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Errata of
M. Hino, A. Maki, and K. Matsuura: Discrete approximation of reflected Brownian
motions by Markov chains on partitions of domains
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On page 6742, lines 8–9 from the bottom, in the calculation of the generalized inverse $A^+(\xi)$ of $A(\xi)$, the matrix $A(\xi)$ was expressed as the product of two auxiliary matrices, $\widetilde{A}(\xi)$ and a diagonal matrix $D(\xi)$ with positive diagonal entries, such that $A(\xi) = \widetilde{A}(\xi)D(\xi)$. Based on this relation, it was claimed that $A^+(\xi) = D(\xi)^{-1}\widetilde{A}^+(\xi)$, but this is incorrect. It was intended that a quantitative estimate for the correction term $c(\xi) = A^+(\xi)b(\xi)$ (see (2.9) on page 6730 for details) would be obtained via this equation. Specifically, $\widetilde{A}^+(\xi)$ is equal to the product of the transpose of $\widetilde{A}(\xi)$ and the inverse of $Q(\xi)$ appearing in Lemmas 3.4 and 3.6, where lower bounds for the minimum eigenvalue of $Q(\xi)$ were provided. However, as mentioned above, the equation $A^+(\xi) = D(\xi)^{-1}\widetilde{A}^+(\xi)$ does not hold in general. Accordingly, we provide the necessary corrections for this error and its consequences as follows.

	Error	Correction
p. 6730, ll. 8–10	We use this expression to define $A(\xi) \in \mathbb{R}^d \otimes \mathbb{R}^{N_\xi}$ whose (i,j)-th element is given by $$(\overline{\eta^{(j)}}_i - \check{\xi}_i)\frac{m(\eta^{(j)})}{m(O_\xi)}. \qquad (2.7)$$	We use this expression to define $A(\xi) \in \mathbb{R}^d \otimes \mathbb{R}^{N_\xi}$ whose (i,j)-th element is given by $$(\overline{\eta^{(j)}}_i - \check{\xi}_i)\sqrt{\frac{m(\eta^{(j)})}{m(O_\xi)}}. \qquad (2.7)$$
p. 6730, ll. −10–−8	For $\xi \in \mathcal{K}$, let $A^+(\xi)$ denote the generalized inverse matrix of $A(\xi)$ and define $c(\xi) \in \mathbb{R}^{N_\xi}$ by $$c(\xi) = A^+(\xi)b(\xi). \qquad (2.9)$$	For $\xi \in \mathcal{K}$, let $A^+(\xi)$ denote the generalized inverse matrix of $A(\xi)$ and define $c(\xi) \in \mathbb{R}^{N_\xi}$ by $$c(\xi) = (D(\xi)^{-1})A^+(\xi)b(\xi), \qquad (2.9)$$ where $D(\xi)$ is the diagonal matrix of size N_ξ whose (k,k)-th element is given by $m(\eta^{(k)})/m(O_\xi)$.
p. 6732, ll. −2–−1	Let $N_\xi^{(n)} = \#\mathcal{N}_\xi^{(n)}$ and define a $d \times N_\xi^{(n)}$ matrix $A^{(n)}(\xi)$ in the same way as (2.7). We write $A^{(n),+}(\xi)$ for the generalized inverse matrix.	Let $N_\xi^{(n)} = \#\mathcal{N}_\xi^{(n)}$ and define a $d \times N_\xi^{(n)}$ matrix $A^{(n)}(\xi)$ in the same way as (2.7). We also define a diagonal matrix $D^{(n)}(\xi)$ of size $N_\xi^{(n)}$ in the same way as (2.9). We write $A^{(n),+}(\xi)$ for the generalized inverse matrix of $A^{(n)}(\xi)$.
p. 6742, ll. 10–13	Let ξ, and let $\mathcal{N}_\xi$ be expressed as $\mathcal{N}_\xi = \{\eta^{(1)}, \dots, \eta^{(N_\xi)}\}$. Let $\widetilde{A}(\xi)$ be the $d \times N_\xi$ matrix whose (i,j)-th element is given by $$(\overline{\eta^{(j)}}_i - \check{\xi}_i)\sqrt{\frac{m(\eta^{(j)})}{m(O_\xi)}}.$$ We write $\widetilde{A}^+(\xi)$ for the generalized inverse matrix of $\widetilde{A}(\xi)$.	Delete.

	Error	Correction
p. 6742, ll. 14–16	Since $\widetilde{A}(\xi){}^t\widetilde{A}(\xi) = Q(\xi)$, Lemmas 3.4 and 3.6 imply that the rank of $\widetilde{A}(\xi){}^t\widetilde{A}(\xi)$ is equal to d. Therefore, we have $\widetilde{A}^+(\xi) = {}^t\widetilde{A}(\xi)(\widetilde{A}(\xi){}^t\widetilde{A}(\xi))^{-1}$ $= {}^t\widetilde{A}(\xi)Q(\xi)^{-1}$.	Since $A(\xi){}^tA(\xi) = Q(\xi)$, Lemmas 3.4 and 3.6 imply that the rank of $A(\xi){}^tA(\xi)$ is equal to d. Therefore, we have $A^+(\xi) = {}^tA(\xi)(A(\xi){}^tA(\xi))^{-1}$ $= {}^tA(\xi)Q(\xi)^{-1}$.
p. 6742, ll. 17–19	Let $D(\xi)$ denote the diagonal matrix of size N_ξ whose (k,k)-th element is given by $\sqrt{m(\eta^{(k)})/m(O_\xi)}$. Then, we have $A(\xi) = \widetilde{A}(\xi)D(\xi)$ and $A^+(\xi) = D(\xi)^{-1}\widetilde{A}^+(\xi)$ $= (D(\xi)^{-1})({}^t\widetilde{A}(\xi))Q(\xi)^{-1}$.	Delete.
p. 6742, ll. 20–22	Applying this equation to (2.9), we obtain that $c(\xi) = A^+(\xi)b(\xi)$ $= (D(\xi)^{-1})({}^t\widetilde{A}(\xi))Q(\xi)^{-1}b(\xi)$. If $\xi \in \mathcal{K} \setminus \partial\mathcal{K}$, the (k,i)-th element of $(D(\xi)^{-1})({}^t\widetilde{A}(\xi))$ is equal to...	Applying this equation to (2.9), we obtain that $c(\xi) = (D(\xi)^{-1})({}^tA(\xi))Q(\xi)^{-1}b(\xi)$. If $\xi \in \mathcal{K} \setminus \partial\mathcal{K}$, the (k,i)-th element of $(D(\xi)^{-1})({}^tA(\xi))$ is equal to...
p. 6743, l. 4	$(D(\xi)^{-1})({}^t\widetilde{A}(\xi))$	$(D(\xi)^{-1})({}^tA(\xi))$
p. 6746, ll. −4–−2	$\langle \nabla f(\bar{\xi}), A(\xi)c(\xi)\rangle$ $= \langle \nabla f(\bar{\xi}), A(\xi)A^+(\xi)b(\xi)\rangle$. Lemma 3.4 implies that the rank of $\widetilde{A}(\xi)$ is equal to d. Thus, the rank of $A(\xi)$ is also equal to d, and it holds that $A(\xi)A^+(\xi)b(\xi) = b(\xi)$.	$\langle \nabla f(\bar{\xi}), A(\xi)D(\xi)c(\xi)\rangle$ $= \langle \nabla f(\bar{\xi}), A(\xi)D(\xi)(D(\xi)^{-1})A^+(\xi)b(\xi)\rangle$ $= \langle \nabla f(\bar{\xi}), A(\xi)A^+(\xi)b(\xi)\rangle$. Lemma 3.4 implies that the rank of $A(\xi)$ is equal to d. Thus, it holds that $A(\xi)A^+(\xi)b(\xi) = b(\xi)$.