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Littlewood-Paley inequality for a diffusion satisfying the logarithmic Sobolev inequality:

Let (X_t) be a symmetric diffusion process on a space $(M, \mathcal{B}(M), \mu)$, where $\mathcal{B}(M)$ is the Borel σ-algebra and μ is a probability measure. We denote the associated Dirichlet form by $\mathcal{E}$. We assume that $\mathcal{E}$ is given as

$$\mathcal{E}(u, v) = \int_M (\nabla u, \nabla v) d\mu(x)$$

for some closed operator $\nabla\colon L^2(\mu) \to L^2(\mu; K)$ which satisfies the derivation property. Here K is a Hilbert space and $L^2(\mu; K)$ may be possibly a L^2 section of a vector bundle over M. We assume that $\mathcal{E}(1, 1) = 0$ and the following logarithmic Sobolev inequality holds: there exist $\alpha > 0$ and $\beta \geq 0$ such that

$$\int_M u^2 \log u/\|u\|_2 \mu(dx) \leq \alpha \mathcal{E}(u, u) + \beta(u, u).$$

We need another semigroup $\{\hat{T}_t\}$ in $L^2(\mu; K)$ with the generator $\hat{L}$. We assume that $|\hat{T}_t \theta|_K \leq T_t |\theta|_K$ and $\nabla L = (\hat{L} - R)\nabla$. We also impose the exponential integrability of the negative part of R. We formulate this condition as follows. Let V be a scalar function satisfying $(R(x)k, k)_K \geq V(x)|k|_K^2$. We denote the negative part of V by V_- and assume that $e^{V_-} \in L^{\infty-} = \bigcap_{p \geq 1} L^p$.

Under these conditions, we have the following theorem:

Theorem 1 *For any $1 < p < q < \infty$, there exist constants C_1, C_2, so that*

$$\|\nabla u\|_p \leq C_1 \|\sqrt{1 - L} u\|_q, \tag{1}$$

$$\|\sqrt{1 - L} u\|_p \leq C_2 (\|\nabla u\|_q + \|u\|_q). \tag{2}$$

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