Mirror Symmetry and Related Topics, Kyoto 2022

Duration: 12-16 December 2022
Venue: Room 127 (Graduate School of Science Bldg no.3), Department of Mathematics, Kyoto University
The workshop website is here.

	9:00-10:00	10:30-11:30	13:30-14:30	15:00-16:00	16:30-17:30
Mon	Iwaki	Osuga	Manabe	Zhou	Yoshida
Tue	Kuwagaki	Saito	Futaki	Kikuta	Satake
Wed	Toda	Hosono			
Thu	Milanov	Song	Hirano	Ouchi	
	9:00-10:00	10:15-11:15	11:30-12:30		
Fri	Tanaka	Ohkawa	Arbesfeld		

Title:

Noah Arbesfeld	*Nested Hilbert schemes and instanton moduli spaces*
Masahiro Futaki	*Equivariant Homological Mirror Symmetry for* $\mathbb{CP}^1$
Yuki Hirano	*Mutations of noncommutative crepant resolutions in GIT*
Shinobu Hosono	*Mirror symmetry of abelian fibered Calabi-Yau manifolds with* $\rho = 2$
Kohei Iwaki	*Topological recursion, BPS structure and Painlevé* τ*-function*
Kohei Kikuta	*Fixed points on the spaces of stability conditions and Thurston compactifications*
Tatsuki Kuwagaki	*Sheaf-theoretic bounding cochain and bulk deformation*
Masahide Manabe	*Recursion Relations for Volumes of Moduli Spaces of Bordered Riemann Surfaces, and Virasoro Constraints*
Todor Milanov	*Reflection vectors and quantum cohomology of blowups*
Ryo Ohkawa	*Wall-crossing for vortex partition function and handsaw quiver variety*
Kento Osuga	*Refined Topological Recursion and the Nekrasov-Shatashivili limit*
Genki Ouchi	*Title: Cubic fourfolds and K3 surfaces with large automorphism groups*
Takahiro Saito	*A description of monodromic mixed Hodge modules*
Ikuo Satake	*Good basic invariants for elliptic Weyl groups and Frobenius structures*
Juae Song	*Rational function semifields of tropical curves*
Yuuji Tanaka	*On a blowup formula for sheaf-theoretic virtual enumerative invariants on projective surfaces*
Yukinobu Toda	*Categorical DT/PT correspondence for local surfaces*
Yutaka Yoshida	*3d indices and quantum K-theory*
Zijun Zhou	*3d mirror symmetry and duality interface*

Organizers: Hiroshi Iritani, Yukiko Konishi, Atsushi Takahashi

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Noah Arbesfeld

Nested Hilbert schemes and instanton moduli spaces

Moduli of Higgs pairs on algebraic curves are of central importance in mirror symmetry and mathematical physics. Recently, moduli of Higgs pairs on algebraic surfaces have also emerged as a subject of active research. Tanaka-Thomas use moduli spaces of stable Higgs pairs on surfaces to formulate an algebro-geometric definition of Vafa-Witten invariants. The invariants are defined by integration over a virtual fundamental class and are formed from contributions of components; the physical notion of S-duality implies conjectural relationships between these contributions.

One component, the so-called "vertical component", can be realized as a nested Hilbert scheme of curves and points on a surface. I will explain work in progress with M. Kool and T. Laarakker, in which we express refined invariants of this vertical component in terms of invariants of a quiver variety, the instanton moduli space of torsion-free framed sheaves on $\mathbb{CP}^2$. We then use a recent blow-up formula of Kuhn-Leigh-Tanaka to obtain formulas conjectured by Göttsche-Kool-Laarakker. One consequence is a formula for vertical refined Vafa-Witten invariants in rank 2, verifying a conjecture of Göttsche-Kool.

Masahiro Futaki

Equivariant Homological Mirror Symmetry for $\mathbb{CP}^1$

The mirror of an n-dimensional toric Fano manifold X is known to be $(\mathbb{C}^*)^n$ equipped with a Laurent polynomial f and is called the Landau-Ginzburg model. The homological mirror symmetry for toric Fano manifold says that the Fukaya category of X is equivalent to the category of matrix factorizations of f (cf. Cho, Hong and Lau 2019).

Givental introduced the equivariant Landau-Ginzburg mirror F by adding logarithmic terms to f.

In this talk we formulate and show a version of equivariant homological mirror symmetry for $\mathbb{CP}^1$ by introducing equivariant Floer A_∞ algebra for toric fibers.

This is a joint work with Fumihiko Sanda (Gakushuin University) and is based on our preprint arXiv:2112.1462.

Yuki Hirano

Mutations of noncommutative crepant resolutions in GIT

For a generic quasi-symmetric representation X of an algebraic torus T, Halpern-Leistner and Sam introduced a periodic locally finite hyperplane arrangement, and they show that certain triangulated subcategories of the

derived category of quotient stack $[X/T]$, which are called magic windows and labeled by each point in the complement of the complexification of the hyperplane arrangement, are all equivalent. In this talk, we explain that the equivalence of magic windows of adjacent points is induced by a tilting module that is obtained by iterated Iyama–Wemyss mutations of modules giving noncommutative crepant resolutions of the affine quotient X/T. This is (ongoing) joint work with Wahei Hara.

Shinobu Hosono

Mirror symmetry of abelian fibered Calabi-Yau manifolds with $\rho = 2$

I will describe mirror symmetry of Calabi-Yau manifolds fibered by $(1, 8)$-polarized abelian surfaces, which have Picard number two. Finding a mirror family over a toric variety explicitly, I observe that mirror symmetry of all related Calabi-Yau manifods arises from the corresponding boundary points, which are not necessarily toric boundary points. Calculating Gromov-Witten invariants up to genus 2, I find that the generating functions are expressed elliptic (quasi-)modular forms, which reminds us the modular anomaly equation found for elliptic surfaces. This talk is based on a published work with Hiromichi Takaki (arXiv:2103.08150).

Kohei Iwaki

Topological recursion, BPS structure and Painlevé τ-function

The topological recursion is a recursive algorithm which computes a family of differential forms associated to a given spectral curve. If the spectral curve is the classical limit of the Gauss hypergeometric equation (or its confluent ones), then the free energy can be written as a weighted sum over the BPS invariants which can be computed through the spectral networks (joint work with O. Kidwai). On the other hand, if the spectral curve is a family of elliptic curves, then the free energy is closely related to the Painlevé τ-function. I'll give a review of these two results, and if time permits, I'll show an observation which implies that the BPS invariants (partially) appear in the Painlevé τ-function.

Kohei Kikuta

Fixed points on the spaces of stability conditions and Thurston compactifications

To study a given group, it is useful to consider its actions on metric spaces of non-positive curvature. In this talk, we consider the actions of autoequivalence groups of K3 surfaces on the spaces of stability conditions, especially focus on fixed points of autoequivalences. Via homological mirror symmetry,

one can see an analogy between spaces of stability conditions and Teichmüller spaces of real surfaces. We also explain Thurston compactifications of the spaces of stability conditions on curves (as toy models for K3 surfaces) in a recent joint work with N. Koseki and G. Ouchi.

Tatsuki Kuwagaki

Sheaf-theoretic bounding cochain and bulk deformation

In Lagrangian intersection Floer theory over the Novikov ring, Fukaya-Oh-Ohta-Ono introduced two kinds of deformations: bounding cochain and bulk deformation. In the study of sheafy symplectic topology over the Novikov ring, we recently found conjectural counterparts of such deformations. In this talk, we explain this and some necessary backgrounds. This talk will be partly based on a work joint with Yuichi Ike.

Masahide Manabe

Recursion Relations for Volumes of Moduli Spaces of Bordered Riemann Surfaces, and Virasoro Constraints

The volumes of moduli spaces of Riemann surfaces, including the Kontsevich-Witten symplectic volumes and the Weil-Petersson symplectic volumes, enjoy three equivalent characteristic relations - the geometric recursion by Andersen-Borot-Orantin, the topological recursion by Chekhov-Eynard-Orantin and the Virasoro constraints. These characteristic relations admit "twists" which provide statistics of lengths of multicurves - closed geodesics on Riemann surfaces. For example, a twist of the Kontsevich-Witten symplectic volumes gives the so-called Masur-Veech volumes of the moduli space of meromorphic quadratic differentials on curves. In this talk, I will present the above mentioned three characteristic relations as well as their twists, and apply them to the $(2, p)$-minimal gravity, labelled by an positive odd integer p, which interpolates the Kontsevich-Witten symplectic volumes for $p = 1$ and the Weil-Petersson symplectic volumes for $p = \infty$. This talk is based on a work in progress with Hiroyuki Fuji.

Todor Milanov

Reflection vectors and quantum cohomology of blowups

Let X be a smooth projective variety with a semi-simple quantum cohomology. It is known that the blowup $Bl(X)$ of X at one point also has semi-simple quantum cohomology. In particular, the monodromy group of the quantum cohomology of $Bl(X)$ is a reflection group. We found explicit formulas for certain generators of the monodromy group of the quantum cohomology of $Bl(X)$ depending only on the geometry of the exceptional divisor.

Ryo Ohkawa

Wall-crossing for vortex partition function and handsaw quiver variety

We investigate partition functions defined from integrals over the handsaw quiver varieties of type A_1 via wall-crossing phenomena. We consider vortex partition functions defined by two types of cohomology classes, and get functional equations for each of them. This gives proofs to formula suggested by physical computations. In particular, we obtain geometric interpretation of formulas for multiple hypergeometric functions including rational limit of the Kajihara transformations formula.

Kento Osuga

Refined Topological Recursion and the Nekrasov-Shatashivili limit

Topological recursion has become known as a power tool to compute a variety of invariants, and it has been applied to many aspects in mirror symmetry. However, a refinement of topological recursion is a long-standing problem, and its mathematical formulation is yet to be established. In this talk, I will propose the definition of the hyperelliptic refined topological recursion, and present its conjectural properties. I will also discuss a couple of possible interesting applications of the hyperelliptic refined topological recursion, in particular in relation to the so-called Nekrasov-Shatashivili limit. This work is partly joint with Omar Kidwai.

Genki Ouchi

Cubic fourfolds and K3 surfaces with large automorphism groups

Relations between cubic fourfolds and K3 surfaces are described by Hodge theory and derived categories. Using Hodge theory and derived categories, we can show that cubic fourfolds and associated K3 surfaces share their symmetries, which are related with Mathieu groups and Conway groups. In this talk, we find pairs of a cubic fourfold and a K3 surface sharing large symplectic automorphism groups via Bridgeland stability conditions on K3 surfaces.

Takahiro Saito

A description of monodromic mixed Hodge modules

The irregular Hodge filtration is expected to be a "Hodge filtration" of a twisted de Rham cohomology of a holomorphic function that appears as a mirror of a Fano manifold. In general, the irregular Hodge filtrations are complicated and it is difficult to deal with them. The twisted de Rham cohomology can be expressed in terms of the Fourier-Laplace transform of a certain regular D-module. One reason that makes the irregular Hodge

filtration difficult is that the Fourier-Laplace transform is irregular in general. I try to make everything clear in the simple case: "monodromic".

For a complex manifold X, regular monodromic D-modules on $X \times \mathbb{C}$ have an important property that "their Fourier-Laplace transforms are regular again". For a mixed Hodge module whose underlying D-module is monodromic (which is called a monodromic mixed Hodge module), it is natural to expect that its Hodge filtration and the irregular Hodge filtration of its Fourier-Laplace transform are "easy". In fact, I clarified the structure of the Hodge filtrations of monodromic mixed Hodge modules, which can be seen as a "Hodge version" of a famous description of perverse sheaves on a complex plane. As an application of this, for a monodromic mixed Hodge module, I described the irregular Hodge filtration of the Fourier-Laplace transform concretely, and endowed the Fourier-Laplace transform with the natural mixed Hodge module structure. I think my result is a good step toward understanding general (irregular) Hodge filtrations.

In this talk, in the first half, I will introduce the theory of mixed Hodge modules. In the second half, I will explain my results on the description of monodromic mixed Hodge modules.

Ikuo Satake

Good basic invariants for elliptic Weyl groups and Frobenius structures

Let G be a finite reflection group which acts on the complex vector space V. Let $\mathbb{C}[V]$ and $\mathbb{C}[V]^G$ be the coordinate rings of V and V/G respectively. By the aid of the Coxeter transformation, we give a graded algebra structure on $\mathbb{C}[V]$ which is the same grading as $\mathbb{C}[V]^G$.

The projection $\pi\colon V \to V/G$ is not an isomorphism but π gives a local isomorphism at $q \in V$, where q is an eigenvector of the Coxeter transformation (because q is outside of the reflection hyperplanes).

This local isomorphism could be described by a filtered algebra morphism $\varphi\colon \mathbb{C}[V]^G \to \mathbb{C}[V]$. The induced morphism $\mathrm{gr}(\varphi)\colon \mathrm{gr}(\mathbb{C}[V]^G) \to \mathrm{gr}(\mathbb{C}[V])$ for the associated graded algebras $\mathrm{gr}(\mathbb{C}[V]^G)$ and $\mathrm{gr}(\mathbb{C}[V])$ gives a graded algebra isomorphism $\psi\colon \mathbb{C}[V]^G \to \mathbb{C}[V]$ and we see that ψ gives a flat structure (the flat coordinates and the flat metric of the Frobenius structure) on V/G.

In this talk, we explain the above (finite reflection group) cases and the elliptic Weyl group cases.

Juae Song

Rational function semifields of tropical curves

Tropical curves are the images of algebraic curves by a limit operation called tropicalization. In this talk, we deal with the more general concept "abstract tropical curves" and consider the following question: "in the classical

algebraic geometry, over an algebraically closed field, the category of nonsingular projective curves and dominant morphisms between them is equivalent to the category of function fields of dimension one and homomorphisms between them. Is there a tropical analogue of this fact?" I explain my progress on this question.

Yuuji Tanaka

On a blowup formula for sheaf-theoretic virtual enumerative invariants on projective surfaces

I'll talk about a blowup formula for sheaf-theoretic virtual enumerative invariants on projective surfaces, which include the Donaldson-Mochizuki invariant (a virtual analogue of the Donaldson invariant), the virtual Euler characteristic or virtual χ_y genus of the moduli space of semistable sheaves on a project surface (they are the instanton parts of the ordinary or K-theoretic Vafa-Witten invariant, respectively), and the virtual Verlinde number and virtual Segre one of the moduli space (the former is a generalisation of the K-theoretic Donaldson invariant and the latter is a generalisation of the Donaldson invariant in the sense adding fundamental matters).

Our blowup formula has exciting applications for these virtual enumerative invariants. For example, we confirm the equivalence of the classical Donaldson invariants and Mochizuki's virtual ones in our setting (namely, for the rank two and the absence of strictly semistable sheaves case). We also prove Göttsche-Kool's conjectural blowup formulae for the generating series of the virtual Euler characteristics and virtual χ_y genera of the moduli spaces, in which modular forms appear in the same way as in Vafa-Witten's original paper in '94.

We prove the blowup formula by considering Nakajima-Yoshioka's m-stable sheaves on a projective surface, which nicely interpolates the moduli spaces of (semi)stable sheaves on the blowup and the original one, but with perfect obstruction theories. We then utilise Mochizuki's enhanced moduli space technique to handle automorphisms on the moduli stacks in a coherent way and apply Kiem-Li's version of master space construction to get around an issue of the absence of GIT descriptions of the moduli spaces in our setting. Interestingly, our blowup formula turns out to be universal, so, e.g. we obtain blowup formulae for the generating series of the virtual Euler characteristics and the virtual χ_y genera of the moduli spaces by studying them in the case for framed sheaves on the projective plane.

This talk is based on joint work arXiv:2107.08155 with Nikolas Kuhn and arXiv:2205.12953 with Nikolas Kuhn and Oliver Leigh.

Yukinobu Toda

Categorical DT/PT correspondence for local surfaces

The DT/PT correspondence is a formula which relates Donaldson-Thomas invariants counting curves in Calabi-Yau 3-folds and Pandharipande-Thomas invariants counting stable pairs on them. In the case of local surfaces, i.e. the total spaces of canonical line bundles on surfaces, I introduced dg-categories which categorify these invariants, called DT/PT categories, based on Koszul duality and singular support quotients. In this talk, I give semiorthogonal decompositions of DT-categories for reduced curve classes which categorify the DT/PT correspondence in this case. The notion of 'quasi-BPS categories' and categorical Hall products play important roles. This is a joint work with Tudor Padurariu.

Yutaka Yoshida

3d indices and quantum K-theory

The genus g topologically twisted index, refined twisted index, and hemisphere index of $3d$ $\mathcal{N}=2$ supersymmetric Chern-Simons-matter theories are thought as trigonometric deformations of $2d$ $\mathcal{N}=(2,2)$ partition functions describing the Kahler potential, genus g Vafa-Intriligator formula, genus zero quasi-map invariants and D-brane central charges. In collaboration with K. Ueda, we have studied the correspondence between $3d$ indices and the quantum K-theory for a Grassmannian $\mathrm{Gr}(N,M)$. We will generalize our work to other manifolds. If we have time, we will also explain the relation between the level correspondence of K-theoretic I-functions by Wen-Dong, $3d$ Seiberg-like dualities, and wall-crossing phenomena of vortex partition functions (Witten indices of handsaw quivers) by Hwang-Yi-YY.

Zijun Zhou

$3d$ *mirror symmetry and duality interface*

Elliptic stable envelope is introduced by Aganagic-Okounkov as a generalization of stable envelopes in equivariant elliptic cohomology. One of their important properties is that they satisfy the $3d$ mirror symmetry - elliptic stable envelopes of a $3d$ mirror pair are transpose to each other. In this talk, I will introduce the idea of a new proof for such 3d mirror symmetry. This is based on joint work in progress with H. Nakajima and A. Okounkov.