Gauge Theory, Moduli Spaces and Representation Theory, Kyoto 2023

In honor of the 60th birthday of Hiraku Nakajima

Dates: 20-24 February 2023
Place: 420 RIMS, Kyoto University
Website:

`https://www.math.kyoto-u.ac.jp/~iritani/Nakajima60/Nakajima60_Kyoto.htm`

Schedule:

	9:30-10:30	11:00-12:00	13:30-14:30	14:50-15:50	16:10-17:10
20 Mon	Yoshioka	Cherkis	Dimofte	Yamakawa	Fukaya
21 Tue	Honda	Kronheimer	Gukov	Finkelberg	Hausel*
22 Wed	Nekrasov*	Göttsche			
23 Thu	Yang	Okounkov	Vasserot	Shan	Kimura
24 Fri	Negut	Gorsky	Braverman	Zhao	McBreen

(∗): Zoom talk

Title:

Alexander Braverman	*Gaiotto conjectures from geometric Langlands point of view*
Sergey Cherkis	*Up and down the bow construction*
Tudor Dimofte	*Vertex algebras in 3d gauge theory*
Michael Finkelberg	*Gaiotto conjectures for basic classical Lie supergroups*
Kenji Fukaya	*Generating function and Floer homology*
Eugene Gorsky	*The affine Springer fiber – sheaf correspondence*
Lothar Göttsche	*(Refined) Verlinde and Segre formulas for Hilbert schemes of points*
Sergei Gukov	*Where Quantum Topology Meets Geometric Representation Theory*
Tamas Hausel	*Mirror symmetry and big algebras*
Ko Honda	*Higher-dimensional Heegaard Floer homology and Hecke algebras*
Yoshiyuki Kimura	*Twist automorphisms and dual canonical bases*
Peter Kronheimer	*Algebraic curves from instanton homology*
Michael McBreen	*Microlocal sheaves on affine Slodowy slices*
Andrei Negut	*Quivers and quantum loop groups*
Nikita Nekrasov	*Lax operators of integrable systems from intersection theory*
Andrei Okounkov	*The Eisenstein Spectrum*
Peng Shan	*Modularity for W-algebras and affine Springer fibers*
Eric Vasserot	*Critical convolution algebras and quantum loop groups*
Daisuke Yamakawa	*Stokes local systems and wild character varieties*
Yaping Yang	*An application of power operations to quantum groups*
Kota Yoshioka	*Moduli of stable sheaves on an elliptic surface*
Gufang Zhao	*Quasimaps to quivers with potentials*

Organizers: Tomoyuki Arakawa, Hiroshi Iritani, Syu Kato, Hiraku Nakajima, Atsushi Takahashi

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Alexander Braverman

Gaiotto conjectures from geometric Langlands point of view

This talk may be viewed as a continuation of M. Finkelberg's talk, but I will try to make it formally independent. In the first half of the talk I will review the formalism of local quantum geometric Langlands correspondence for general reductive groups. In the 2nd half of the talk I will discuss certain specific examples of this correspondence having to do with super-groups whose even part is our original reductive group (these examples are mostly due to D. Gaiotto). I will explain how one can derive the category equivalences from Finkelberg's talk from these conjectures. In addition, I will explain the relationship between the above conjectures and coset constructions of certain super-analogs of vertex W-algebras.

Sergey Cherkis

Up and down the bow construction

We give an overview of the bow construction of instantons on Asymptotically Locally Flat hyperKähler manifolds, with the view to its generalizations. From this point of view, defining corresponding bow varies constitutes only a first step in a chain of generalizations of the Nakajima ' s quiver varieties. In the second half of the talk we focus on completeness of the bow construction and on demonstrating that it establishes an isometry between bow moduli spaces and instanton moduli spaces. These results were obtained in the joint work with Andres Larrain-Hubach and Mark Stern.

Tudor Dimofte

Vertex algebras in 3*d gauge theory*

Topological and/or holomorphic twists of $3d$ supersymmetric gauge theories admit boundary vertex algebras. The vertex algebras can be used to access features of the bulk theories — much as the chiral WZW model was used in the 80's and 90's to access Chern-Simons TQFT. For example, it was shown by Costello-Creutzig-Gaiotto that boundary vertex algebras in $3d$ $\mathcal{N} = 4$ theories can recover the watershed Braverman-Finkelberg-Nakajima construction of bulk Coulomb branches. I will review constructions of these boundary vertex algebras, and present recent work on their role in defining braided tensor categories of line operators in $3d$ $\mathcal{N} = 4$ theories, and proving 3d mirror symmetry thereof. (Based on joint work with T. Creutzig, N. Garner, and N. Geer; and A. Ballin, T. Creutzig, and W. Niu.)

Michael Finkelberg

Gaiotto conjectures for basic classical Lie supergroups

In the framework of the Gaiotto-Witten S-duality of boundary conditions in $\mathcal{N} = 4$ super Yang-Mills theory, the boundary conditions arising from the basic classical Lie supergroups are S-dual to the ones arising from certain equivariant slices. D. Gaiotto has upgraded this S-duality to a (conjectural) equivalence of the representation category of the corresponding quantum group and a certain category of twisted D-modules on the corresponding affine Grassmannian on the S-dual side.

The Gaiotto equivalences specialize to certain particular cases of the Fundamental Local Equivalence (Gaitsgory-Lurie) and of the local version of the Periods–L-functions duality (Ben-Zvi-Sakellaridis-Venkatesh). I will report on our joint work with A. Braverman, V. Ginzburg and R. Travkin in this direction.

Kenji Fukaya

Generating function and Floer homology

To study Lagrangian submanifolds of the cotangent bundle there are two ways, one is using Floer homology and the other uses generating functions. The relation between these two methods are clarified by Viterbo and Milkovic-Oh in the case of exact embedded Lagrangian submanifolds. Including immersed cases there are many works in one and two dimensional cases. In this talk based on a joint work with A. Daemi I will explain a way to study in the case of higher dimensional exact immersion.

Eugene Gorsky

The affine Springer fiber – sheaf correspondence

Given a semisimple element in the loop Lie algebra of a reductive group, we construct a quasi-coherent sheaf on a partial resolution of the trigonometric commuting variety of the Langlands dual group. The construction uses affine Springer theory and Coulomb branch algebras of Braverman-Finkelberg-Nakajima. We also consider a quantization of this construction for homogeneous elements. This is a joint work with Oscar Kivinen and Alexei Oblomkov.

Lothar Göttsche

(Refined) Verlinde and Segre formulas for Hilbert schemes of points

This is joint work with Anton Mellit. Segre and Verlinde numbers of Hilbert schemes of points have been studied for a long time. The Segre numbers are evaluations of top Chern and Segre classes of so-called tautological bundles on Hilbert schemes of points. The Verlinde numbers are the holomorphic Euler characteristics of line bundles on these Hilbert schemes. We give the generating functions for the Segre and Verlinde numbers of Hilbert schemes of points. The formula is proven for surfaces with $K_S^2 = 0$, and conjectured in general. Without restriction on K_S^2 we prove the conjectured Verlinde-Segre correspondence relating Segre and Verlinde numbers of Hilbert schemes. Finally we find a generating function for finer invariants, which specialize to both the Segre and Verlinde numbers, giving some kind of explanation of the Verlinde-Segre correspondence.

Sergei Gukov

Where Quantum Topology Meets Geometric Representation Theory

Tamas Hausel

Mirror symmetry and big algebras

First we recall the mirror symmetry identification of the coordinate ring of certain very stable upward flows in the Hitchin system and the Kirillov algebra for

the minuscule representation of the Langlands dual group via the equivariant cohomology of the cominuscule flag variety (e.g. complex Grassmannian). In turn we discuss a conjectural extension of this picture to non-very stable upward flows in terms of a big commutative subalgebra of the Kirillov algebra, which also ringifies the equivariant intersection cohomology of the corresponding affine Schubert variety.

Ko Honda

Higher-dimensional Heegaard Floer homology and Hecke algebras

Hecke algebras are ubiquitous in number theory and geometric representation theory. In this talk we describe the appearance of various Hecke algebras such as the affine Hecke algebra and the double affine Hecke algebra (DAHA) in Floer theory, through the higher-dimensional analog of Heegaard Floer homology. This is joint work with Yin Tian and Tianyu Yuan.

Yoshiyuki Kimura

Twist automorphisms and dual canonical bases

Originally, twist automorphisms for unipotent cells are introduced by Berenstein-Fomin-Zelevinsky in the study of total positivity of Schubert varieties. In the talk, we study the quantum analogue of the twist automorphism of unipotent cells and their generalization to (upper) cluster algebras and quantization. In particular, we show that the quantum twist automorphism preserves the dual canonical bases and their generalization (called common triangular bases). This is a joint work with Hironori Oya; and with Fan Qin and Qiaoling Wei.

Peter Kronheimer

Algebraic curves from instanton homology

Instanton homology assigns to a knot or link K in a 3-manifold Y a vector space $I(Y, K)$, and in a version with local coefficients this is a module for a ring of Laurent polynomials in a variable T. It also carries certain natural operators related to the homology of Y and K. In this talk we will describe a computation of this instanton homology, as a module over the polynomial algebra generated by T and these natural operators, in the case of the very simplest braids K in $S^1 \times S^2$. This builds on earlier work (with $\mathbb{Q}$ coefficients) by Street and Xie-Zhang, and on the work of Munoz for the case that K and T are absent. Our answer expresses $I(Y, K)$ in these cases as the structure sheaf of an algebraic curve with an interesting singular structure. This is joint work with Tom Mrowka, where one eventual aim is to answer topological questions about immersed surfaces in 4-manifolds. A sample question is whether a smooth quintic surface can contain a rational curve of large degree.

Michael McBreen

Microlocal Sheaves on Affine Slodowy Slices

I will describe certain moduli of wild higgs bundles on the line, and explain why they are affine analogues of Slodowy slices. I will then describe an equivalence between microlocal sheaves on a particular such space and a block of representations of the small quantum group. Joint work with Roman Bezrukavnikov, Pablo Boixeda Alvarez and Zhiwei Yun.

Andrei Negut

Quivers and quantum loop groups

We propose a general framework that starts with a quiver whose arrows are labeled by arbitrary elements of a ground field, and outputs a quantum loop group. The gist of the construction is to use shuffle algebras to motivate the appropriate relations one needs to impose between the generators of such a quantum loop group (for quivers of finite type, these are the famous Drinfeld-Serre relations). The construction is completely explicit in many cases of interest, such as Hall algebras of curves over finite fields, localized equivariant K-theoretic Hall algebras, quantum loop groups associated to symmetric Cartan matrices, and the quiver quantum toroidal algebras from mathematical physics.

Nikita Nekrasov

Lax operators of integrable systems from intersection theory

Classical (and quantum) integrable systems of Garnier (Gaudin) and elliptic Calogero-Moser type have been long expected to capture the Seiberg-Witten geometry of A-type quiver $\mathcal{N} = 2$ gauge theories in four dimensions. The traditional approach to integrable systems casts them in the Lax form. We present the construction of the associated Lax operators from the intersection theory on the moduli space of framed parabolic sheaves on the projective plane. In gauge theory terms, this is related to the studies of surface defects in (Omega-deformed) linear or cyclic quiver $\mathcal{N} = 2$ theory. Based on the works with I. Krichever and A. Grekov.

Andrei Okounkov

The Eisenstein Spectrum

This will be a report about our ongoing work with David Kazhdan devoted to the spectral analysis of the span of the Eisenstein series.

Peng Shan

Modularity for W-algebras and affine Springer fibers

We will explain a bijection between admissible representations of affine Kac-Moody algebras and fixed points in affine Springer fibers. We will also explain how to match the modular group action on the characters with the one defined by Cherednik in terms of double affine Hecke algebras, and extensions of these relations to representations of W-algebras. This is based on joint work with Dan Xie and Wenbin Yan.

Eric Vasserot

Critical convolution algebras and quantum loop groups

We introduce a new family of algebras attached to quivers with potentials, using critical K-theory and critical Borel-Moore homology, which generalize the convolution algebras attached to quivers defined by Nakajima. We give some applications to cohomological and K-theoretical Hall algebras, to shifted quantum loop groups, to Kirillov-Reshetikhin and to prefundamental representations.

Daisuke Yamakawa

Stokes local systems and wild character varieties

By the Riemann-Hilbert correspondence, regular singular meromorphic connections on a compact Riemann surface are classified by the corresponding local systems on the surface minus singular points. In the irregular singular case, connections are classified by the corresponding Stokes local systems, which is the rephrasing of the Riemann-Hilbert-Birkhoff correspondence. The wild character varieties are defined as the affine GIT quotients of the "wild representation varieties" parameterizing framed Stokes local systems, modulo the change of framing. Thus points on the wild character varieties correspond to closed orbits in the wild representation varieties. I will review our joint work with Philip Boalch on the wild character/representation varieties.

Yaping Yang

An application of power operations to quantum groups

I will talk about the quantum group associated to a quiver and an oriented cohomology theory A^*, which is constructed via the A^*-cohomology of the moduli of representations of the preprojective algebra of the quiver. I will explain power operations in cohomology theories, which in special cases are Steenrod operations in mod-p cohomology and Adams operations in K-theory. I will give one application of power operations in the setting of quiver with automorphism. In particular, it recovers Lusztig's quantum Frobenius homomorphism from the quantum group at roots of 1 to the enveloping algebra. This is based on my joint work with Gufang Zhao.

Kota Yoshioka

Moduli of stable sheaves on an elliptic surface

Bridgeland studied moduli of stable sheaves on elliptic surfaces by using relative Fourier-Mukai transforms. In particular a suitable relative Fourier-Mukai transform induces a birational morphism between a moduli space of stable sheaves and the Hilbert scheme of points. On the other hand, the bitarional morphism is not isomorphic in general, and the obtained moduli space parameterizes rank 1 sheaves with torsions. By considering various Fourier-Mukai transforms, we have a collections of moduli spaces of rank 1 sheaves with torsions. I would like to talk about relations of these moduli spaces.

Gufang Zhao

Quasimaps to quivers with potentials

This talk concerns non-compact GIT quotient of a vector space, in the presence of an abelian group action and an equivariant regular function (potential) on the quotient. We define virtual counts of quasimaps from prestable curves to the critical locus of the potential. The construction borrows ideas from the theory of gauged linear sigma models as well as recent development in shifted symplectic geometry and Donaldson-Thomas theory of Calabi-Yau 4-folds. Examples of virtual counts arising from quivers with potentials are discussed. This is based on work in progress, in collaboration with Yalong Cao.