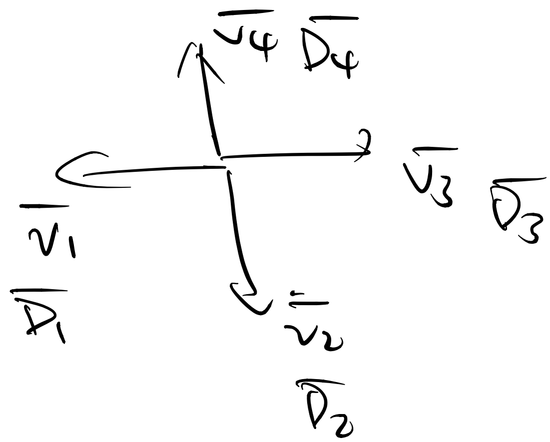
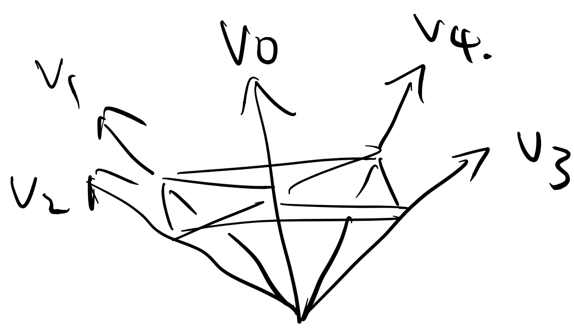
① $X_{\overline{\sigma}} = \mathbb{P}^1 \times \mathbb{P}^1.$

$\overline{\sigma}$:



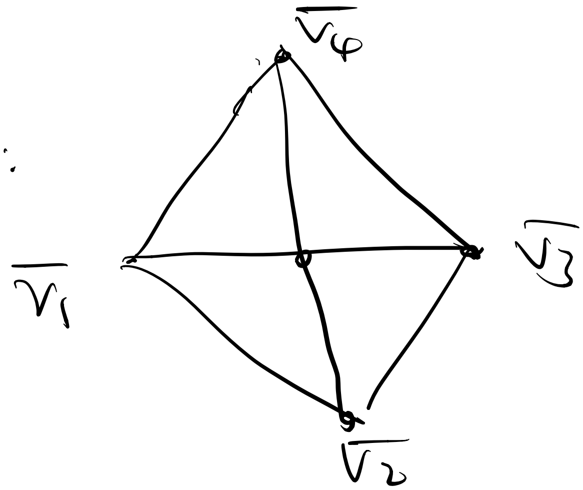
$K_{X_{\overline{\sigma}}} = X_{\Sigma}$

Σ :



$\{D_1, D_2\}$ basis of $\mathrm{Pic}(K_{\mathbb{P}^1 \times \mathbb{P}^1})$

Δ:



B:

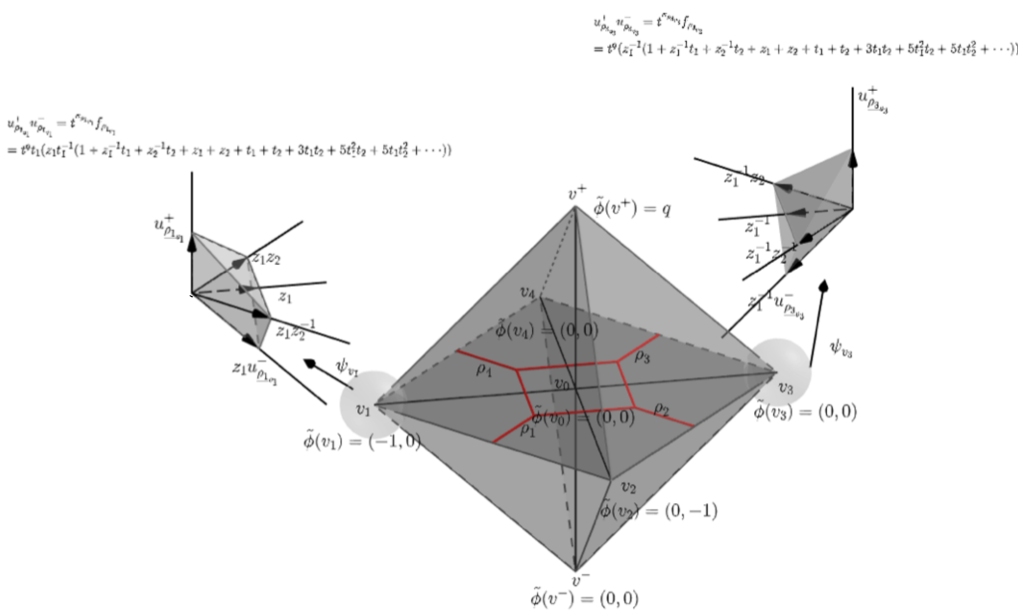
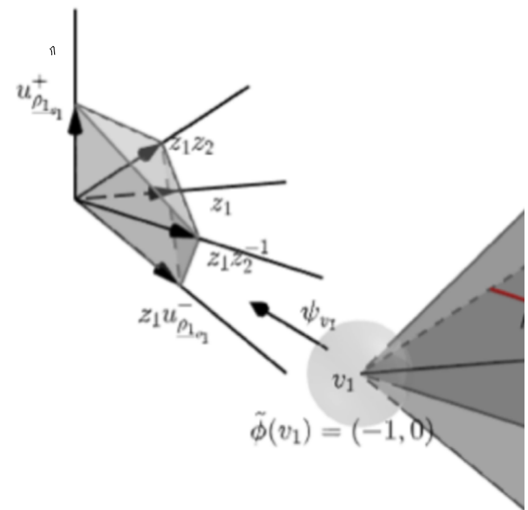


Figure 5: The mirror of $K_{\mathbb{P}^1\times\mathbb{P}^1}$

(U_{v_1}, ψ_{v_1}) :



$$f_{\underline{\rho}_{v_0}} = \underbrace{1 + z_1^{-1} t_1 + z_2^{-1} t_2 + z_1 z_2}_{\text{Classical local mirror symmetry}} + \underbrace{t_1 + t_2 + 3 t_1 t_2 + \cdots}_{\text{Extra terms to satisfy the normalization condition}}$$

Lemma 1:

$$ch(i_* \mathcal{O}_C) = [D]\cdot[D_0] + \frac{1}{2}[D_0]\cdot[-D_0-D]\cdot[D]$$

Lemma 2:

$$[D_0]\cdot[D]\cdot[D_j] = \ell(F_j)$$

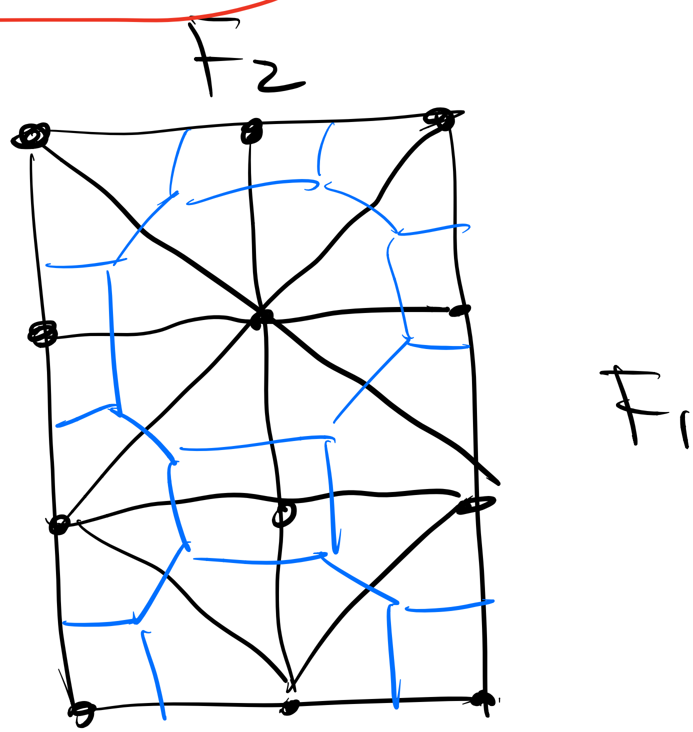
$$\int_{K_{X_{\bar\sigma}}} \underbrace{t^{-\vec{D}}}_{\prod_{j=1}^{k} t_j^{[D_j]}} \cdot \underbrace{\hat{\Gamma}_{K_{X_{\bar\sigma}}}}_{\exp\left(-\gamma \underbrace{c_1(K_{X_{\bar\sigma}})}_{=0} + \sum_{k=2}^{\infty}(-1)^k \zeta(k)(k-1)!\, ch_k(TK_{X_{\bar\sigma}})\right)} \underbrace{(2\pi i)^{\deg/2} ch(i_* \mathcal{O}_C)}$$

$C \subseteq \mathbb{P}^1 \times \mathbb{P}^1$

$C \sim 2\overline{D_1} + 3\overline{D_2}$

$$I_t(i_* \mathcal{O}_C) = (2\pi i)^2 (3\log t_1 + 2\log t_2) + (2\pi i)^3 (-1)$$

D_C:



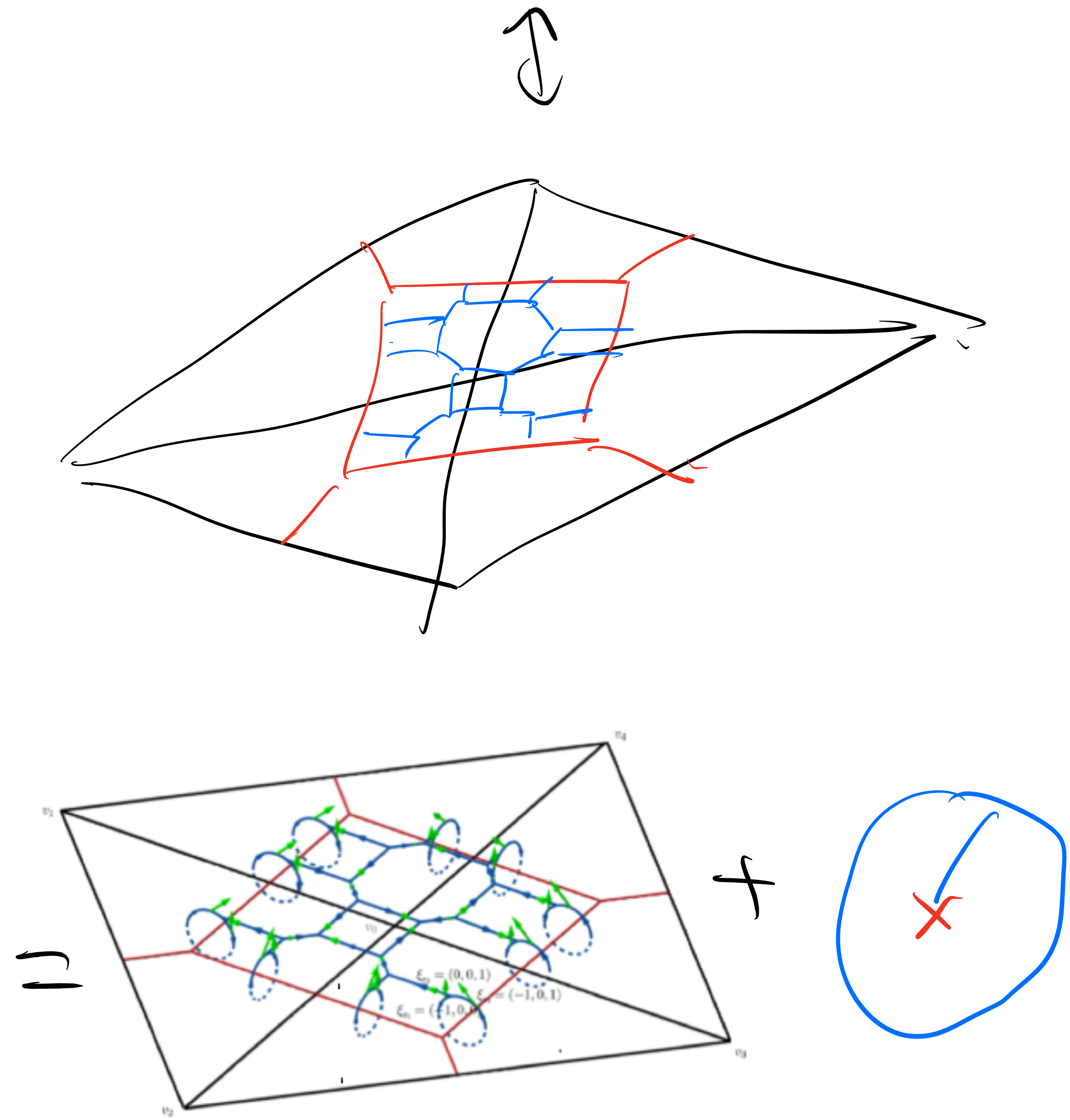
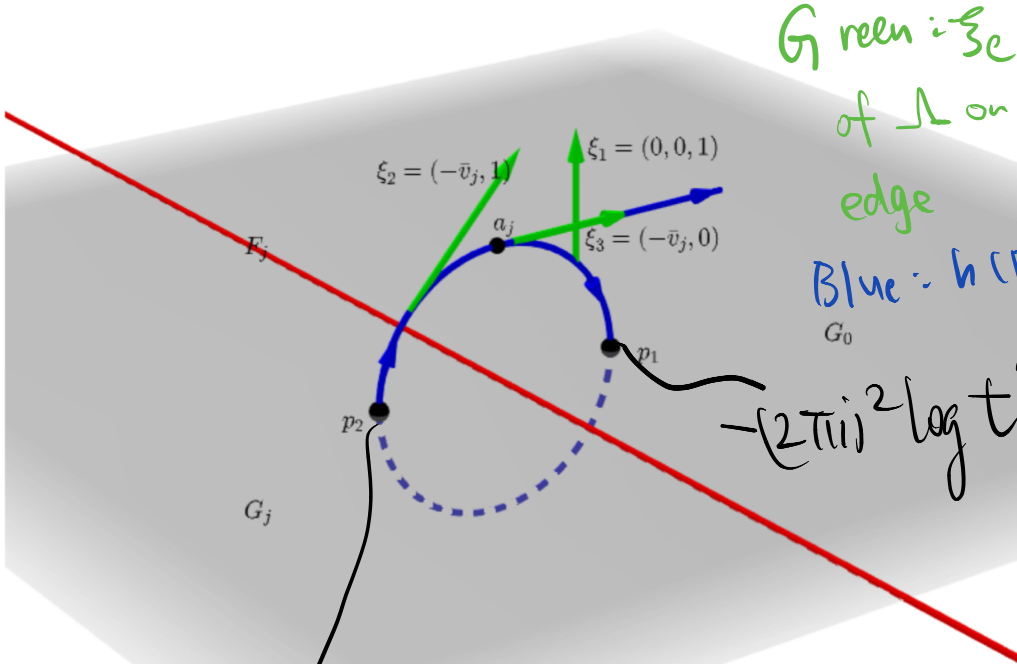


Figure 6: A tropical 1-cycle in the Gross-Siebert model of the mirror of $K_{\mathbb{P}^1\times\mathbb{P}^1}$

Green: ξ_e, section of Λ on the edge

Blue: $h(\Gamma)$



$-(2\pi i)^2 \log t^q$

$(2\pi i)^2 \log(t^{K_v}) = (2\pi i)^2(\log t^q - \log t^{\tilde{\phi}(v)})$

$$\int_{\beta_t} \Omega_t = (2\pi i)^2 (3\log t_1 + 2\log t_2) + (2\pi i)^3 \left(\frac{1}{2}(10) - \frac{1}{2}(121)\right)$$

$$\mu_6 : \mathrm{Spec}(\mathbb{C}[\Lambda_6]) \cong (\mathbb{C}^\times)^3$$

$$\downarrow$$

$$\mathrm{Int}(6) \quad T^3 = \mathrm{Hom}(\Lambda_6, S^1)$$

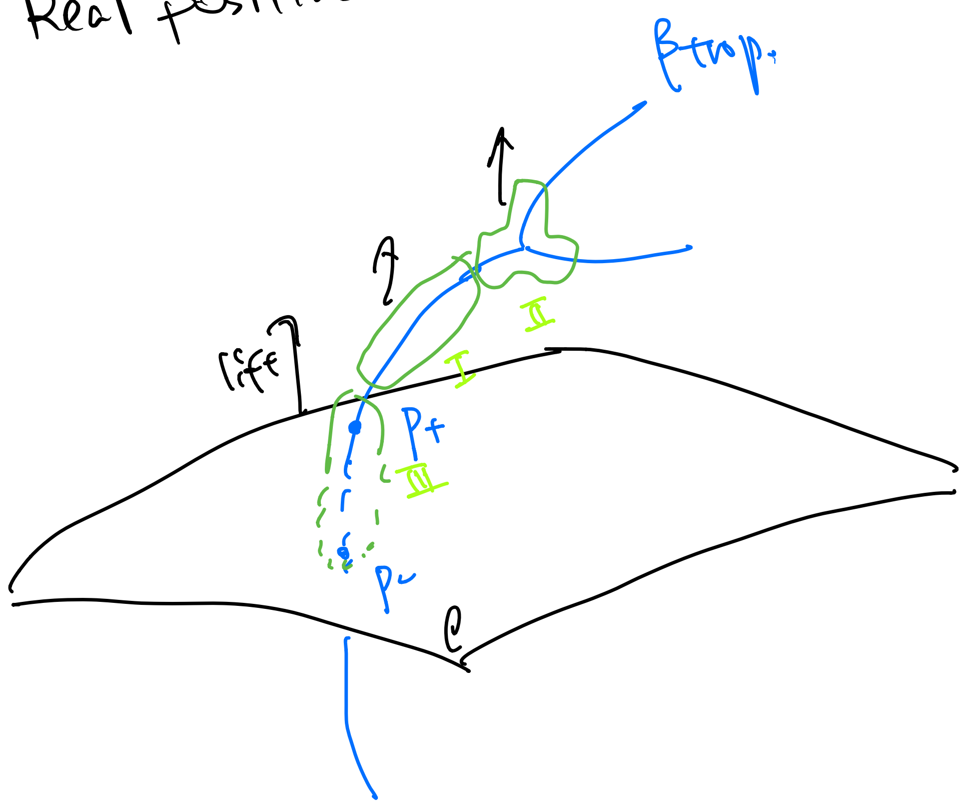
$$\cap$$

$$\mathrm{Spec}(\mathbb{C}[\Lambda_6]) \cong \mathrm{Hom}(\Lambda_6, \mathbb{C}^\times)$$

$$S_6 : \mathrm{Int}(6) \to (\mathbb{C}^\times)^3 \cap (\mathbb{R}_{>0})^3$$

$\uparrow$

Real positive section.



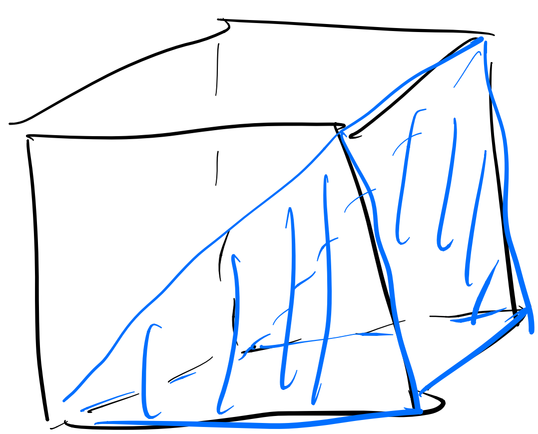
Type I:

$$\mathbb{T}_e := \{ \theta \mid \theta(\xi_e) = 1 \} \cong \mathbb{T}^2 \subseteq \mathbb{T}^3$$

$$\mathbb{T}_{e,6} = \mathbb{T}_e \cdot S_6(\beta_{e,6}) \times \{t\}$$

Type II:

$$\left.\begin{array}{l} T_{e_1} \cdot S_6(v) \\ T_{e_2} \cdot S_6(v) \\ T_{e_3} \cdot S_6(v) \end{array}\right\} \text{bound a 3-cycle in the fiber of } h(v)$$



P_v

Type IV:

$$S_{6+}(p_+) \times \{t\} = (r_1, r_2, r_3, t)$$
$$\in \mathrm{Spec}\ \mathbb{C}[\Lambda_{6+}] \times \{t\}$$

$$\underbrace{S_{6-}(p_-) \times \{t\} = (r_1', r_2', r_3', t)}$$
$$\in \mathrm{Spec}\ \mathbb{C}[\Lambda_{6-}] \times \{t\}$$

Represent it the chart.

$$\mathrm{Spec}\ \mathbb{C}[\Lambda_{6+}] \times \{t\}$$

$$\underbrace{\left(r_1', r_2', r_3' t^{k_{\underline{e}_v}} f_{\underline{e}_v}(r_1', r_2', t), t\right)}$$

(r_1, r_2, r_3, t) (r_1', r_2', r_3', t)

$T_{e,\ell v} = T_e \cdot \ell$

For each pt

$(z_1, z_2, z_3, t) \in T_e \cdot (r_1', r_2', r_3', t)$

Choose

$\gamma_z : [0,1] \to (\operatorname{Spec} \mathbb{C}[\Lambda_\sigma^+]) \times \{t\}$

$\lambda \mapsto (z_1, z_2, z_3(1+\lambda \tilde{f}_{\ell v}(z_1, z_2, t)), t)$

$$\Gamma_{\ell v} = \bigcup_{z \in T_e \cdot (r_1', r_2', r_3')} \gamma_z$$

For each pt $(z_1, z_2, z_3, t) \in \Gamma_{\underline{\ell}v}$

Choose

$$\tilde{\gamma}_z : [0,1] \to \left(\mathrm{Spec}\, \mathbb{C}[\Lambda_{\sigma^+}]\right) \times \{t\}$$

$$\lambda \mapsto \left(z_1, z_2, z_3 (t^{\kappa_{\underline{\ell}v}})^{\lambda}, t\right)$$

$$\tilde{\Gamma}_{\underline{\ell}v} = \bigcup_{z \in \Gamma_{\underline{\ell}v}} \tilde{\gamma}_z$$

$$\Rightarrow \int_{\beta_{f,1}} \Omega_t$$

$$= \underbrace{\int_{T_{e,0}}}_{0} + \underbrace{\int_{\Gamma_v}}_{-\frac{1}{2}(2\pi i)^3} + \underbrace{\int_{T_{e,\ell_v}}}_{0} + \underbrace{\int_{\Gamma_{\ell_v}}}_{0} + \underbrace{\int_{\widehat{\Gamma}_{\ell_v}}}_{\mathrm{sgn}(p)\{\xi_e\}_3 \log(t^{\kappa_v})}$$