Lec 第3回目 ①

Poincaré duality $\Longleftrightarrow$ Residue Pairing

Residue Pairing

Assume

$W(x;-)$ is Morse

$$\mathrm{Jac}\, W(x;-)$$

$$= \bigoplus_{\xi \in \mathrm{Crit}\, W(x;-)} \Lambda_0$$

$1 \in \mathbb{C}_\xi$ $\qquad \langle 1, 1 \rangle = \dfrac{1}{\sqrt{\det \mathrm{Hess}_\xi W}}$

③

$$T_\gamma H(X) \cong \mathrm{Jac}\, W(\gamma; -)$$

$$\frac{\partial}{\partial z_i} \longmapsto \frac{\partial W}{\partial z_i}$$

$$\left(\gamma - \sum z_i \theta_i\right)$$

Thm 1

$$\left\langle \frac{\partial}{\partial z_i}, \frac{\partial}{\partial z_j} \right\rangle_{pp} = \left\langle \frac{\partial W}{\partial z_i}, \frac{\partial W}{\partial z_j} \right\rangle_{Res}$$

$$T_\gamma H(X) \cong H(X)$$

③

Sketch of the proof

Put $\mathfrak{b}=0$ for simplicity

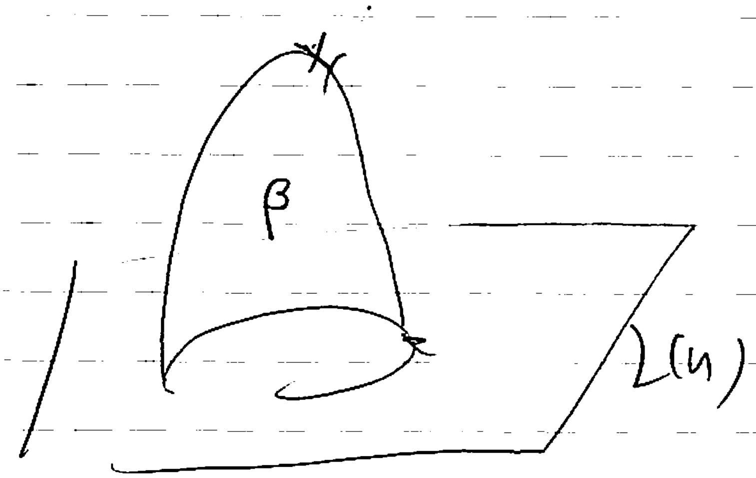
$u \in \mathrm{Int}\, P$ $\qquad$ $b \in H^1(L(u); \Lambda_0 \setminus \Lambda_+)$

$HF((L(u), b)) \neq 0$

$$
\begin{tikzcd}
Q_1 \arrow[r, "\psi"] \arrow[d, phantom, "\cap"] & \mathrm{Jac}\, W \arrow[r, "\text{Projection}"] & \mathrm{Jac}\, W_b \arrow[d, phantom, "\parallel"] \\
H(X) & & \Lambda_P
\end{tikzcd}
$$

is given as follows

$\mathcal{M}_{1,1}(\beta)$ moduli of



$$\mathcal{M}_{1,1}(\beta) \xrightarrow{(ev_{int},\ ev_0)} X \times L$$

evaluation map

dim $= n + \mu(\beta) -$

Consequ

$\deg PD\, Q_1 = (\mathrm{codim}\ Q_1) = \mu(\beta)$

Put

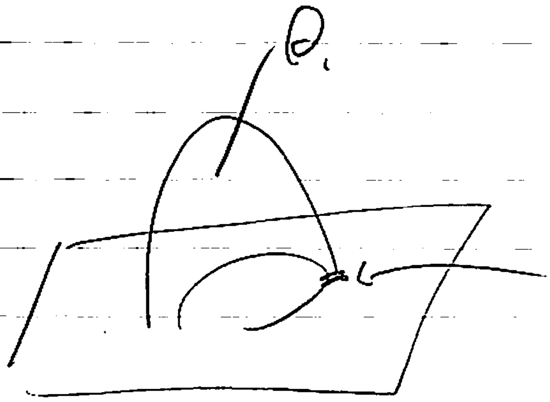
$$C(\beta,\Theta_i) = ev_0!\left(\mathcal{M}_{1,1}(\beta) \times_{ev_{int}} \Theta_i\right) \cap [L] \in \mathbb{Q}$$

$$\Psi(\Theta_i) \text{ at } (u,b) \overset{def}{=} \Psi_{u,b}(\Theta_i)$$

$$= \sum T^{\beta\cap\omega} \rho_y(\partial\beta)\, C(\beta,\Theta_i) \in \Lambda_0$$

Roughly

count



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$$H(X) \xrightarrow{\Psi_{u,b}} \Lambda_0$$

dfn

$$\Psi^*_{u,b} : \Lambda_0 \longrightarrow H(X)$$

$$\langle \Psi^*_{u,b}(pt), \Theta \rangle_{PD}$$

$$= \Psi_{u,b}(\Theta)$$

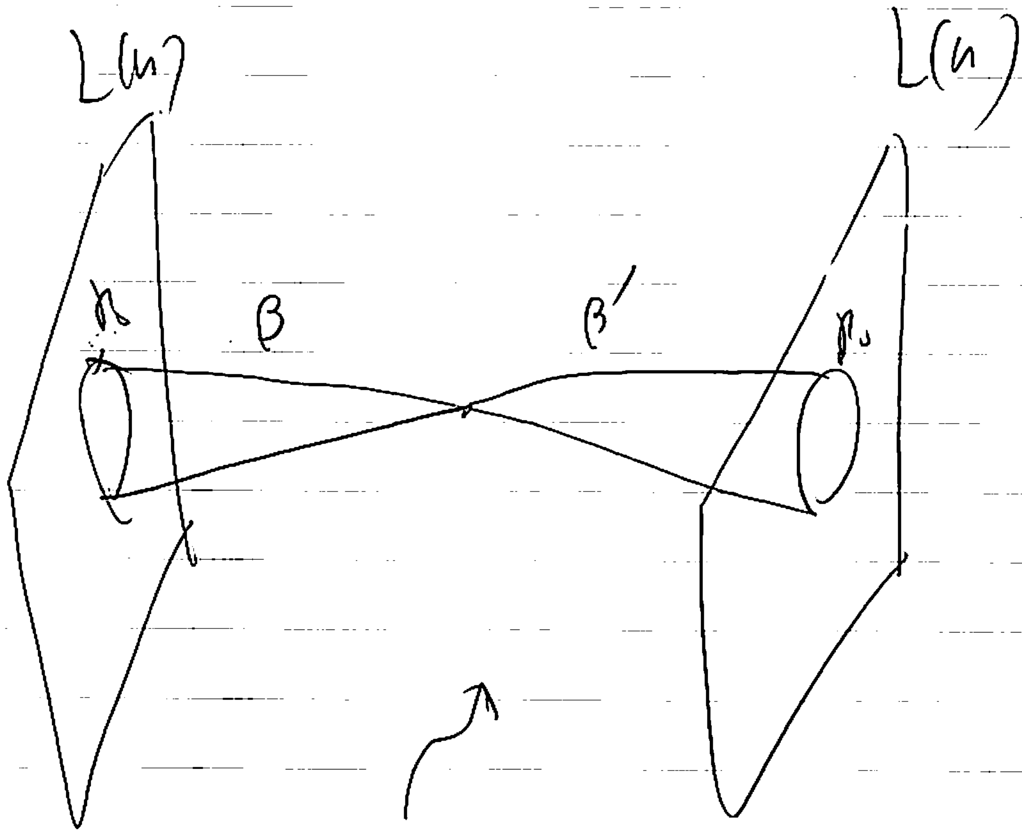
We want to calculate

$$\langle \Psi^*_{u,b}(pt), \Psi^*_{u,b}(pt) \rangle_{PD} \in \Lambda_0$$

0

Lem

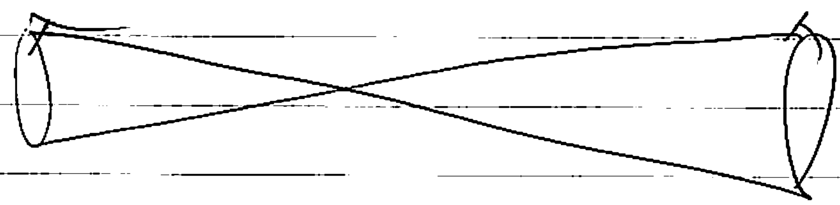
0 is given



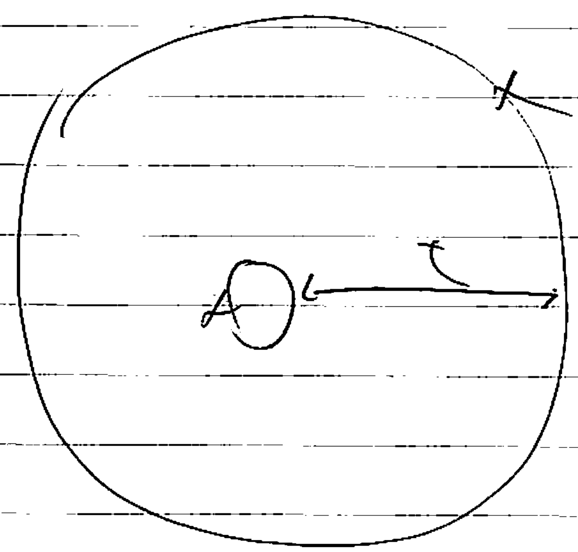
count $C_{\beta\beta'}$

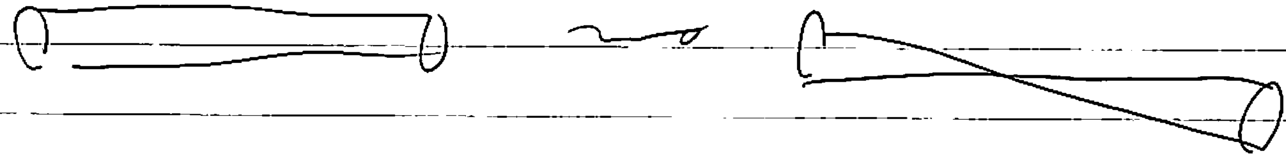
$$0 = \sum \prod^{(\beta+\beta')\cap W} P_y(\partial(\beta+\beta'))\, C_{\beta,\beta'}$$

∵) easy from def

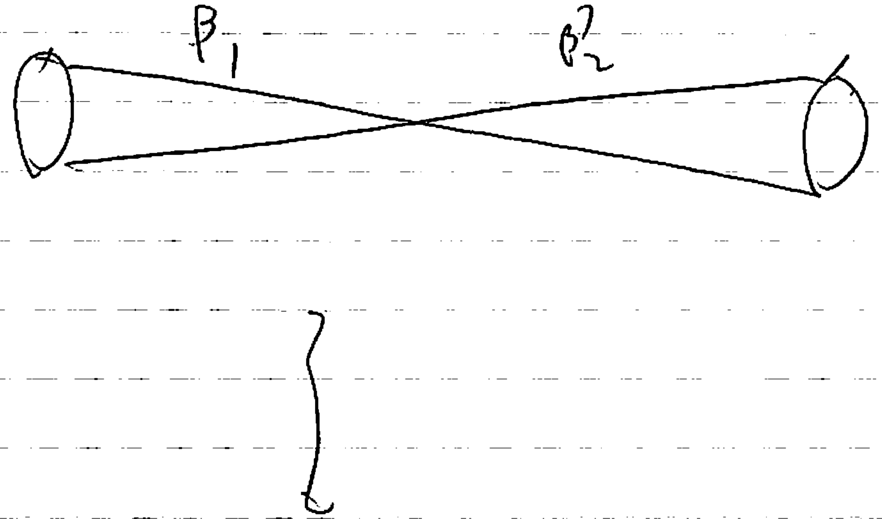


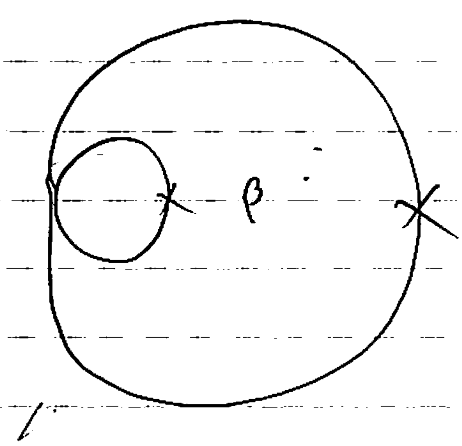
$\lim_{t\to\infty}$





Change modular param of domain

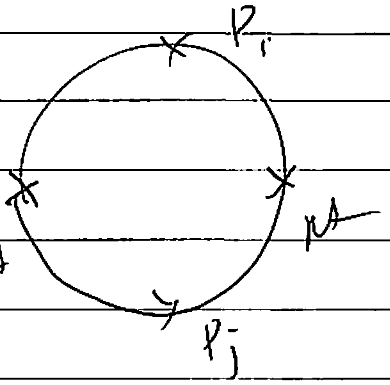




$\beta = \beta_1 + \beta_2$

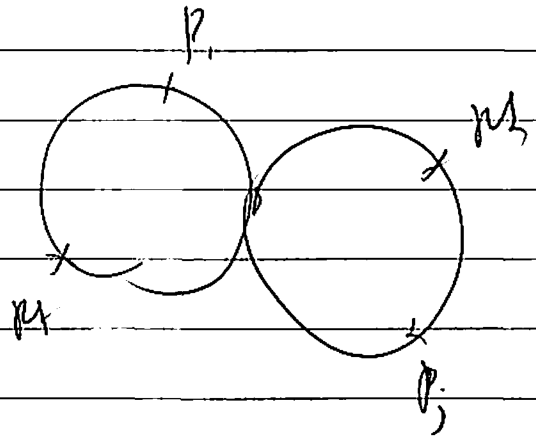
Th is

$= \sum_{(i)}$



$\delta^{(i)}$

$= \sum_{(i)}$



$\delta^{(i)}$

$$= \sum t\langle m_2(pt, p_i), m_2(pt, p_j)\rangle \delta^{(i)}$$

Lemma

$$e = \sum \langle m_2^{y}(\gamma_v, P_i), m_2^{y}(\gamma_v, P_j) \rangle g^{ij}$$

P_i : basis of $H(T^n)$

$$g^{ij} = \left(\langle P_i, P_j \rangle_{PD(T^n)} \right)^{-1}$$

m_2^{y} prod. str. on $HF(L(u), y))$

Lemma (Cho)

If W is Morse at y

then $HF(L(u), y)$ is clifford alg

ass to Hess W

Lemma ⇒ Thm (PD = Res pairing)

$$H^n(L;\Lambda_{0,nov})^{\otimes 2} \longrightarrow H(X;\Lambda_{0,nov})^{\otimes 2} \xrightarrow{PD} \Lambda_{0,nov}$$

$$\begin{array}{ccc} \Downarrow & & \\ pt,\ pt & \longmapsto & ? \end{array}$$

$$? = \sum_{I \subseteq \{1,\cdots,n\}} \langle m_2(pt, e^I), m_2(pt, e^{I^c}) \rangle$$

$$\left(\begin{array}{ll} I = \{i_1 \cdots i_k\} & e^I = e_{i_1} \cup \cdots \cup e_{i_k} \\ I^c = \{1,\cdots,n\} \setminus I & \end{array} \right)$$

$$= \sum_{I \subseteq \{1\cdots n\}} \langle m_2(e_1 \cdots e_n, e^I), m_2(e_1 \cdots e_n, e^{I^c}) \rangle$$

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$$\frac{\partial W}{\partial x_i \partial x_j} = d_i \delta_{ij}$$

$$\Rightarrow \quad m_2(e_i, e_j) = e_i e_i + d_i \delta_{ij} 1$$

$$m_2(e_1 \cdots e_n, e^I)$$

$$= d_{i_1} \cdots d_{i_k} e^{I^c} \qquad I = \{i_1 \cdots i_k\}$$

$$? = \sum_{I \subset 2^n} d_{i_1} \cdots d_{i_k} d_{j_1} \cdots d_{j_{n-k}}$$

$$\{j_1 \cdots j_{n-k}\} = \{1 \cdots n\} \setminus I$$

$$= 2^n \det \operatorname{Hess} W$$