Gauge Theory, Moduli Spaces and Representation Theory, Kashiwa 2023

In honor of the 60th birthday of Hiraku Nakajima

Dates: 27 February– 3 March 2023
Place: Lecture Hall, Kavli IPMU
Website:

`https://www.math.kyoto-u.ac.jp/~iritani/Nakajima60/Nakajima60_IPMU.htm`

Schedule on Mon.:

	10:30-11:30	lunch break	13:30-14:30	tea time	15:30-16:30
27 Mon	Nakajima 1		Saito		Zhou

Schedule on Tue. & Wed. (Changed):

	10:00-11:00	11:30-12:30	lunch break	14:00-15:00	tea time	16:00-17:00
28 Tue	Nakajima 2	Cao		Nekrasov		Finkelberg
1 Wed	Tanaka	Arbesfeld		Sun		Negut

Schedule on Thu. & Fri. (same as Mon.):

	10:30-11:30	lunch break	13:30-14:30	tea time	15:30-16:30
2 Thu	Bershtein		Ikeda		Muthiah
3 Fri	Zhao		Ito		Sala

Title:

Noah Arbesfeld	*Vertical Vafa-Witten invariants and instanton moduli spaces*
Mikhail Bershtein	*Blowup equation, isomonodromic deformations and quantum spectral problems*
Yalong Cao	*From curve counting on Calabi-Yau 4-folds to quasimaps for quivers with potentials*
Michael Finkelberg	*TBA*
Takeshi Ikeda	*Equivariant Schubert Calculus of Affine Grassmanniann of Symplectic Group*
Yukari Ito	*From 3-dimensional McKay correspondence*
Dinakar Muthiah	*Fundamental monopole operators and affine Grassmannian slices*
Hiraku Nakajima	*Coulomb branches of Orthosymplectic quiver gauge theories*
Andrei Negut	*Quivers on the torus and quantum loop groups*
Nikita Nekrasov	*TBA*
Yoshihisa Saito	*On elliptic root systems*
Francesco Sala	*Cohomological Hall algebras and affine Yangians*
Kaiwen Sun	*A survey of recent progress on blowup formulas*
Yuuji Tanaka	*A blowup formula for sheaf-theoretic virtual enumerative invariants on projective surfaces and its applications*
Yu Zhao	*The derived birational geometry of nested quiver varieties*
Zijun Zhou	*3d mirror symmetry and duality interface*

Organizers: Tomoyuki Arakawa, Naoki Genra, Hiroshi Iritani, Syu Kato, Hiraku Nakajima, Atsushi Takahashi

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Noah Arbesfeld

Vertical Vafa-Witten invariants and instanton moduli spaces

Tanaka-Thomas gave an algebro-geometric definition of Vafa-Witten invariants using moduli spaces of stable Higgs pairs on surfaces. The invariants are formed from contributions of components; S-duality implies conjectural symmetries between these contributions.

I'll explain work in preparation with M. Kool and T. Laarakker on the "vertical" (or "monopole") component, which can be regarded as a nested Hilbert scheme. Namely, we use a blow-up identity of Kuhn-Leigh-Tanaka for instanton moduli space to obtain constraints on vertical Vafa-Witten invariants predicted by G ö ttsche-Kool-Laarakker. One consequence is a formula for refined vertical invariants in rank 2.

Mikhail Bershtein

Blowup equation, isomonodromic deformations and quantum spectral problems

I will talk about the (mainly conjectural) blowup relations for Nekrasov partition functions of U(2) 4d gauge theory with adjoint matter. I will concentrate on two (related) applications of these relations, namely the spectrum of a 2-particle elliptic Calogero-Moser operator and formulas for tau functions of the isomonodromic deformation of a rank 2 linear system on a torus.

Based on joint work with P. Gavrylenko and A. Grassi.

Yalong Cao

From curve counting on Calabi-Yau 4-folds to quasimaps for quivers with potentials

I will start by reviewing a joint work with Davesh Maulik and Yukinobu Toda on relating Gromov-Witten, Gopakumar-Vafa and stable pair invariants on compact Calabi-Yau 4-folds. For non-compact CY4 like local curves, similar invariants can be studied via the perspective of quasimaps to quivers with potentials. In a joint work in progress with Gufang Zhao, we define a virtual count for such quasimaps and prove a gluing formula. Computations of examples will also be discussed.

Michael Finkelberg

TBA

Takeshi Ikeda

Equivariant Schubert Calculus of Affine Grassmanniann of Symplectic Group

We study torus equivariant cohomology of the affine Grassmannian of symplectic group. The aim of our work is to give a concrete realization of the ring and its Schubert basis in term of symmetric functions. In fact we proved that the equivariant cohomology ring of the symplectic affine Grassmannian is isomorphic to the image of the ring of Schur Q-functions with coefficient in the equivariant scalars, whose kernel is explicitly described. We also proved that some special Schubert classes are represented by Ivanov' s factorial Q-functions. These results are "affine" analogue of the corresponding results for the Lagrangian Grassmannian, and also the equivariant generalization of non-equivariant results by Lam, Schilling, and Shimozono. Our map can be also understood via folding constructions of Dynkin diagrams. The talk is based on joint work with Yusuke Nakayama and Mark Shimozono.

Yukari Ito

From 3-dimensional McKay correspondence

There are several descriptions of 3-dimensional McKay correspondence which gives a relation between a crepant resolution of a quotient singularity and the group representations. The orbifold Euler characteristics of Calabi-Yau quotients in superstring theory is a topological Euler number of a crepant resolution. However, the existence of a crepant resolution is not trivial.

In this talk, I will introduce the G-Hilbert scheme for finite groups as a crepant resolution of a quotient singularity in dimension three and the combinatorial properties.

Dinakar Muthiah

Fundamental monopole operators and affine Grassmannian slices

Affine Grassmannians are objects of central interest in geometric representation theory. For example, the geometric Satake correspondence tells us that their singularities carry representation theoretic information. In fact, it suffices to work with affine Grassmannian slices, which retain all of this information.

Recently, Braverman, Finkelberg, and Nakajima showed that affine Grassmannian slices arise as Coulomb branches of certain quiver gauge theories. Remarkably, their construction works in Kac-Moody type as well. Their work opens the door to studying affine Grassmannians for Kac-Moody groups. Unfortunately, it is difficult at present to do any explicit geometry with the Coulomb branch definition. For example, a basic feature is that affine Grassmannian slices embed into one another. However, this is not apparent from the Coulomb branch definition. In this talk, I will explain why these embeddings are necessarily subtle. Nonetheless, I will show a way to construct the embeddings using fundamental monopole operators.

This is joint work with Alex Weekes.

Hiraku Nakajima

Coulomb branches of Orthosymplectic quiver gauge theories

I will report an on-going joint project with Hanany and Finkelberg. We identify Coulomb branches of orthosymplectic quiver gauge theories with orthosymplectic bow varieties. Then we use this identification to realize closures of nilpotent orbits for SO, and more as Coulomb branches.

Andrei Negut

Quivers on the torus and quantum loop groups

Deeper structures behind BPS counting on toric Calabi-Yau 3-folds have recently been realized mathematically in terms of the quantum loop group associated to a certain quiver drawn on a torus, which is endowed with an action on the BPS vector space via crystal melting. In this talk, we identify the annihilator of the aforementioned action, thus leading to the definition of a reduced quantum loop group associated to a toric Calabi-Yau 3-fold satisfying a certain consistency condition we call "shrubbiness".

Nikita Nekrasov

TBA

Yoshihisa Saito

On elliptic root systems

In the middle of 1980's, motivated by study of singularity theories, K. Saito introduced the notion of "elliptic root systems". Roughly speaking, they are root systems with two null directions. Furthermore, he had classified elliptic root systems R with one-dimensional subspace of G of two dimensional null directions, under the assumption that the quotient affine root system R/G is reduced. In our joint work with A. Fialowski and K. Iohara, we take off the assumption above, and give the classification of the pair (R, G) with no assumption. In addition, we give an overview of the theory of elliptic root systems in this talk, and certain applications of this theory are also discussed.

Francesco Sala

Cohomological Hall algebras and affine Yangians

After a brief introduction to the theory of (2-dimensional) cohomological Hall algebras (COHAs) of quivers, curves, and surfaces, I will discuss the COHA(S, Z) of coherent sheaves on a smooth quasi-projective complex surface S set-theoretically supported on a closed subscheme Z. When S is the minimal resolution of an ADE singularity and Z is the exceptional divisor, I will describe how to characterize COHA(S, Z) via the Yangian of the corresponding affine ADE quiver. (joint project with Emanuel Diaconescu, Mauro Porta, Oliver Schiffmann, and Eric Vasserot.)

Kaiwen Sun

A survey of recent progress on blowup formulas

I give a survey of the recent progress on Nakajima-Yoshioka's blowup formulas since 2015. I will discuss the generalizations of blowup equations to various supersymmetric gauge theories in 4d, 5d, 6d that are cohomological, K-theoretic, elliptic respectively, and to refined topological string theory on local Calabi-Yau threefolds. I will also discuss various applications of blowup equations such as in the exact quantization of mirror curves and isomonodromic systems. I also discuss the relation between blowup equations and (refined) holomorphic anomaly equations which are two universal approaches to solve enumerative invariants of local Calabi-Yau threefolds.

Yuuji Tanaka

A blowup formula for sheaf-theoretic virtual enumerative invariants on projective surfaces and its applications

I'll talk about a blowup formula for sheaf-theoretic virtual enumerative invariants on projective surfaces and its applications at the level of the generating series of those invariants. For instance, we obtain blowup formulae for the generating series of the virtual Euler characteristics and virtual χ_y-genera of the moduli spaces, in which modular forms appear in the same way as in Vafa-Witten's original paper in '94, as Goettsche and Goettsche-Kool conjectured. These enable one to compute some of universal functions in the generating series of the instanton part of the Vafa-Witten invariants on a projective surface, provided they existed as Goettsche-Kool and Goettsche-Kool-Laarakker conjectured. This talk is based on joint work arXiv:2107.08155 with Nikolas Kuhn and arXiv:2205.12953 with Nikolas Kuhn and Oliver Leigh.

Yu Zhao

The derived birational geometry of nested quiver varieties

Given a quiver, Professor Nakajima introduced the quiver variety and the Hecke correspondence, which is a closed subvariety of Cartesian products of quiver varieties. In this paper, we consider two nested quiver varieties, which are the fiber products of Hecke correspondences along natural projections. We prove that after blowing up the diagonal, they are isomorphic to a quadruple moduli space which Negut observed for the Jordan quiver. In order to prove it, we introduce the recent work of Hekking on the derived blow-up and show its influence on birational geometry and enumerative geometry.

Zijun Zhou

3d mirror symmetry and duality interface

Elliptic stable envelope is introduced by Aganagic-Okounkov as a generalization of stable envelopes in equivariant elliptic cohomology. One of their important properties is that they satisfy the 3d mirror symmetry - elliptic stable envelopes of a 3d mirror pair are transpose to each other. Further observation indicates that those classes may arise from a common class, called duality interface, on the product of the mirror pair. In this talk, I will discuss such an aspect of 3d mirror symmetry. This is based on joint work in progress with H. Nakajima and A. Okounkov.