Exponential convergence of Markov Processes

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Abstract: Let $\{T_t\}$ be a Markovian semigroup in $L^2(M, m)$. We also assume that its dual $\{T_t^*\}$ is Marikovian. Then $\{T_t\}$ defines a Marikovian semigroup in L^p for any $p \in [1, \infty)$. We assume that μ is an invariant probalility measeure and $T_t 1 = T_t^* 1 = 1$. We are interested in the exponential convergence rate of $T_t f$ to $\int_M f\,dm$ as $t \to \infty$. To be precise, set

$$\gamma_{p \to q} = -\limsup \frac{1}{t} \log \|T_t - m\|_{p \to q}$$

where m stands for an operator $f \mapsto m(f) = \int_M f\,dm$ and $\|\ \|_{p \to q}$ denotes an operator norm from L^p to L^q. We are interested in how $\gamma_{p \to q}$ depends on p and q.

We show that under the assumption of hyper-contractivity of the semigroup, $\gamma_{p \to q}$ does not depend on p and q (p, $q > 1$). Moreover, if we assume the symmetry, we can show that L^p spectrum of the generator are independent of $p > 1$. Without the hyper-contractivity, we can constract an example of which the spectrum depends on p. We can also discuss the convergence rate in the setting of the Zygmund space $L \log L$.

This is a joint work with Seiichiro Kusuoka.

References

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