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Non-symmetric diffusions on a Riemannian manifold

Abstract:

Let (M, g) be a complete Riemannian manifold. We deonte the Riemannian volume by $m = \text{vol}$. We consider a diffusion process generated by $\mathfrak{A} = \frac{1}{2}\triangle + b$. Here $\triangle$ is the Laplace-Beltrami operator and b is a vector field on M. We are interested in a semigroup generated by $\mathfrak{A}$ in $L^2(m)$ or $L^p(m)$. We give some sufficient conditions. In L^2 case, we can give a precise generator domain as well.

The other topic is the ultracontractivity. The ultracontractivity is well-discussed for symmetric Markovian semigroups. Necessary and sufficient conditions, e.g., Sobolev inequality and Nash inequality, etc., are known. We can extend them to non-symmetric semigroups and apply them to non-symmetric diffusions on a Riemannian manifold. If the manifold is compact, we can give an estimate of convergence rate of the fundamantal solution converging to the invariant measure.

References

[1] Shigekawa, I.: Non-symmetric diffusions on a Riemannian manifold. preprint.