Automorphic Representations and Related Topics

RIMS Conference

Organizers: Atsushi Ichino (Kyoto University)
Taku Ishii (Seikei University)

Date : January 21 (Mon.), 2013 – January 25 (Fri.), 2013
Venue : Room 420
Research Institute for Mathematical Sciences
Kyoto University, Kyoto, JAPAN.
webpage : http://www.math.kyoto-u.ac.jp/~ichino/rims.html

Titles and Abstracts (As of January 10, 2013)

Jan.21 (Mon)

9:40-9:45 Opening address

9:45-10:45 **市野 篤史 Atsushi Ichino** (Kyoto University)

Title: *Periods of automorphic forms: the case of* $(\mathrm{GL}_{n+1} \times \mathrm{GL}_n, \mathrm{GL}_n)$

Abstract: Following Jacquet, Lapid and Rogawski, we define a regularized period of an automorphic form on $\mathrm{GL}_{n+1} \times \mathrm{GL}_n$ relative to GL_n, and we express it in terms of the zeta integral studied by Jacquet, Piatetski-Shapiro and Shalika. This extends the theory of the Rankin-Selberg integral representation for $\mathrm{GL}_{n+1} \times \mathrm{GL}_n$ to all automorphic forms on $\mathrm{GL}_{n+1} \times \mathrm{GL}_n$. The proof uses an idea of Lapid and Rogawski. This is a joint work with Shunsuke Yamana.

11:00-12:00 **山名 俊介 Shunsuke Yamana** (Kyushu University)

Title: *Periods of automorphic forms: the case of* $(\mathrm{U}_{n+1} \times \mathrm{U}_n, \mathrm{U}_n)$

Abstract: Following Jacquet, Lapid and Rogawski, we define a regularized period of an automorphic form on $\mathrm{U}_{n+1} \times \mathrm{U}_n$ relative to U_n and express the regularized periods of Eisenstein series in terms of Bessel periods of inducing data. Moreover, we will give a formula for the period of certain residual representations and apply it to the Gan-Gross-Prasad conjecture. This is a joint work with Atsushi Ichino.

13:30-14:30 **若槻 聡 Satoshi Wakatsuki** (Kanazawa University)

Title: *On coefficients of unipotent orbital integrals for the symplectic group of rank 2*

題目: 階数２のシンプレクティック群に関するユニポテント軌道積分の係数について

Abstract: This is a joint work with Werner Hoffmann. The geometric side of the Arthur trace formula is expressed as a linear combination of weighted orbital integrals. In the expansion, coefficients of unipotent orbital integrals are not understood in general. First, we

review some known results on coefficients for GL(2), SL(2), GL(3), and SL(3). In such the cases, the coefficients are expressed by the constant term of the Laurent expansion of the Dedekind zeta function at s=1, the special values of Hecke L-functions at s=1, and so on. Next, we mention our main result on coefficients of unipotent orbital integrals for the symplectic group of rank 2. We show that the coefficients are expressed by the constant term of the Laurent expansion of the Shintani zeta function for the space of binary quadratic forms at s=3/2 in addition to the Dedekind zeta function and Hecke L-functions. Furthermore, we explain relations between these results and stabilization.

14:45-15:45 **Nadya Gurevich** (Ben-Gurion University of the Negev)

Title: *The standard L-function of the group G_2 and its poles*

Abstract: we introduce a new family of Rankin-Selberg integrals all representing the standard L-function of degree seven of the exceptional group G2. We also discuss the location of the poles of the L-function that are governed by the poles of degenerate Eisenstein series on the family of quasi-split groups of type D4.

16:00-17:00 **Wee Teck Gan** (National University of Singapore)

Title: *The regularized Siegel-Weil formula: a second term identity*

Abstract: In joint work with Yannan Qiu and Shuichiro Takeda, we established the second term identity of the regularized Siegel-Weil formula (a la Kudla-Rallis) for general reductive dual pairs. Using this, we obtained the general Rallis inner product formula. Together with earlier work on the first term identity (by Kudla-Rallis, Ikeda, Ichino and Yamana), and the work of Yamana on the local theory of the doubling zeta integrals, one has a quite satisfactory local-global criterion for the nonvanishing of global theta liftings.

Jan. 22 (Tues)

9:45-10:45 **宮崎 琢也　Takuya Miyazaki** (Keio University)

Title: *On confluent hypergeometric functions and real analytic Siegel Eisenstein series*

題目: 合流型超幾何関数と実解析的 *Siegel Eisenstein* 級数について

Abstract: I will discuss the confluent hypergeometric functions attached to K-finite vectors of a reducible degenerate principal series of Sp(n,R). I will explain that the non-vanishing of such functions will bring us a notion of the signature defined for each irreducible submodule. In particular, when n=2, I will describe the Fourier-Jacobi expansion of a Siegel Eisenstein series attached to the signature (1,1)-submodule.

アブストラクト: Sp(n,R) の可約な退化主系列表現の K-finite ベクトルに関する合流型超幾何関数を考える。それが 0 にならない条件をみると、退化主系列の既約部分加群それぞれに”符号数”を割り振ることが出来るのがわかる。また特に次数 2 の場合、符号数 (1,1) の部分加群に関する実解析的 Eisenstein 級数の Fourier-Jacobi 展開の記述について説明する。

11:00-12:00 **宮崎 直　Tadashi Miyazaki** (Kitasato University)

Title: *Matrix coefficients of discrete series representations of SU(2,1) and SU(3,1)*

題目: *SU(2,1)* と *SU(3,1)* の離散系列表現の行列係数

Abstract: In this talk, we give expressions of matrix coefficients of discrete series representations of SU(2,1) and SU(3,1) in terms of the generalized hypergeometric series. Applying Vidunas's result to them, we can describe their aymptotic behavior, explicitly. This is a joint work with Proffessors Takahiro Hayata, Harutaka Koseki and Takayuki Oda.

アブストラクト: SU(2,1) と SU(3,1) の離散系列表現の行列係数の動径成分に対して，一般化超幾何級数を用いた表示を与える．また，この表示に Vidunas 氏の結果を適用する事で得られる行列係数の漸近挙動に関する結果についても紹介する．また，本研究は織田孝幸氏，古関春隆氏，早田孝博氏との共同研究である．(上記の講演内容は 2011 年 1 月 17〜21 日に行われた RIMS 研究集会「保型形式と関連する跡公式，ゼータ関数の研究」での織田先生，古関先生，早田先生の講演「SU(3,1) の大きな離散系列表現の再生核の明示公式」の訂正と追加にあたります．定式化や明示公式の形を大きく変更し、漸近挙動等の新しい結果を追加したものです．)

13:30-14:30 **Wen-Wei Li** (Chinese Academy of Sciences)

Title: *On the inner forms of SL(n)*

Abstract: The automorphic representations of the inner forms of SL(n) provide a good test ground for the Arthur-Langlands conjectures. They can be studied by using restriction from the corresponding inner forms of GL(n). For the cuspidal spectrum, this has been achieved by Hiraga and Saito. On the other hand, Arthur gives a blueprint to establish the multiplicity formula for the discrete spectrum in his Endoscopic Classification of Representations, which is worked out for the classical groups. In this talk, I will try to compare these two approaches and give some speculations.

14:45-15:45 **Omer Offen** (Technion-Israel Institute of Technology)

Title: *On the distinguished automorphic spectrum for* $(Sp(2n), Sp(n) \times Sp(n))$

Abstract: We introduce the notion of the automorphic spectrum of a group G distinguished by a subgroup H and discuss cases where we can provide some information on the distinguished space. For $(GL(2n), Sp(n))$ we describe the distinguished spectrum completely by recent results of Sunshuke Yamana. We further discuss joint work with Erez Lapid on the case $(Sp(2n), Sp(n) \times Sp(n))$ where we provide a partial description of the distinguished spectrum and its relation with the descent construction of Ginzburg-Rallis-Soudry.

16:00-17:00 **Erez Lapid** (The Weizmann Institute of Science)

Title: *Whittaker Fourier coefficients of certain classical groups*

Abstract: We use the descent method of Ginzburg-Rallis-Soudry to study the analogue of the Ichino-Ikeda conjecture for Whittaker coefficients and reduce it to a local conjecture. In the case of metaplectic groups we prove the local conjectute in the non-archimedean case. Joint work with Zhengyu Mao

Jan.23 (Wed.)

9:45-10:45 **石井 卓　Taku Ishii** (Seikei University)

Title: *Archimedean L-factors for standard L-functions attached to non-holomorphic Siegel modular forms of degree 2*

Abstract: Bump, Friedberg and Ginzuburg (Math. Ann. 1999) introduced a two variable zeta integral which interpolates the standard and the spinor L-functions on GSp(2) simaltaneously. We compute the archimedean part of this zeta integral by using explicit formulas for archimedean Whittaker functions. For some generic representations of GSp(2,R), we show that the arichimedean zeta integrals coincide with the product of two archimedean L-factors.

11:00-12:00 **杉山 真吾　Shingo Sugiyama** (Osaka university)

Title: *Asymptotic behaviors of means of central values of automorphic L-functions for GL(2)*

題目: *GL(2)* の保型 *L* 関数の中心値の平均の漸近的な振る舞い

Abstract: Ramakrishnan and Rogawski gave an asymptotic formula for a mean of central L-values attached to elliptic holomorphic cusp forms with prime level as the level tends to infinity. Tsuzuki proved a result similar to that of Ramakrishnan and Rogawski for Hilbert cuspidal waveforms with square free level. In this talk, we generalize Tsuzuki's asymptotic formula to Hilbert cuspidal waveforms with arbitrary level.

アブストラクト: Ramakrishnan と Rogawski は、素数レベルの楕円正則カスプ形式に付随する保型 L 関数の中心値の平均に関して、レベルを無限大にした時の漸近公式を与えた。また都築氏は square free レベルの Hilbert カスプ波動形式に対する同様の漸近公式を与えた。本講演では、都築氏の漸近公式の一般のレベルの Hilbert カスプ波動形式への一般化について話す。

13:30-14:30 **市川 尚志　Takashi Ichikawa** (Saga University)

Title: *Selberg zeta values of Schottky groups and the Mumford isomorphisms*

題目: *Schottky* 群の *Selberg* ゼータ値と *Mumford* 同型

Abstract: As a higher genus version of Ramanujan's delta function,we consider the arithmetic property of Selberg zeta values for Schottky groups. By using the Mumford isomorphism, we apply this property to showing the arithmeticity of Selberg zeta values for Schottky groups which uniformize algebraic curves defined over number fields.

アブストラクト: Ramanujan デルタ関数の高種数版として、Schottky 群の Selberg ゼータ関数の数論的性質を考察する。その応用として、代数体上定義された代数曲線を一意化する Schottky 群について、Mumford 同型を用いることにより Selberg ゼータ値の数論性を示す。

14:45-15:45 **伊藤 哲史　Tetsushi Ito** (Kyoto University)

Title: *Supercuspidal representations in the cohomology of the Rapoport-Zink space for the unitary group in three variables*

Abstract: We study the l-adic cohomology of the Rapoport-Zink space for the unitary group in three variables, and show how the local Langlands correspondence for supercuspidal representations is realized in it. We use both geometric methods such as p-adic uniformization of Shimura varieties and automorphic methods such as the structure of L-packets and A-packets. This is a report on a joint work with Yoichi Mieda (Kyoto).

16:00-17:00 **Jiu-Kang Yu** (The Chinese University of Hong Kong)

Title: *On the dimension datum of a subgroup and its applications*

Abstract: Let G be a complex reductive group. The dimension datum of a subgroup H of G is the information $\dim V^H$, for all irreducible representation V of G. It is a common technique to use the dimension datum in attempts to determine H, in particular Langlands proposed to do so in his "Beyond endoscopy" program. I will show that some expectation of Langlands is true while some other widely believed expectation is false. This is a joint work with Jinpeng An and Jun Yu.

17:00 **Orginizers**

Title: *On the next RIMS conference*: Discussion and announcement

18:00- Banquet at *Camphora*

Jan.24 (Thurs.)

9:45-10:45 **Hatice Boylan** (Max Planck Instutut für Mathematik)

Title: *Finite Quadratic Modules and Weil Representations over Number Fields*

Abstract: In the study of Hilbert, Jacobi and orthogonal modular forms of low weight over number fields it is essential to understand the representations of Hilbert modular groups or of certain two-fold central extensions. In the case of the field of natural numbers it is known that the key to the study of all representations of the modular group $SL(2, Z)$ which are interesting in the mentioned context are the Weil representations associated to finite quadratic modules. In analogy to the case of the field of rational numbers we developed a theory of finite quadratic modules and their associated Weil representations over arbitrary number fields. In this talk we report about the main features of this new theory, about interesting new phenomena arising in the general theory over arbitrary number fields, and we indicate applications to the explicit construction of automorphic forms over number fields.

11:00-12:00 **Siegfried Boecherer** (University of Tokyo, University of Mannheim)

Title: *On the growth of Fourier coefficients of Siegel modular forms*

Abstract: W.Kohnen showed that elliptic modular forms, whose Fourier coefficients satisfy a "Hecke bound" $a(n) << n^{\frac{k}{2}}$ are cuspidal and he suggested to investigate the analogue for Siegel modular forms. Meanwhile, the degree 2 case was settled by Kohnen/Martin and Mizuno independently. I will discuss the general degree case, including modular forms for congruence subgroups, where several Fourier expansions appear (joint work with S.Das, Tata

Institute).

13:30-14:30 **Andrew Booker** (University of Bristol, RIMS)

Title: *Some remarks on the GL(2) converse theorem*

Abstract: I will discuss some ways of weakening the hypotheses of the GL(2) converse theorem over number fields. In particular, I will describe a version of Weil's classical converse theorem that allows for arbitrary poles for almost all twists. If time permits, I will mention an application to Saito-Kurokawa lifts.

14:45-15:45 **Jasmin Matz** (University of Bonn)

Title: *An explicit bound for global coefficients in Arthur's trace formula for GL(n)*

Abstract: We will discuss an explicit upper bound for the so-called global coefficients appearing on the geometric side of Arthur's trace formula in the case of GL(n). These coefficients are in general left unspecified, but a better understanding of them is essential for applications. We will also discuss an anticipated application to the Weyl's law for Hecke operators on GL(n).

16:00-17:00 **Tobias Finis** (Freie Universität Berlin)

Title: *Congruence subgroups of arithmetic lattices and the limit multiplicity property*

Abstract: We study the limiting behavior of the discrete spectra of congruence subgroups of an irreducible arithmetic lattice in a semisimple Lie group G. Assuming that the subgroups in question do not contain any non-trivial central elements, one expects their spectra to converge to the Plancherel measure of G (the limit multiplicity property). We are able to prove this property for the lattices $\mathrm{SL}(n, \mathfrak{o}_F)$, where F is a number field, and obtain conditional results in the general case. The focus lies on the case of non-compact quotients, where the spectra have a continuous part.

There are two main parts of the proof, which is based on Arthur's trace formula. First, we prove some general results on congruence subgroups of arithmetic lattices and their intersections with conjugacy classes. Second, we reduce the control of the continuous spectrum to two conjectural properties of intertwining operators, one global and one local, which we can verify for the groups $\mathrm{GL}(n)$ and $\mathrm{SL}(n)$. This is joint work with Erez Lapid (Jerusalem). The second part of the argument is also joint with Werner Müller (Bonn).

Jan.25 (Fri)

9:45-10:45 **永野 中行　Atsuhira Nagano** (Waseda Unibversity)

Title: *Hilbert modular functions via the period mapping for K3 surfaces)*

題目: *K3* 曲面の周期写像を通して見たヒルベルト・モジュラー関数について

Abstract: In the classical theory of elliptic functions, there is a close relationship among elliptic curves, modular functions and the Gauss hypergeometric equation. In this talk, we give an extension of this classical story to the Hilbert modular case for $\mathrm{Q}(\sqrt{5})$ using a

family of K3 surfaces. Namely, the inverse correspondence of the period mapping for our family defines the Hilbert modular functions. Moreover, we will mention the Shioda-Inose structure of our K3 surface and the corresponding Kummer surface.

アブストラクト: 古典的な楕円関数論においては、楕円曲線族と楕円モジュラー関数及びガウスの超幾何微分方程式とが密接に結びついています。この古典理論の拡張を、ある K3 曲面族と $\mathrm{Q}(\sqrt{5})$ のヒルベルト・モジュラー関数論の関係において見いだしました。則ち、この K3 曲面族の周期写像の逆対応としてヒルベルト・モジュラー関数を実現します。更に、K3 曲面の塩田・猪瀬耕造と、対応するクンマー曲面についても触れたいと思います。

11:00-12:00 **志賀 弘典　Hironori Shiga** (Chiba University)

Title: *Explicit modular map for Abelian surfaces by way of K3 surfaces*

題目: *K3* 曲面族を用いた *Abel* 曲面族の明示的モジュラー写像

Abstract: We show an explicit modular map for the family of Abelian surfaces by considering a special family of K3 surfaces expressed in a normal form with simple parameters. We use recent results of Kumar, Clingher —Doran and Nagano together with those of Hashimoto-Murabayashi, Igusa, Rosenhein to get it.

アブストラクト: Abel 曲面族と、ホッジ構造の変形族として同値な K3 曲面族が考えられる。このような族を明示的パラメーターによって標準表示でき、周期写像の逆写像がテータ関数を用いて表示される。近年の Kumar, Clingher-Doran, Nagano らの研究と古典的な Igusa, Rosenhein, Hashimoto-Murabayashi らの研究の相互参照を主な方法とする。ある種の実乗法を持つ族へも応用される。

13:30-14:30 **森本 和輝　Kazuki Morimoto** (Osaka City University)

Title: *On theta correspondences for* $(\mathrm{GSp}_4, \mathrm{GSO}_{4,2})$

題目: $(\mathrm{GSp}_4, \mathrm{GSO}_{4,2})$ のテータ対応について

Abstract: I will talk about both local and global theta correspondences for $(\mathrm{GSp}_4, \mathrm{GSO}_{4,2})$. Using the accidental isomorphism between $\mathrm{PGU}_{2,2}$ and $\mathrm{PGSO}_{4,2}$, we can prove a relationship between Shalika periods on $\mathrm{GU}_{2,2}$ and the theta correspondence for automorphic representations of GSp_4 and $\mathrm{GU}_{2,2}$. We shall consider a similar relationship over nonarchimedean local fields.

アブストラクト: GSp(4) と GSO(4,2) の間の局所・大域テータ対応について話す。PGU(2,2) と PGSO(4,2) の間の同型を用いて、GU(2,2) の Shalika 周期と、GSp(4) と GU(2,2) の間の保型表現のテータリフトとの関係を得た。同様の問題を局所体上においても考察する。

14:45-15:45 **古澤 昌秋　Masaaki Furusawa** (Osaka City University)

Title: *On a certain simple relative trace formula for* GSp(4)

題目: GSp(4) のある *simple relative trace formula* について

Abstract: In a previous work with Martin and Shalika which is to appear in Memoirs of the AMS, we extended the fundamental lemmas to the whole spherical Hecke algebra for two relative trace formulas concerning central critical values of twists of spinor L-functions

for GSp(4). As an application of this result, assuming supercuspidality at one finite place, we established a simple version of the first relative trace formula. As a consequence, under the same assumption, we obtain an equivalence between the non-vanishing of the L-function and the non-vanishing of the Bessel period. This is a joint work with Kimball Martin at University of Oklahoma.

アブストラクト: ［Kimball Martin (University of Oklahoma) との共同研究です.］GSp(4) のスピノル L 函数の中心での特殊値に関係する二つの相対跡公式について，Martin と Shalika との共同研究で，基本補題を Hecke 環全体に拡張した．その応用として，一つの有限素点に関する supercuspidality を仮定することによって，第一の相対跡公式について，その simple version の成立を示した．その帰結として，上記の仮定のもとで，L 函数の non-vanishing と Bessel period の non-vanishing の間の関係が得られた．

16:00-17:00 **伊吹山 知義　Tomoyoshi Ibukiyama** (Osaka University)

Title: *Structures and dimensions of vector valued Jacobi forms, liftings, and conjectures of Shimura type and Harder type*

Abstract: The Harder conjecture on congruences between vector valued Siegel modular forms and elliptic modular forms is reformulated as a congruence between half-integral weight symplectic cusp forms and Eisenstein series through a conjectural Shimura type correspondence between integral weight and half-integral weight of the symplectic group of rank two proposed by the speaker. The advantage of this reformulation as well as decisive evidences and explicit examples by using explicit structure theorem of Jacobi forms of degree two will be explained.

アブストラクト: 次のうちから白馬での講演とあまり重ならないような題材について述べたい。
(1) ２次の正則ベクトル値ヤコービ形式で指数が１のものについての構造定理をウェイトが十分大のときにベクトル値のジーゲル保型形式に帰着すること。
(2) 応用として対馬によって予想されていた次元公式を証明すること。
(3) ベクトル値の半整数ウェイトジーゲル保型形式への吉田リフトを証明すること。また、類似のリフトについて予想を述べること。
(4) これを根拠に、２次ジーゲル保型形式で整数ウェイトと半整数ウェイトのものの間の志村型の対応予想の新しい版をのべること。
(5) 保型形式の合同に関する Harder 予想の半整数ウェイト版について述べ、すべての素点でのオイラー因子について合同が証明できる実例を与える。

17:00 Closing address