Mirror Symmetry for Fano Manifolds and Related Topics

Time: 10-14 December 2018
Venue: Room 127 (Graduate School of Science Bldg no.3), Department of Mathematics, Kyoto University

	9:00-10:00	10:30-11:30	13:30-14:30	15:00-16:00	16:30-17:30
Mon	Cho	Cho	Vianna	Ikeda	
Tue	Vianna	Vianna	Cho	Harder	Ikeda
Wed	Kasprzyk	Smirnov			
Thu	Harder	Harder	Kasprzyk	Kasprzyk	
Fri	Smirnov	Smirnov	Shamoto	Shamoto	

Title:

Cheol-Hyun Cho	1. *HMS for toric Fano manifolds*
	2. *Mirror symmetry for orbi-spheres*
	3. *Pairings and Mirror Symmetry*
Andrew Harder	*Fibrations and mirror symmetry*
Akishi Ikeda	*q-stability conditions and q-quadratic differentials*
Alexander Kasprzyk	*Mirror symmetry for Fano manifolds*
Yota Shamoto	*Hodge structures on tame compactified Landau-Ginzburg models*
Maxim Smirnov	*Quantum cohomology and derived categories of rational homogeneous spaces*
Renato Vianna	*Lagrangian fibrations and open Gromov-Witten invariants*

See the conference page for more information.

Organizers: Hiroshi Iritani, Yukiko Konishi, Atsushi Takahashi

Acknowledgments: This workshop is supported by Kiban-S 16H06337 (Atsushi Takahashi) and Kiban-C 16K05127 (Hiroshi Iritani).

Cheol-Hyun Cho

Lecture 1. *HMS for toric Fano manifolds*

We explain how to construct explicit A-infinity functor for toric Fano manifold which derives homological mirror symmetry between its Fukaya category and matrix factorization category of its Landau-Ginzburg mirror. It is based on Floer theoretic construction which uses formal Maurer-Cartan elements and curved Yoneda embedding. This is a joint work with Hansol Hong and Siu-Cheong Lau.

Lecture 2. *Mirror symmetry for orbi-spheres*

We consider a sphere with 3 orbifold points. Its mirror is given by a cusp singularity whose closed mirror symmetry has been studied by Rossi, Satake, Takahashi, Ishibashi and Shiraishi. We explain (homological) mirror symmetry for orbi-spheres using Lagrangian Floer theory. Using an immersed circle (called Seidel Lagrangian), we define homological mirror functor as well as Kodaira-Spencer map to prove (homological) mirror symmetry. This is based on joints works with Lino Amorim, Hansol Hong and Siu-Cheong Lau.

Lecture 3. Pairings and Mirror symmetry

Given a (homological) mirror symmetry between a symplectic manifold and its Landau-Ginzburg mirrors, we may ask whether mirror symmetry preserves pairing structures. For a symplectic manifold, Poincare duality provides pairings for both open and closed theories. A matrix factorization category has the Kapustin-Li pairing and Jacobian ring has a residue pairing. We introduce what we call, multi-crescent Cardy identity, which is used to compare these pairings. We will find that interesting conformal factor arises between these pairings in mirror symmetry. This is a joint work with Sangwook Lee and Hyungseok Shin.

Andrew Harder

Fibrations and mirror symmetry

My first two talks will focus on describing how fibrations on Calabi-Yau manifolds are related to degenerations under mirror symmetry. In my first talk, I will discuss elliptic fibrations on K3 surfaces and how, in Dolgachev ' s lattice polarized mirror symmetry, they relate to type II boundary components in the moduli space of mirror K3 surfaces. My second talk will describe how the same ideas, applied to K3 fibrations on Calabi-Yau threefolds and Tyurin degenerations, lead to classification theorems analogous to Kodaira ' s classification of elliptic fibrations, for certain K3 fibered Calabi-Yau threefolds.

My final talk will change direction slightly. I will discuss new conjectures, developed with Katzarkov, Pantev, Przyjalkowski and others, about how the weight filtration on a the cohomology log Calabi-Yau variety is related to the perverse Leray filtration on the mirror manifold associated to its affinization map, and I will describe a collection of examples in which these conjectures have been verified.

Akishi Ikeda

q-stability conditions and q-quadratic differentials

For a given triangulated category, we can associate a complex manifold, called the space of Bridgeland stability conditions. By the work of Bridgeland-Smith (BS) and Haiden-Katzarkov-Kontsevich (HKK), for a certain class of triangulated categories associated with marked bordered surfaces, the space of stability conditions on them can be identified with the moduli spaces of quadratic differentials on Riemann surfaces.

On the recent work with Yu Qiu, we introduced the notion of q-stability conditions and showed that the space of q-stability conditions on a certain double graded triangulated category constructed from a marked bordered surface can be identified with the moduli space of q-quadratic differentials (which are multi-valued quadratic differentials with certain conditions) on the corresponding Riemann surface. This result looks Calabi-Yau "s" analogue (s is a complex number) of the work of BS for Calabi-Yau 3 categories and also clarifies the relationship between the works of BS and HKK. In the talk, I will introduce about this work.

Alexander Kasprzyk

Mirror symmetry for Fano manifolds

We will review the approach being developed by Coates, Corti, Kasprzyk, and others to potentially allow for the classification of Fano manifolds via mirror symmetry. We explain what we can do so far, and what we hope to be able to do in the near future. The focus will be on the combinatorial aspects of the theory.

Yota Shamoto

Hodge structures on tame compactified Landau-Ginzburg models

A tame compactified Landau-Ginzburg model is a pair (X, f) of a smooth projective complex variety X, and a flat projective morphism f from X to the projective line satisfying some conditions. In this talk, we will consider several kinds of Hodge structures associated with this model. In particular, we discuss the relation to some conjectures proposed by Katzarkov-Kontsevich-Pantev.

Maxim Smirnov

Quantum cohomology and derived categories of rational homogeneous spaces

As the title suggests these lectures are devoted to the quantum cohomology and derived categories of coherent sheaves of rational homogeneous spaces G/P. I will begin by a gentle introduction to quantum cohomology and derived categories and illustrate both in some simple examples. Then we will discuss some results on the structure of the small and big quantum cohomology of a rational homogeneous space G/P (e.g. (non-)semisimplicity), and some results the structure of the derived category of coherent sheaves (e.g. existence of full exceptional collections). Finally, we are going to relate both sides via Dubrovin 's conjecture and homological mirror symmetry. These lectures are based on papers joint with (subsets of) J.A. Cruz Morales, S. Galkin, A. Kuznetsov, A. Mellit, N. Perrin, and some further work in progress.

Renato Vianna

Lagrangian fibrations and open Gromov-Witten invariants

The first two talks we will address problems of symplectic topology, such as, what is the space of Lagrangian tori modulo Hamiltonian isotopy in a given symplectic manifold and what are the shapes of Weinstein neighbourhoods of a given Lagrangian torus. For that, we view Lagrangian tori as fibre of almost toric fibrations (or what we call Gelfand-Cetlin fibrations) and we study their open Gromov-Witten invariants and their Fukaya algebra. This will be based on recent work with Egor Shelukhin and Dmitry Tonkonog as well as previous work. As an application of the recent work, we show, under certain assumptions, the unobstructedness of SYZ fibres in a symplectic Calabi-Yau.

For the third talk we will explore how relate the superpotential of a monotone Lagrangian inside a Donaldson divisor D of X to the superpotential of its Biran's lift in X. The relative Gromov-Witten invariants of the pair (X, D) plays a role. Applications include proving existence on infinitely many monotone Lagrangian tori in symplectic manifolds of arbitrary dimensions, providing relations for relative Gromov Witten invariants for some complete intersections, and, via the relationship proven by Tonkonog between periods of the superpotential of a monotone Lagrangian torus and quantum periods, we recover a formula of Coates-Corti-Galkin-Kasprzyk relating quantum periods of D and X, but in a slightly different setting.