§ Mirror for $(\mathbb{P}^2, \triangle)$

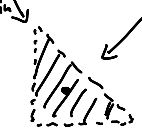
$D \subseteq \mathbb{P}^2$ toric boundary $\quad D = \triangle$

$\mathbb{P}^2 \setminus D \qquad\qquad U \subseteq (\mathbb{C}^*)^2 \xrightarrow{W} \mathbb{C}$

Lagrangian moment map

log|·|

T-dual fibration



W is the generating fct of Maslov 2 holomorphic disc in $\mathbb{P}^2$ w boundary on a moment map fibre

$U = \{ \mathcal{L} = (L, \nabla) \}$

$$W(\mathcal{L}) = \sum_{\substack{\beta \in \pi_2(\mathbb{P}^2, L) \\ \mu(\beta) = 2}} n_\beta(\mathcal{L}) \cdot z^\beta(\mathcal{L})$$

FOOO, A

$$z^\beta(\mathcal{L}) = e^{-\int_\beta \omega} \cdot \mathrm{hol}_{\partial\beta}(\nabla) \in \mathbb{C}^*$$

For $\mathbb{P}^2$ we get

$$W = x + y + \frac{t^3}{xy}$$

where x, y are coord. of $(\mathbb{C}^*)^2$

t = complex modulus of (U, W)

= complexified Kähler param. of $\mathbb{P}^2$

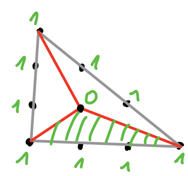
§ Mirror for $(\mathbb{P}^2, E)$

E is smooth elliptic curve

Q: How to produce the mirror dual, how to produce a torus fibration?

A: Use a toric degeneration of $(\mathbb{P}^2, E)$

This works by subdividing the moment polytope of $\mathbb{P}^2$



central subdivision $\xrightarrow{\check{\varphi}} \mathbb{R}$

$(\check{B}, \check{\mathcal{P}}, \check{\varphi})$

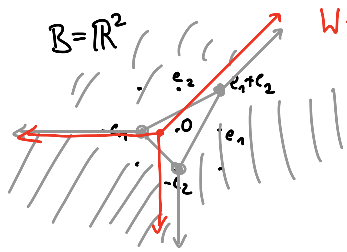
pa function

collection of polytopes of subdivision

polytope

discrete Legendre transform $(B, \mathcal{P}, \varphi)$

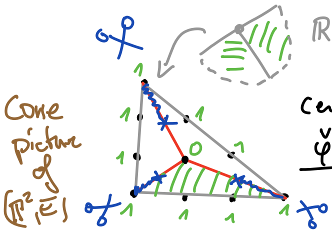
$B = \mathbb{R}^2$



$W = x + y + \frac{t^3}{xy}$

$\mathbb{P}^2 \rightsquigarrow X_0 \xrightarrow{\text{moment map}} \Delta$

§ Degeneration of $(\mathbb{P}^2, E)$



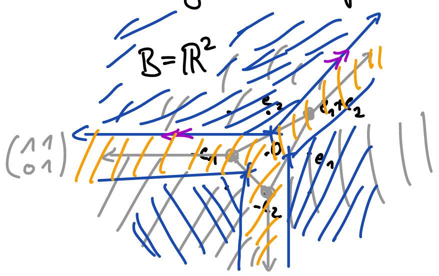
central subdivision $\check{\varphi} \to \mathbb{R}$

$(\check{B}, \check{\mathcal{S}}, \check{\varphi})$ ← pa function

← collection of polytopes of subdivision

← polytope

discrete Legendre transform $(B, \mathcal{S}, \varphi)$



fan picture of $(\mathbb{P}^2, E)$

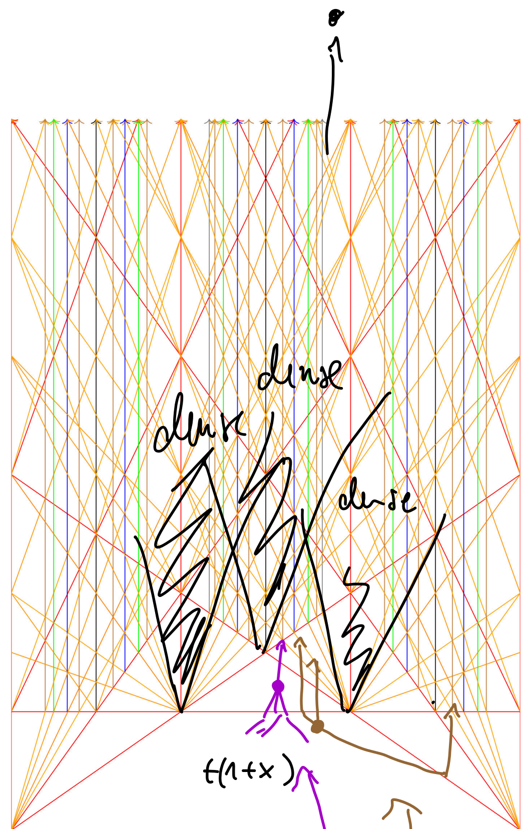
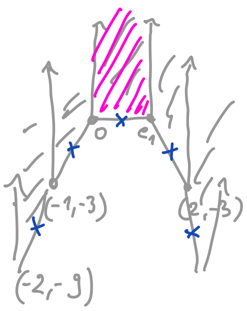
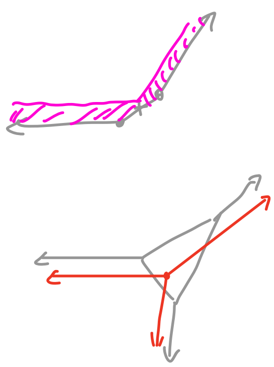


FIGURE 1. The wall structure of $(\mathbb{P}^2, E)$ consistent to order 6.





$$W = x + y + \frac{t^3}{xy}$$

1. change of coords $x \mapsto tx$, $y \mapsto ty$
2. change of coordinates by $x \mapsto x^{-1}y$, $y \mapsto y$ yields

$$W = t\left(y + x^{-1} + \frac{x}{y^2}\right)$$

crossing first wall yields

$$W = y + \frac{t^3}{y^2}\left(x^{-1} + 2 + x\right)$$

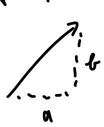
mutation:

$\mathbb{C}[x^{\pm 1}, y^{\pm 1}][\![t]\!]$

$x \mapsto x f^{-b}$

$y \mapsto y f^{a}$

for $f = 1 + c x^a y^b$



Thm: Gräfnitz, R, Zaslow

base point near E:

$$y^{-1} W = 1 + 2Q + 5Q^2 + 32Q^3 + 286Q^4 + \ldots$$

$$Q = (t/y)^3$$