# Moduli of K3 via hyperKähler metrics

Mirror sym. conference "at Kyoto"

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10th Dec (2020)

J.w.w. Y. Oshima

arXiv: 1805.01724 (announce/summary 12p), 1810.07685 175p ] general prep, mainly K3 type III; 2010.00416, [Osh] in prep. ] K3, type II.

Background idea

Moduli "b'dry" of polarized Calabi-Yau varieties $(X, L)$ (L: ample l.b (or Kähler class))

⟷ Understand the Relation!

Asymp. Behaviour of Ricci-flat Kähler / Calabi-Yau / Kähler-Einstein metric

Apply → Canonical Cpt'fication of Moduli of CY.

Apply → (More) general lim of CY metric.

※ we also observe "Mirror sym" phenomena in the course.

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Apply

Canonical Cptification of Moduli of CY.

Apply

(More) general lim of CY metric.

So far, most succesful in $\boxed{\text{HyperKähler}}$ case, notably K3.

(E.g. KS conj for $K3_{AV}$; new inv for type II degen ... )

Asymp. Behaviour of Ricci-flat Kähler / Calabi-Yau / Kähler-Einstein metric

## Elliptic curve case

$$X_t = \mathbb{C}^*/t^{\mathbb{Z}} \quad (\text{for } |t|<1) \qquad \text{"Tate curve"}$$

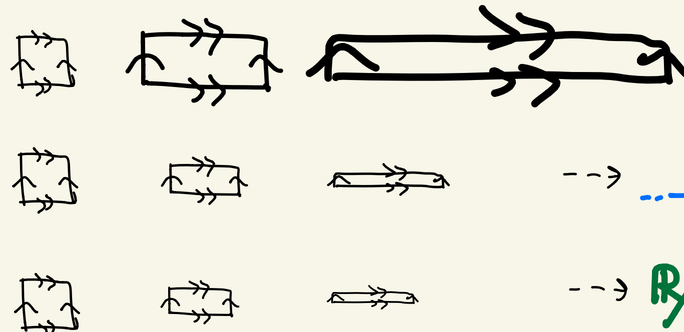
$$= \mathbb{C}/2\pi i\mathbb{Z} + \log t\, \mathbb{Z}$$

for $t \to 0$

① fix inj. rad (rescale)

② fix vol

③ fix diam (rescale)



Néron model

⇓

$\dashrightarrow \mathbb{C}^*$

$\dashrightarrow \mathbb{R}$

$\dashrightarrow \mathbb{R}/\mathbb{Z} = S^1$

trop. ell curve ⇑

Konts-Soib "conj."

= dual int. cpx (of $I_N$-degen)

= ess. skeleton

Focus on

# § K3 surface case.

Recall

$\circ$ $\mathcal{F}_{2d} = \left\{ (X,L) \;\middle|\; \begin{array}{l} X: \text{possibly } \underline{\text{ADE}} \text{ K3} \\ L: \text{ample \& primitive.} \\ \quad \text{w/ } (L^2) = 2d \end{array} \right\}$ $\left( \supset \mathcal{F}^\circ_{2d} = \left\{ \text{''} \;\middle|\; \begin{array}{l} X: \\ \text{smooth} \\ \text{K3} \end{array} \right\} = \mathcal{F}_{2d} \setminus \text{finite Heegner divs.} \right)$

Zar open dense.

$\xrightarrow{\sim} \underset{\text{discrete arith. group}}{\overset{\Gamma_{2d}}{\widetilde{O}^+(\Lambda_{2d})}} \backslash \boxed{\left\{ \mathbb{C}\sigma \;\middle|\; \begin{array}{l} \sigma \in \Lambda_{2d} \otimes \mathbb{C} \setminus \{0\} \\ \sigma^2 = 0,\ (\sigma, \bar{\sigma}) > 0 \end{array} \right\}^\circ}\ \mathcal{D}_{\Lambda_{2d}}$

loc. Herm. Sym. space of IV/orthog. typ

( Pyatetski-Shapiro; Shafarevich "Torelli" type thm )

- I. Satake constructed finite compactifications
  around 1957-60
  to each loc. sym. sp ($\Gamma \backslash \mathcal{D} = G/K$) (associated to "types" of (highest wt rep) irr **Rep.** of $G$)

- probably most famous example is "SBB"
  when $G/K$ is $\boxed{\text{hermitian (i.e. } \exists\ \mathbb{C}\text{-str)}}$ & Baily-Borel 1964-66
  --- the cptif is projective (var)

$$\partial \overline{\mathcal{F}_{2d}}^{\text{Sat, adjoint rep}} = \coprod_{\ell:\ \text{isotrop line} \subset \Lambda_{2d}\otimes\mathbb{Q}} \Gamma_{2d}\cap \mathrm{stab}(\ell) \Big\backslash \left\{ v \in \ell^{\perp}/\ell \otimes\mathbb{R} \;\middle|\; v^2>0 \right\}$$

"type III"

$\mathcal{F}_{2d}(\ell)$

(real) 18-dim ball quot

(NON variety)

$$\amalg \coprod_{p:\ \text{isotrop plane} \subset \Lambda_{2d}\otimes\mathbb{Q}} \text{od } \Gamma_{2d})\ \Gamma_{2d}\cap \mathrm{stab}(p) \backslash 1\text{-pt}$$

"type II"

$\boxed{1\text{-pt}}$ $\mathcal{F}_{2d}(p)$ 0

compare

SBB cptf

$$\partial \overline{\mathcal{F}_{2d}}^{SBB} = \coprod \left( \boxed{\text{Modular curve}}\ 2 \qquad \mathrm{Stab}(p) \backslash \mathbb{H} \right)$$

type II degen.

(variety)

$$\amalg \coprod_{\ell:\ \text{isotrop line} \subset \Lambda_{2d}\otimes\mathbb{Q}} (1\text{-pt})$$

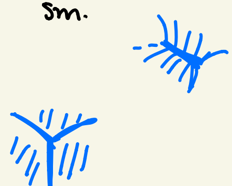
type III degen.

**Equivalent alternative to Sat, adj cptif.**

(via "Tropical.")

## Morgan-Shalen type compactif.

$$\left[\ \underset{\text{smooth}}{U} \subset \underset{\text{sm.}}{(X,\ \overset{\text{SNC div}}{D})}\ \right]_{/\mathbb{C}\text{ or nA field.}} \rightsquigarrow \left[\ U^{an} \subset \overbrace{U^{an} \sqcup \underset{\text{dual int cpx}}{\Delta(D)}}^{=\ \overline{U^{an}}^{MSBJ}}\ \right]$$



# Equivalent alternative to Sat. adj cptif.

(via "Tropical.")

## Morgan-Shalen type compactif.

$$\overline{U^{an}}^{MSBJ}$$

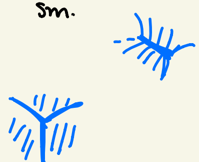
$$\Big\|$$

$U$ smooth $\subset$ $(X, D)$ sm., $D$: SNC div

/$\mathbb{C}$ or nA field.

$\rightsquigarrow$ $U^{an} \subset \overline{U^{an} \sqcup \Delta(D)}$

$\Delta(D)$: dual int cpx



**Thm** [OO18, §2 Thm2.1]

For $\forall$ Shimura var $U$

$\subset \forall$ toroidal cptif $(X, D)$. (AMRT)

$$\overline{U^{an}}^{MSBJ} \overset{!}{=} \text{Satake adj. cptif}$$

Geometric realization map $\Phi_{alg} : \overline{\mathcal{F}_{2d}}^{Sat,adj} \to \{$ metric $\overset{cpt}{}$ sp (+ additional str) $\}$

$\cup$

(38) $\mathcal{F}_{2d} \longrightarrow$

$\Psi$

$(X,L) \longmapsto$ KE met on $X^4$ / diameter

(hyper Kähler.)

metric lim ("collapse")

"type III" $\mathcal{F}_{2d}(\ell) \to \{$ certain (explicit) metrized $S^2$ ($\cong \mathbb{CP}^1$) $\}$

(18)

"type II" $\mathcal{F}_{2d}(p) \to$ [0,1]

(0)

metric lim (collapse)

Cor Gross-Wilson / Kontsevich-Soibelman Conj. for AV/K3.

[ Conj whole $\Phi_{alg}$ is $\boxed{\text{conti (w.r.t. GH top)}}$ ]

Thm (0018) true for $\swarrow$ direction (also true for AV)

## $\boxed{\text{sketch proof}}$ of K3 $\rightsquigarrow$ $S^2$

continuity
(collapsing)

If $(X_i, L_i) \in \mathcal{F}_{2d}$ approaches to $\mathcal{F}_{2d}(\ell)$
$i=1,2,\dots$

e.g. type III degeneration

(e.g. $[x_0x_1x_2x_3 + t\,F_4 = 0]$  )

$\Rightarrow$ for $i \gg 0$

$\exists$ (canonical) special Lagrangian fibration

$$X_i \twoheadrightarrow S^2$$

$$\left( \Leftrightarrow \text{"hK rot"} \atop \text{(different } \mathbb{C}\text{-str)} \right) \quad \overset{\shortparallel}{X_i^\vee} \twoheadrightarrow \overset{\shortparallel}{\mathbb{CP}^1} \Bigg)$$

$\boxed{\text{ellip. K3}}$
$\pi_i$

(as expected in Mirror symmetry)

Moreover we can specify $\boxed{\text{Explicit nbhd}}$ of $\mathcal{F}_{2d}(\ell)$

[0018, §4.4 4.14, 4.18]

## sketch proof of K3 → $S^2$ continuity (collapsing)

If $(X_i, L_i) \in \mathcal{F}_{2d}$ approaches to $\mathcal{F}_{2d}(\ell)$
$i = 1, 2, \dots$

e.g. type III degeneration

(e.g. $[x_0 x_1 x_2 x_3 + t F_4 = 0]$  )

$\Rightarrow$ for $i \gg 0$

$\exists$ (canonical) special Lagrangian fibration $\quad$ (as expected in Mirror symmetry)

$$\begin{array}{ccc} X_i & \overset{\pi_i}{\twoheadrightarrow} & S^2 \\ \| & & \| \\ X_i^{\vee} & & \mathbb{CP}^1 \end{array}$$

$\Rightarrow$ if $i \to \infty$, the $\pi_i$-fibers shrink! $\rightsquigarrow S^2$!

after Yau,
Gross–Wilson / Gross–Tosatti–Zhang…
(geom. analysis

Recent developments : "type II" case (cf. arXiv:2010.00416 (O) & [Osh, in preparation])
(w/ Oshima)

- associate (explicit) convex PL density fun $V : [0,1] \to \mathbb{R}_{\geq 0}$ to type II degen family / seq.
- partially prove it captures limit measure of the KE seq

basic shape is classified b Dynkin diags.

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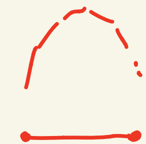

basic shape is classified by Dynkin diags.

**Abst. Existence Thm** (Honda - Sun - Zhang '19)

For $\{X_i : K3\}_{i=1,2,\dots} \xrightarrow[\lim]{\text{mGH}}$ $X_\infty$ $[0,1]$ (roughly, $\exists \varphi_i : X_i \to X_\infty$ map almost keeping $\left\{ \begin{array}{l} \text{measure} \\ \text{metric} \end{array} \right.$ )

$\exists$ PL fun $V$ s.t. limit metric $= \sqrt{V}\, dx$

$\exists$ nat aff str $\nabla_{HSZ}$ ($\Leftrightarrow dx$) limit measure $= V\, dx$.

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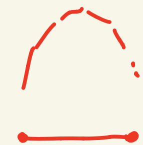

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$\exists$ PL fun $V$ s.t. limit metric $= \sqrt{V}\, dx$

$\exists$ nat aff str $\nabla_{HSZ}$ ($\Leftrightarrow dx$) limit measure $= V\, dx$.

"mirror"?

(Berkovich type aff. str. $\neq$)

What we did:

- consider the real (17)-dim ball quot

$$\underbrace{O(\Lambda_{seg})\backslash \left\{ v \in \Lambda_{seg}\otimes\mathbb{R} \;\middle|\; v^2>0 \right\} / \mathbb{R}_{>0}}$$

$\left(=: \mathcal{M}_{K3}(d)^{\tau} \text{ in op. cit}\right)$

isot. plane $P \subset \Lambda_{K3}$ $(3,19)$

$\Lambda_{seg} := P^{\perp}/P = \mathrm{II}_{1,17}$

and $\underbrace{O(\Lambda_{seg})\backslash \left\{ v \in \Lambda_{seg}\otimes\mathbb{R} \;\middle|\; v^2>0 \right\} / \mathbb{R}_{>0}}_{\text{real (17)-dim ball quot}} \xrightarrow{\widetilde{\overline{\Phi}}} \{V\}$

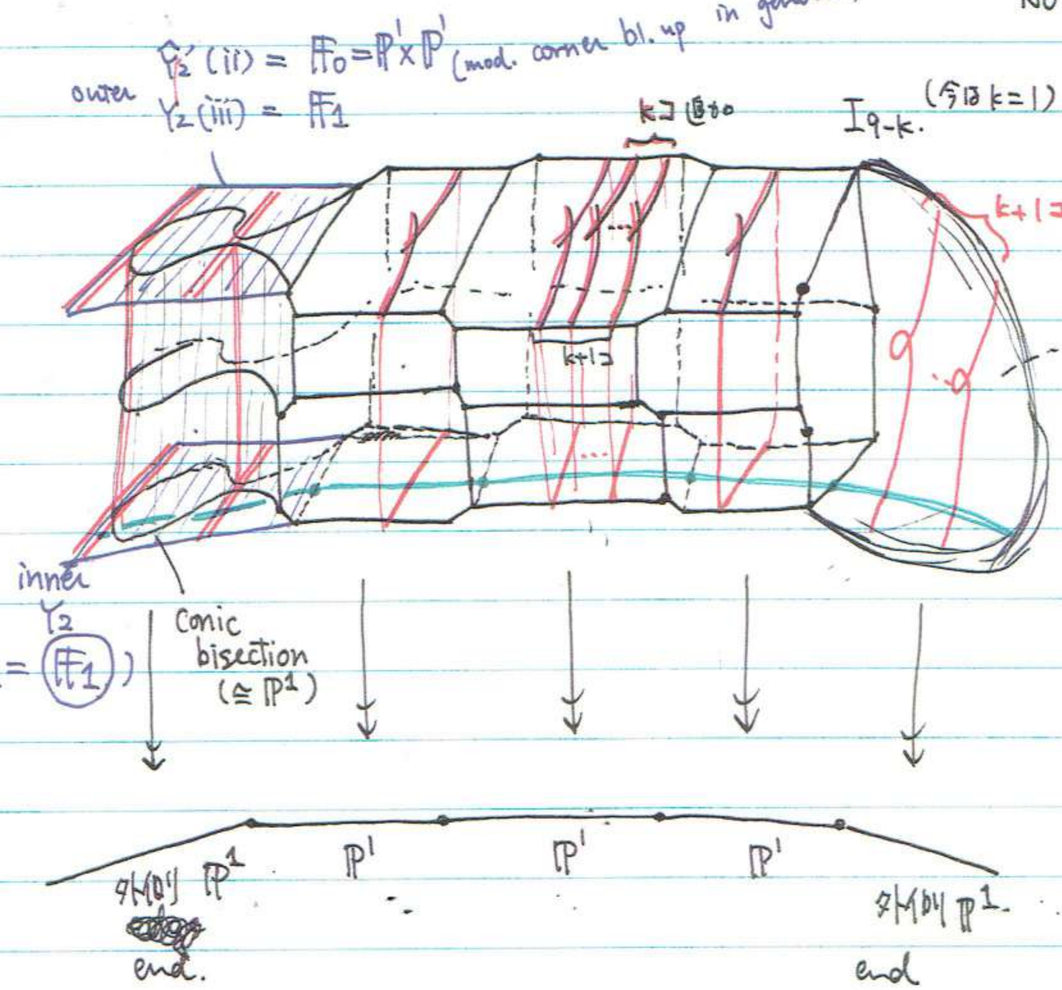
def by $\overbrace{\text{Alexeev-Brunyate-Engel '20 \& Osh}}^{\text{for Moduli cptif of Ellip. K3s. (log KSBA)}}$ (independent)

for this (lim. measure) purpose.

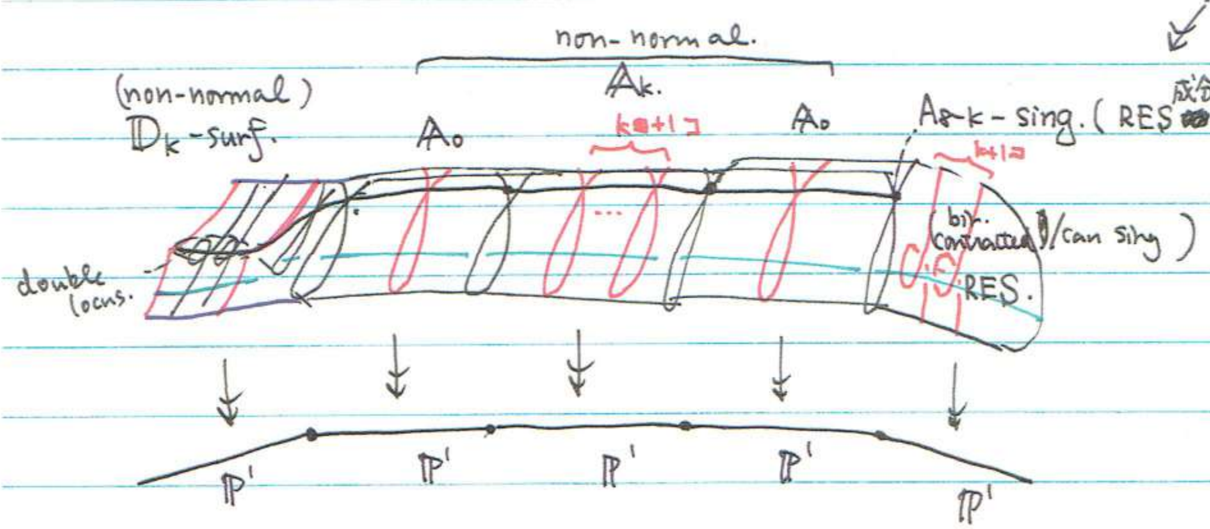
# (NOT dual) intersection complex. of $\boxed{\text{Kulikou model}}$

NOT unique.
i.e. allow flops.

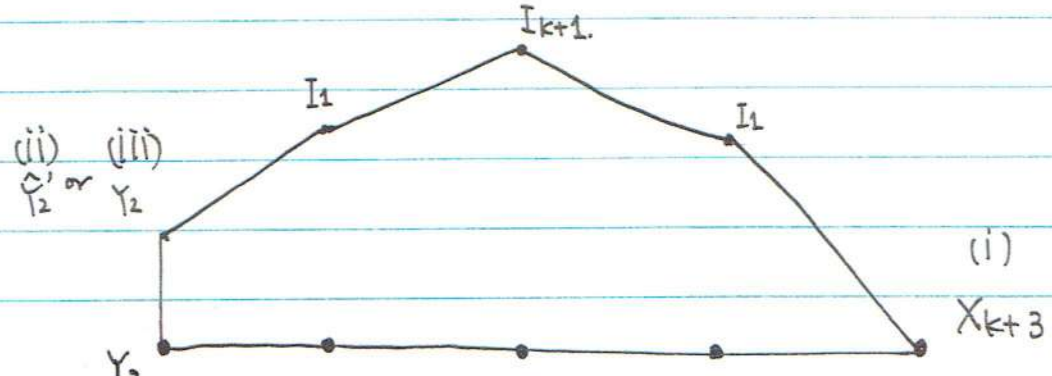
I. ⟷
II. ⟷




この底面部分のみ残して他をcontract
(log KSBA)
w/ b'dry = 緑 + 赤
(section …)
($\times \varepsilon$, $\varepsilon \ll 1$)




log KSBA model (unique)



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$$O(\Lambda_{seg}) \backslash \{ v \in \Lambda_{seg} \otimes \mathbb{R} \mid v^2 > 0 \} / \mathbb{R}_{>0}$$

(=: $\mathcal{M}_{K3}(a)^{\tau}$ in op. cit)

and $O(\Lambda_{seg}) \backslash \{ v \in \Lambda_{seg} \otimes \mathbb{R} \mid v^2 > 0 \} / \mathbb{R}_{>0} \xrightarrow{\tilde{\Phi}} \{V\}$ def by Alexeev-Brunyate-Engel 20 & Osh (independent)

real (17)-dim ball quot

- for any type II degen / seq (of K3) polarized

  $(\mathcal{X}^*, \mathcal{L}^*)$ $\{X_i\}$

$\rightsquigarrow$ we take limit in $\square$ & $V(\mathcal{X}^*, \mathcal{L}^*) := \tilde{\Phi}(X_\infty)$

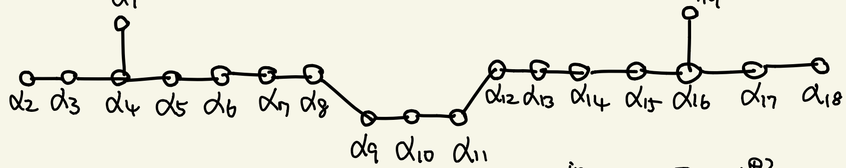
$X_\infty$

# Some picture of $\Phi$

.. controlled by

the roots

$\alpha_1 = \frac{1}{9}(-8e_1 + e_2 + \cdots + e_9)$




$\alpha_9 = e_8 - e_9 + e$, $\alpha_{10} = -e+f$, $\alpha_{11} = e_8' - e_9' + e$

in $\Lambda_{seg} = E_8(-1)^{\oplus 2} \oplus U$

$\subset \langle e_1, \dots e_9 \rangle$ $\langle ', \dots \,' \rangle$ $\quad \langle e, f \rangle$

&

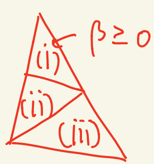
$3\beta := \alpha_1 - 2\alpha_2 - \alpha_3.$

real (17)- dim ball quot

- we identify $O(\Lambda_{seg}) \backslash \{ v \in \Lambda_{seg} \otimes \mathbb{R} \mid v^2 > 0 \} / \mathbb{R}_{>0}$ $=$ Left-Right invol $\Big\backslash \Big\{ \lambda \in \Lambda_{seg, \mathbb{R}} \;\Big|\; (\alpha_i, \lambda) \geq 0 \; (\forall i) \;\; \lambda^2 > 0 \Big\}$

(by Vinberg)

- divide  into $3 \times 3$ chambers.
  (L) (R)



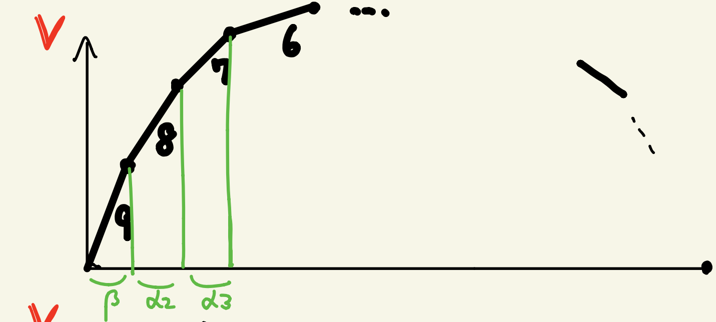

$\begin{Bmatrix} (i)_L \\ (ii)_L \\ (iii)_L \end{Bmatrix} \times \begin{Bmatrix} (i)_R \\ (ii)_R \\ (iii)_R \end{Bmatrix}$

(L) decides left end of V

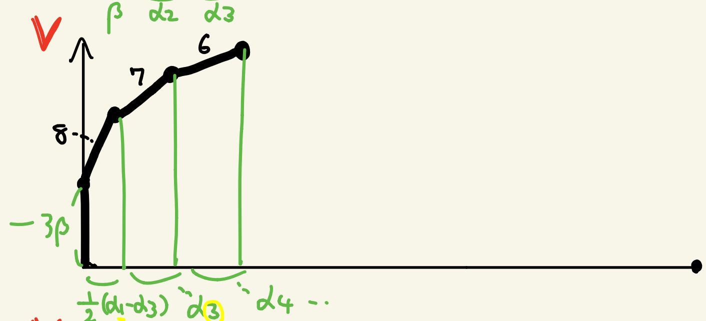
(R) " right "

part (i)$_L$

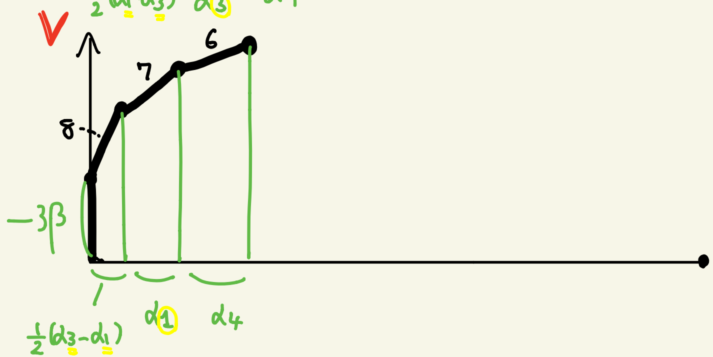
$(\beta, \lambda) \geq 0$




(ii)$_L$

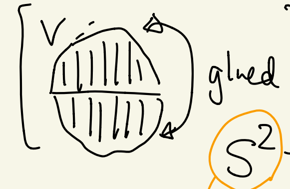



(iii)$_L$




Rmk

[ABE] used




"dumpling"

for Ellip K3 degen.

the same for Right side.

the way of taking limit $X_\infty$ in $\square$

--- Consider in general Kähler setting :

$$\overline{\mathcal{F}_{2d}}^{38} \longrightarrow \mathcal{M}_{K3}^{57} := \left\{ \text{all } \left(\begin{matrix}\text{KE-metrized}\\ \text{possibly ADE}\end{matrix}\right) \text{ Kähler K3} \right\} \Big/ \begin{matrix}\text{certain}\\ \text{change of } \mathbb{C}\text{-str}\\ (\text{hK rot})\end{matrix}$$

$$\bigcup_{\substack{\text{all}\\ K3}} \text{Kähler cone} \Big/ \sim$$

$$\| $$

$$\underset{(\text{Kobayashi-Todorov})}{\cong} \quad SO^\circ(3,19) \big/ SO(3) \times SO(19)$$

the way of taking limit $X_\infty$ in $\square$

--- Consider in

general Kähler setting :

$$\mathcal{F}_{2d} \xrightarrow{} \mathcal{M}_{K3} := \left\{ \begin{array}{l} \text{all } \left(\begin{array}{l}\text{KE-metrized} \\ \text{possibly ADE}\end{array}\right) \\ \text{Kähler K3} \end{array} \right\} \Big/ \begin{array}{l}\text{certain} \\ \text{change of } \mathbb{C}\text{-str} \\ (\text{hK rot})\end{array}$$

(38) (57)

$\bigcup_{\text{all K3}}$ Kähler cone $/\sim$

$\mathcal{M}_{K3} \cong SO^\circ(3,19)/SO(3)\times SO(19)$ (Kobayashi-Todorov)

$(\bigcirc : \mathbb{R}\text{-dim})$

$$\overline{\mathcal{M}_{K3}}^{\text{Satake, adj. rep}} \twoheadleftarrow \overline{\mathcal{M}_{K3}}^{\text{Satake}, \tau}$$

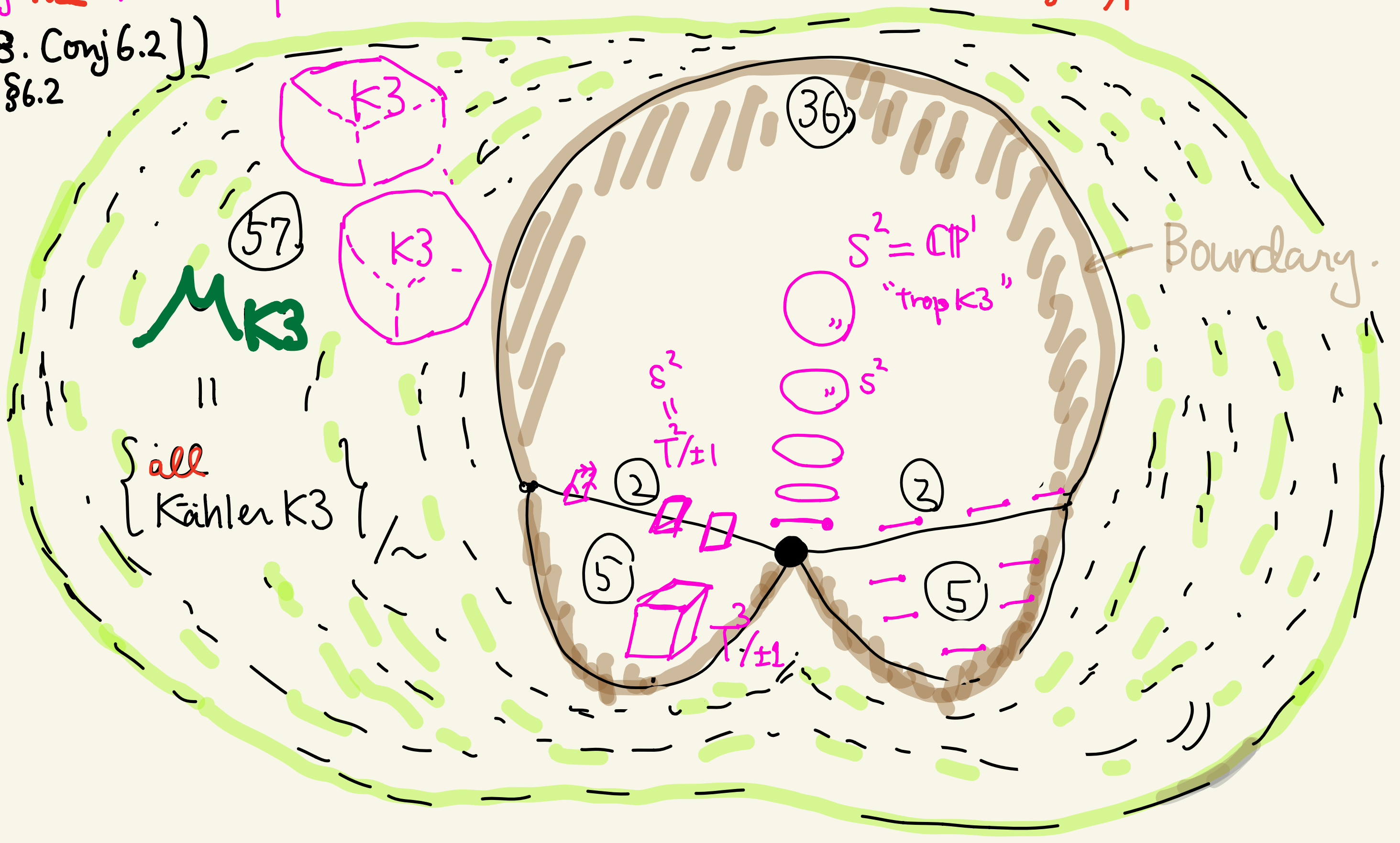
( 2 compactifications )

Geometric realization of **ALL** K3 collapse.

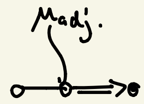
([OO18. Conj 6.2]) §6.2

Thm 6.5 ~ 6.8

$\overline{M_{K3}}^{\text{Satake, adj. rep}}$ (Satake cptif adj. type)

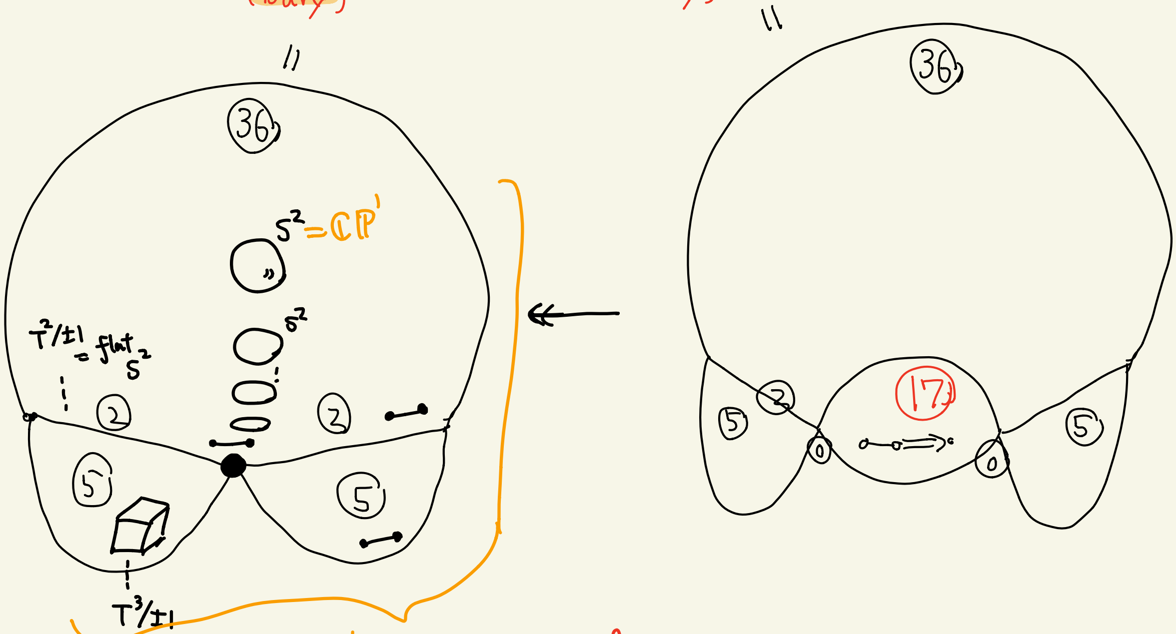


# picture





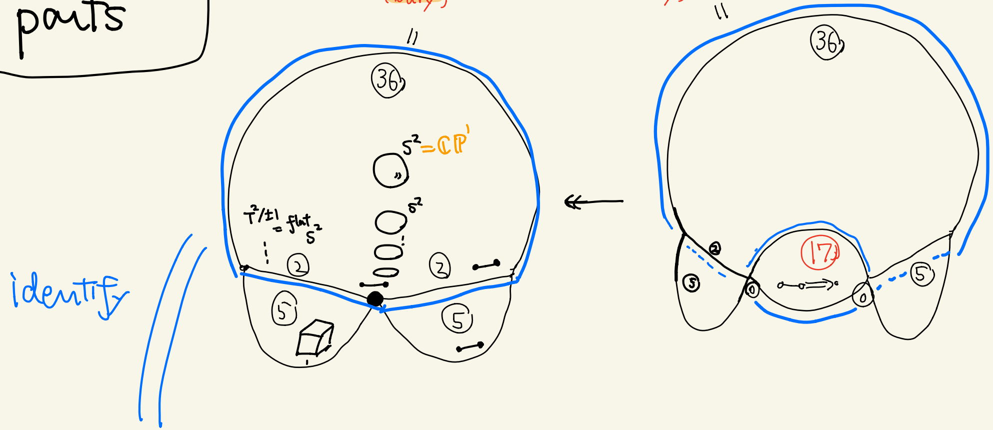
$$\underset{(\text{bdry})}{\partial \overline{M_{K3}}^{\text{Satake, adj. rep}}} \twoheadleftarrow \underset{(\text{bdry})}{\partial \overline{M_{K3}}^{\text{Satake}, \tau}}$$




[OO'18 §6] gives this geometric realization.
(& proved continuity at 36)

**Extract Blue parts**

$\partial \overline{M_{K3}}^{\text{Satake, adj. rep}}$ (b'dry) $\twoheadleftarrow$ $\partial \overline{M_{K3}}^{\text{Satake}, \tau}$ (b'dry)




(Weierstrass)

Ellip. K3's Moduli

$M_W$

18-dim normal $\mathbb{Q}$-pr. var.

(loc. HSD)

& its Satake-Baily-Borel compactification!

(GIT

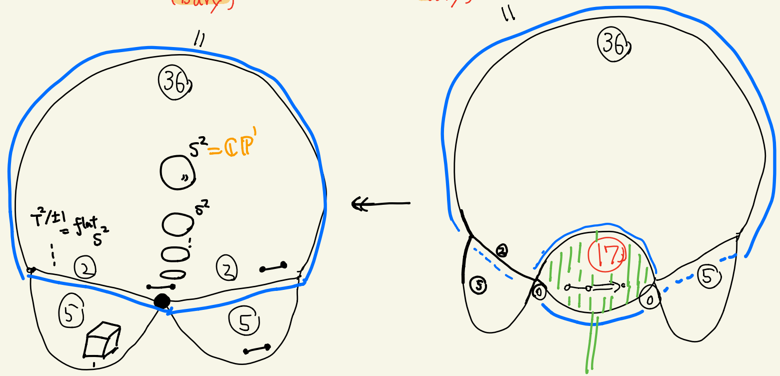
⇕ 0018 §7)

**Extract Blue parts**

$\partial \overline{M_{K3}}^{\text{Satake, adj. rep}} \twoheadleftarrow \partial \overline{M_{K3}}^{\text{Satake}, \tau}$

(bdry) (bdry)

$\parallel$ $\parallel$




identify

$\mathcal{M}_{K3}(d)^\tau$ ← Nothing but our concerned parameter sp of $V$

(Weierstrass)

Ellip. K3's Moduli

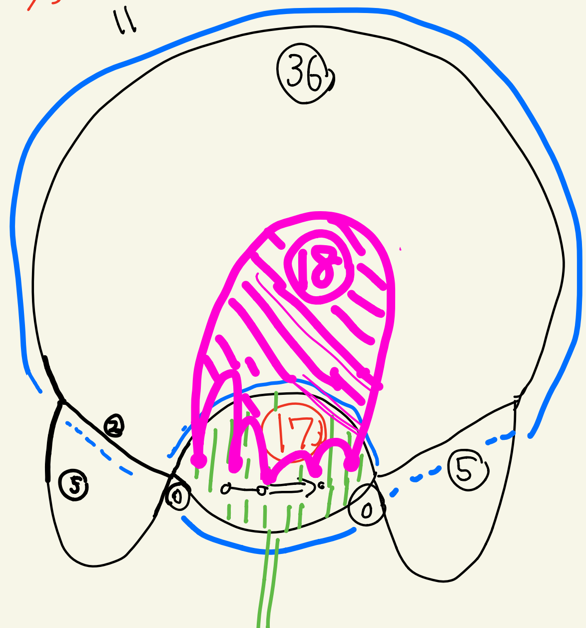
$M_W$

18-dim normal $\mathbb{Q}$-pr. var.

(loc. HSD)

$\partial \overline{\mathcal{M}_{K3}}^{\text{Satake},\tau}$ (bdry)

$=$




$\mathcal{M}_{K3}(d)^{\tau}$

**Limit locus of $\mathcal{F}_{2d}$ (Alg. K3s)**

$=$

for each $d$.

**# ( Its $\cap$ w/ 17 ) $< \infty$**

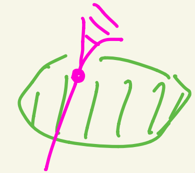
"but become dense when $\bigcup_d$"

$\boxed{\text{Associated lattice}}$ ("weaker info than V")

$\Lambda_{per}$ for type II degen

$$\begin{array}{ccc} (\mathcal{X}^*, \mathcal{L}^*) & \subset & (\mathcal{X}, \mathcal{L}) \\ \downarrow & & \downarrow \\ \Delta^* & \subset & \Delta \end{array}$$

$\rightsquigarrow$



$[v_{2d} \in P^{\perp}/P]$

$$:= \left( v_{2d}^{\perp} \subset P^{\perp}/P \simeq \Lambda_{seg} \right)$$

$\cup$

$\mathbb{D}\mathbb{A}\mathbb{A}\cdots\mathbb{A}\mathbb{D}$

or

$\mathbb{D}\mathbb{A}\cdots\mathbb{A}\mathbb{E}$

or

$\mathbb{E}\mathbb{A}\cdots\mathbb{A}\mathbb{E}$

Speculated Geom. meaning (7.1 of op cit [O'20 arXiv 201000416])

$\begin{matrix} V_0 \\ \hline V_1 \end{matrix} = \mathcal{X}_0$, then term. obj ('s dim) of MMP w/ scaling $\mathcal{L}|_{V_i}$ on $V_i$ may decide $\mathbb{D}$ or $\mathbb{E}$?

(OK for $d=1$: Friedman 80s)

[EAE-type]

**Ex**

If $\mathcal{X}_0 = \boxed{V_0} \cup \cdots \cup \boxed{V_1}$

w/ $V_i$ : (Anti-can) Del Pezzo surf

$\Rightarrow$ $\mathcal{X}_t$ : (probably) close to glued HK space by Hein-Sun-Viaclovsky-Zhang '18

Rem

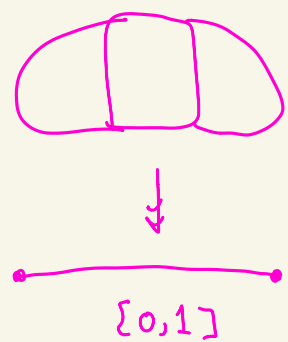
Around the Ends compo (E type) $V_0, V_1,$

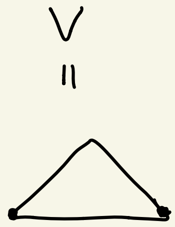
the metric & phenomenon (rel. w/ [ABE])

we observe

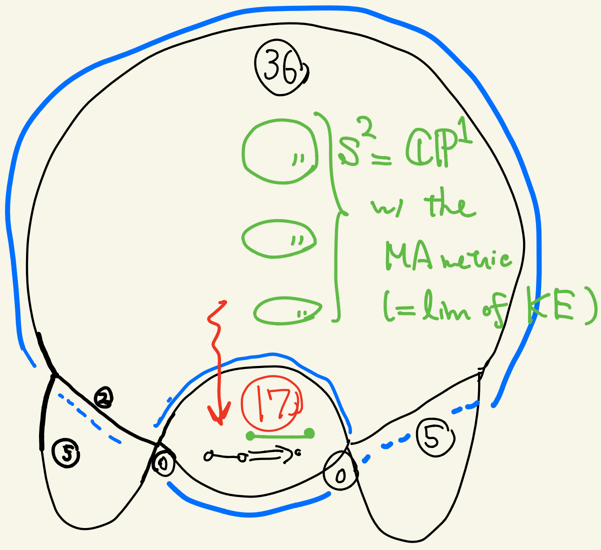
should be conn to Y-S. Lin's talk.

K3



## Main Thm (of Part II of op. cit) also [Osh]




$\tilde{\Phi}$ i.e. the V-fns indeed describes (at least) the $\boxed{\text{limit measure}}$ (on •—• ) of $(S^2, \text{MA}^{\mathbb{R}}\text{ met})$ seq.

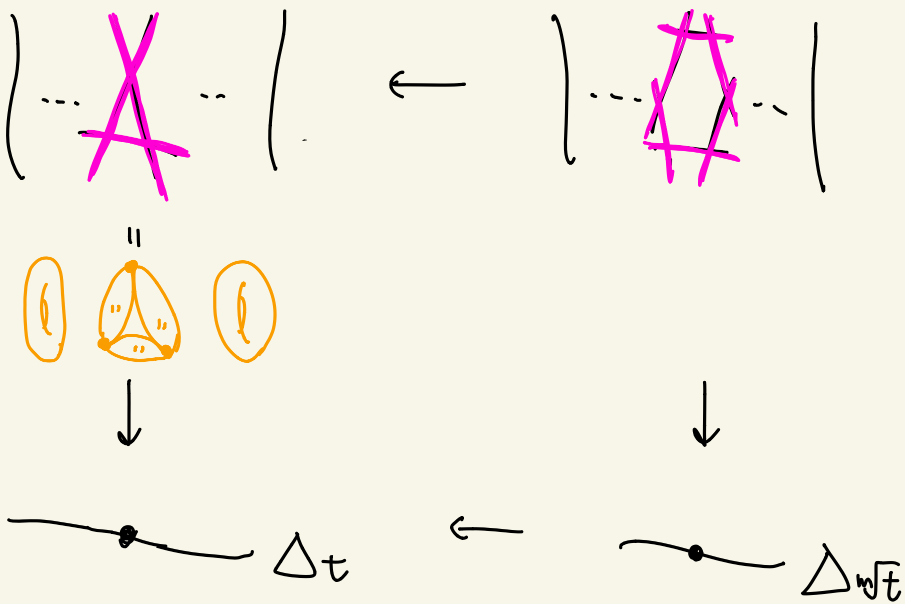
[Conj: of (K3, KE) as well] $\in M_{K3}$.

proof ingredients

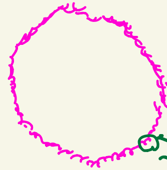
- Asym. behaviour of $\text{MA}^{\mathbb{R}}$ metrics (cf. [OO18, §7], [Osh])
- 17-dim (open) b'dry strata = dual int. cpx of $\partial \overline{M}_W^{ABE,\nu}$ (toroidal)
- [ABE] & [O, Part I] stable reduction.

## § Brief intro to "Galaxy" (arXiv : 2011.12748)

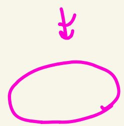
For $\boxed{I_d -}$ degen (in Kodaira's sense) of Ellip. curve $\qquad$ $I_{md}$ - degen.



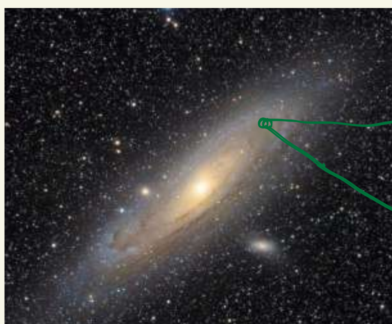
$$X_\infty \supset \coprod_{a\in\mathbb{Q}/\mathbb{Z}} \boxed{\left(\mathbb{P}^1\setminus\{0,\infty\}\right)} \sqcup \left(\coprod_{\substack{a:\,\text{irr}\\ \text{mod}.\mathbb{Z}}} \bullet\right)$$



scale up !



$S^1 = \mathbb{R}/\mathbb{Z}$



scale up !

shape of the totality.

# § Brief intro to Open K-polystability

(Details: arXiv: 2009.13876)

.. Non-cpt version of K-polystability

More precisely, motivated by many

Complete Ricci-flat Kähler metrics (e.g. $\dim_{\mathbb{C}} = 2$, ALG, ALH grav. instantons)

(KE)

w/ vol. growth dim $\leq \dim_{\mathbb{C}} X^\circ$

on $(X^\circ, L^\circ)$

$\cap$

$((\underbrace{X, D}_{\text{log CY}}), L)$

Thm

seems true for many known examples

? $\Updownarrow$ ?

We introduced Open K-polystabilities of $(X^\circ, L^\circ)$

Thank you
for listening.

Please keep
safe & healthy

until "the next spring"
春