Semigroups that preserve a convex set in a Banach space

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KEY WORDS: semigroup, convex set, Markov process, excessive function

MATHEMATICAL SUBJECT CLASSIFICATION: 60J25, 47D07

Abstract:

Let B be a Banach space and $\{T_t\}$ be a semigroup in B. Suppose we are given a convex set C in B. We are interested in when the semigroup preserves the convex set C, i.e., $T_tC \subseteq C$ for any $t \geq 0$. This kind of issue was discussed by Brezis-Pazy in Hilbert space case and we extend it to Banach space case.

This theorem covers the following cases:

1. positivity preserving
2. Markov property
3. L^1 contraction
4. excessive function
5. invariant function

In each case, the necessary and sufficient conditions are rather well-known but the point of the talk is that they can be treated in a unified way. We also discussed Hilber space case. In that case, the conditions are described in terms of bilinear forms.

References

[1] H. Brézis, "*Opérateurs maximaux monotones et semi-groupes de contractions dans les espaces de Hilbert*," North-Holland, Amsterdam-London, 1973.

[2] H. Brezis and A. Pazy, Semigroups of non linear contractions on convex sets, *J. Funct. Anal.*, **6** (1970), 237–281.

[3] W. Farkas, N. Jacob and R. Schilling, Feller semigroups, L^p-sub-Markovian semigroups, and applications to pseudo-differential operators with negative definite symbols, Forum Math. **13** (2001), 51–90.

[4] Z.-M. Ma, L. Overbeck and M. Röckner, Markov processes associated with semi-Dirichlet forms, *Osaka J. Math.*, **32** (1995), 97–119.

[5] E. Ouhabaz, Invariance of closed convex sets and domination criteria for semigroups, *Potential Analysis*, **5** (1996), 611–625.

[6] E. Ouhabaz, L^p contraction semigroups for vector valued functions, *Positivity*, **3** (1999), 83–93.