, vertices are the contributions from an affine toric chart to the enumerative invariants of Donaldson-Thomas (DT) or Pandharipande-Thomas (PT) moduli spaces. Unlike partition functions, vertices are fundamentally torus-equivariant objects, and they carry a great deal of combinatorial complexity, particularly in equivariant K-theory. In joint work with Nick Kuhn and Felix Thimm, we give two different proofs of the K-theoretic DT/PT vertex correspondence. Both proofs use equivariant wall-crossing in a setup originally due to Toda. A crucial new ingredient is the construction of *symmetrized* pullbacks of symmetric obstruction theories on Artin stacks, using Kiem-Savvas' étale-local notion of almost-perfect obstruction theory.

Slides available at `https://member.ipmu.jp/henry.liu/research/DT-PT-slides.pdf`

Keiho Matsumoto

On a smooth proper family of dg-categories and $\mathbb{Z}_p$-local system

For a smooth proper $\mathbb{C}$-algebra A and an A-linear smooth proper dg-category T, the periodic homology $HP(T/A)$ admits a flat connection, which is a non-commutative analogue of Gauss-Manin connection. Without the assumption of that T/A admits a geometrical realization, it is still an unsolved problem what the $\mathbb{C}$-local system corresponding to $HP(T/A)$ under the Riemann-Hilbert correspondence will be, and it is also unknown whether that $\mathbb{C}$-local system has a natural $\mathbb{Z}$-structure.

In my talk, I will first explain the p-adic analogue of the discussion above. Then, for a smooth algebra A over a p-adic field, and for an A-linear smooth proper dg category, I will construct a $\mathbb{Z}_p$ local system corresponding to $HP(T/A)$ under the p-adic Riemann-Hilbert correspondence using topological periodic homology.

Hayato Morimura

Homological mirror symmetry for complete intersections in algebraic tori

The Fukaya category of a symplectic manifold is a highly nontrivial invariant, mainly because of the global nature of pseudo-holomorphic disks. However, under some assumptions it is expected to show good local-to-global behavior. Namely, the Fukaya category there can be computed by gluing that of local pieces. Recently, Ganatra–Pardon–Shende established sectorial descent, the cosheaf property of partially wrapped Fukaya categories of stopped Weinstein sectors with respect to Weinstein sectorial covers. On the other hand, following the suggestion by Seidel, Lee and Pascaleff–Sibilla independently with different methods proved HMS for curves in algebraic 2-tori by inductively gluing wrapped Fukaya categories of pairs-of-pants which appear in their pants decompositions. In this talk, we present a new local-to-global principle for very affine hypersurfaces combining a sort of sectorial descent and an extension of the inductive argument by Pascaleff–Sibilla. This allows us to remove the assumption from HMS for very affine hypersurfaces proved by Gammage–Shende which gives a concrete global skeleton, key input for another local-to-global principle via microlocal sheaf theory. Moreover, our method is compatible with the combinatorics governing the geometric duality between Hori–Vafa mirrors. As a consequence, one can deduce HMS for complete intersections, both of whose Weinstein sectorial covers and global skeleta are in general unknown to compute, from the hypersurface case due to their local product structures of pairs-of-pants. This is joint work with Nicolo Sibilla and Peng Zhou.

Hiroshi Ohta

Note on Stokes curves, spectral curves and holomorphic curves

I will discuss about Stokes curves in exact WKB analysis and holomorphic curves in Floer theory. This is based on an ongoing joint work with Kohei Iwaki and Tatsuki Kuwagaki.

Yota Shamoto

Mirror symmetry and Stokes structure of difference modules

In mirror symmetry, we encounter some examples of linear difference equations. Motivated by such examples, we introduce the notion of Stokes filtered locally free sheaves of "solutions" of liner difference equations, or difference modules. In this talk, we first explain some easy examples related to mirror symmetry. Then we will explain a general result (and a conjecture) related to the Riemann-Hilbert problem.

Hiro Lee Tanaka

Toward a higher algebra of symplectic geometry

Any ring gives us a rich linear category: its category of modules. More generally, any scheme, any stack, and (tautologically) any non-commutative space does, too. The incredible surprise is that symplectic manifolds also give rise to rich linear categories through the Fukaya category construction. Even better, mirror symmetry should allow us to recast and study all the examples in the first two sentences through symplectic techniques. But we're not there yet. In this talk, I'll talk about recent progress in proving that the flimsy, geometric setting of symplectic geometry actually displays some of the structured and homotopically convenient properties that homotopy theorists and algebraists know and love. In a sentence: The ∞-category of Weinstein sectors is a localization of an ordinary category, and the ∞-category admits localizations that behave very much like inverting prime numbers (and more generally, nullifying finite stable homotopy types like Moore spectra). This is joint work with Oleg Lazarev and Zachary Sylvan.

Yuki Tsutsui

Graded modules associated with permissible C^∞-divisors on tropical manifolds

Tropical geometry is a field that studies the convex geometric analogies of algebraic varieties. It is a natural problem to investigate the tropical geometric counterpart of the Hirzebruch–Riemann–Roch theorem for tropical line bundles on compact tropical manifolds. Recently, thanks to the efforts of Medrano–Rincón–Shaw, the Todd class for any tropical manifold has been defined, making it possible to define the Riemann–Roch number $\mathrm{RR}(X; L)$ for a tropical line bundle L on a compact tropical manifold X. In this talk, I will discuss the conjecture that, if a tropical line bundle L on a compact tropical manifold is represented by a permissible C^∞-divisor s, then the Euler characteristic of the graded module defined by s coincides with $\mathrm{RR}(X; L)$. I will also discuss examples where this conjecture is indeed true.

Slides available at `https://www.math.kyoto-u.ac.jp/~iritani/mirrorsymmetry2023/Tsutsui_2023-12.pdf`

Kazushi Ueda

Talk 1: *Homological mirror symmetry for Milnor fibers of simple elliptic hypersurface singularities*

We discuss homological mirror symmetry for complements of smooth anticanonical hypersurfaces in projective spaces. The case of the projective plane is a triple cover of the Milnor fiber of simple elliptic singularity of type E_6.

Slides available at `https://researchmap.jp/uedakazushi/presentations/44206837`

Talk 2: *Homological mirror symmetry for maximally degenerate Calabi-Yau manifolds*

We discuss homological mirror symmetry between complements of ample divisors in Calabi-Yau manifolds and maximal degenerations of their mirrors.

Slides available at `https://researchmap.jp/uedakazushi/presentations/44219474`

Zhe Wang

Linear reciprocal transformation of bihamiltonian integrable hierarchies

Reciprocal transformations transform an evolutionary PDE into another one by performing a certain type of transformation of independent variables. Those transformations date back to the study of gas dynamics and play important roles in the theory of integrable hierarchies. In this talk, we will introduce the recent results on the study of linear reciprocal transformation of

bihamiltonian integrable hierarchies. In the semisimple setting, we give a full set of invariants characterizing equivalent classes of integrable PDEs under linear reciprocal transformations. We will give examples from the Gromov-Witten theory and talk about how the linear reciprocal transformations can be applied to study the relation between the 2D topological field theory and the integrable hierarchies. The talk is based on a joint work with Si-Qi Liu and Youjin Zhang.

Yuto Yamamoto

The Gross–Siebert program and non-archimedean SYZ fibrations

For a maximally degenerate Calabi–Yau variety, the Berkovich retraction associated with a (good) minimal dlt model is regarded as an SYZ fibration in non-archimedean geometry. In general, the integral affine structure induced on the base space of the fibration differs from the one defined for the dual intersection complex of a toric degeneration in the Gross–Siebert program. In this talk, using tropical geometry, we construct non-archimedean SYZ fibrations whose bases are integral affine manifolds appearing in the Gross–Siebert program for Calabi–Yau complete intersections of Batyrev–Borisov.