ON LOG CANONICAL SINGULARITIES

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ABSTRACT. In this short note, we study log canonical singularities. We consider when an isolated log canonical singularity with the index one is Cohen-Macaulay or not.

CONTENTS

1. INTRODUCTION

In this paper, we will work over $\mathbb{C}$, the complex number field. Let us recall the following vanishing theorem obtained in [F3].

Theorem 1.1. *Let X be a projective variety with only log canonical singularities and L an ample line bundle. Then $H^i(X, \mathcal{O}_X(K_X)\otimes L) = 0$ for any $i > 0$.*

In Theorem 1.1, if X is log terminal, then X has only rational singularities. In particular, X is Cohen-Macaulay. Therefore, $H^j(X, L^{-1}) = 0$ for any $j < \dim X$ by the Serre duality. However, in general, $H^j(X, L^{-1}) \neq 0$ for some $j < \dim X$. It is because X is not necessarily Cohen-Macaulay. So, it is an interesting problem to consider when a log canonical singularity $P \in X$ becomes Cohen-Macaulay. In this short paper, we treat the case when $P \in X$ is an isolated log

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canonical singularity with the index one. This paper is a continuation of my paper: [F2].

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We will make use of the standard notation and definition as in [KM].

2. Preliminaries

In this section, we prove some preliminary results.

2.1. **A criterion of Cohen-Macaulayness.**

We prepare two lemmas on Cohen-Macaulayness.

Lemma 2.1. *Let X be a normal variety with an isolated singularity $P \in X$. Let $f : Y \to X$ be any resolution. If X is Cohen-Macaulay, then $R^i f_*\mathcal{O}_Y = 0$ for $0 < i < n-1$, where $n = \dim X$.*

Proof. Without loss of generality, we can assume that X is projective. We consider the following spectral sequence

$$E_2^{p,q} = H^p(X, R^q f_*\mathcal{O}_Y \otimes L^{-1}) \Rightarrow H^{p+q}(Y, f^*L^{-1})$$

for an ample line bundle L on X. By the Kawamata-Viehweg vanishing theorem, $H^{p+q}(Y, f^*L^{-1}) = 0$ for $p+q < n$. On the other hand, $E_2^{p,0} = H^p(X, L^{-1}) = 0$ for $p < n$ since X is Cohen-Macaulay. By using the exact sequence $0 \to E_2^{1,0} \to E^1 \to E_2^{0,1} \to E_2^{2,0} \to E^2 \to \cdots$, we obtain $E_2^{0,1} \simeq E_2^{2,0} = 0$ when $n \geq 3$. This implies $R^1 f_*\mathcal{O}_Y = 0$. We note that $\mathrm{Supp} R^i f_*\mathcal{O}_Y \subset \{P\}$ for any $i > 0$. Inductively, we obtain $R^i f_*\mathcal{O}_Y \simeq H^0(X, R^i f_*\mathcal{O}_Y \otimes L^{-1}) = E_2^{0,i} \simeq E_\infty^{0,i} = 0$ for $0 < i < n-1$. □

Lemma 2.2. *Let X be a normal projective n-fold and let $f : Y \to X$ be a resolution. Assume that $R^i f_*\mathcal{O}_Y = 0$ for $0 < i < n-1$. Then X is Cohen-Macaulay.*

Proof. It is sufficient to prove $H^i(X, L^{-1}) = 0$ for any ample line bundle L on X for all $i < n$ (see [KM, Corollary 5.72]). We consider the spectral sequence

$$E_2^{p,q} = H^p(X, R^q f_*\mathcal{O}_Y \otimes L^{-1}) \Rightarrow H^{p+q}(Y, f^*L^{-1}).$$

As before, $H^{p+q}(Y, f^*L^{-1}) = 0$ for $p+q < n$ by the Kawamata-Viehweg vanishing theorem. By the exact sequence $0 \to E_2^{1,0} \to E^1 \to E_2^{0,1} \to E_2^{2,0} \to E^2 \to \cdots$, we obtain $H^1(X, L^{-1}) = H^2(X, L^{-1}) = 0$ if $n \geq 3$. Inductively, we can check that $H^i(X, L^{-1}) = E_2^{i,0} \simeq E_\infty^{i,0} = 0$ for $i < n$. We finish the proof. □

Combining the above two lemmas, we obtain the next corollary.

Corollary 2.3. *Let $P \in X$ be a normal isolated singularity and $f : Y \to X$ a resolution. Then X is Cohen-Macaulay if and only if $R^i f_* \mathcal{O}_Y = 0$ for $0 < i < n-1$, where $n = \dim X$.*

Proof. We shrink X and assume that X is affine. Then we compactify X and can assume that X is projective. Therefore, we can apply Lemmas 2.1 and 2.2. □

2.2. **Basic properties of dlt pairs.** In this subsection, we prove supplementary results on dlt pairs. The following proposition generalizes [FA, 17.5 Corollary], where it was only proved that S is semi-normal and S_2.

Proposition 2.4. *Let X be a normal variety and $S+B$ a boundary $\mathbb{R}$-divisor such that $(X, S+B)$ is dlt, S is reduced, and $\llcorner B \lrcorner = 0$. Let $S = S_1 + \cdots + S_k$ be the irreducible decomposition and $T = S_1 + \cdots + S_l$ for $1 \leq l \leq k$. Then T is semi-normal, Cohen-Macaulay, and has only Du Bois singularities.*

Proof. Let $f : Y \to X$ be a resolution such that $K_Y + S' + B' = f^*(K_X + S + B) + E$ with the following properties: (i) S' (resp. B') is the strict transform of S (resp. B), (ii) $\mathrm{Supp}(S' + B') \cup \mathrm{Exc}(f)$ and $\mathrm{Exc}(f)$ are simple normal crossing divisors on Y, (iii) f is an isomorphism over any generic point of any lc center of $(X, S+B)$, and (iv) $\ulcorner E \urcorner \geq 0$. We write $S = T + U$. Let T' (resp. U') be the strict transform of T (resp. U) on Y. We consider the following short exact sequence $0 \to \mathcal{O}_Y(-T' + \ulcorner E \urcorner) \to \mathcal{O}_Y(\ulcorner E \urcorner) \to \mathcal{O}_{T'}(\ulcorner E|_{T'} \urcorner) \to 0$. Since $-T' + E \sim_{\mathbb{R}, f} K_Y + U' + B'$ and $E \sim_{\mathbb{R}, f} K_Y + S' + B'$, we have $-T' + \ulcorner E \urcorner \sim_{\mathbb{R}, f} K_Y + U' + B' + \{-E\}$ and $\ulcorner E \urcorner \sim_{\mathbb{R}, f} K_Y + S' + B' + \{-E\}$. By the vanishing theorem, $R^i f_* \mathcal{O}_Y(-T' + \ulcorner E \urcorner) = R^i f_* \mathcal{O}_Y(\ulcorner E \urcorner) = 0$ for any $i > 0$. Note that we used the vanishing theorem of Reid-Fukuda type. Therefore, we have $0 \to f_* \mathcal{O}_Y(-T' + \ulcorner E \urcorner) \to \mathcal{O}_X \to f_* \mathcal{O}_{T'}(\ulcorner E|_{T'} \urcorner) \to 0$ and $R^i f_* \mathcal{O}_{T'}(\ulcorner E|_{T'} \urcorner) = 0$ for all $i > 0$. Note that $\ulcorner E \urcorner$ is effective and f-exceptional. Thus, $\mathcal{O}_T \simeq f_* \mathcal{O}_{T'} \simeq f_* \mathcal{O}_{T'}(\ulcorner E'|_{T'} \urcorner)$. Since T' is a simple normal crossing divisor, T is semi-normal. By the above vanishing result, we obtain $Rf_* \mathcal{O}_{T'}(\ulcorner E|_{T'} \urcorner) \simeq \mathcal{O}_T$ in the derived category. Therefore, the composition $\mathcal{O}_T \to Rf_* \mathcal{O}_{T'} \to Rf_* \mathcal{O}_{T'}(\ulcorner E|_{T'} \urcorner) \simeq \mathcal{O}_T$ is a quasi-isomorphism. Apply $R\mathcal{H}om_T(\underline{\quad}, \omega^\bullet_T)$ to the quasi-isomorphism $\mathcal{O}_T \to Rf_* \mathcal{O}_{T'} \to \mathcal{O}_T$. Then the composition $\omega^\bullet_T \to Rf_* \omega^\bullet_{T'} \to \omega^\bullet_T$ is a quasi-isomorphism by the Grothendieck duality. By the vanishing theorem (see, for example, [F3, Lemma 5.1]), $R^i f_* \omega_{T'} = 0$ for $i > 0$. Hence, $h^i(\omega^\bullet_T) \subseteq R^i f_* \omega^\bullet_{T'} \simeq R^{i+d} f_* \omega_{T'}$, where $d = \dim T = \dim T'$.

Therefore, $h^i(\omega_T^\bullet) = 0$ for $i > -d$. Thus, X is Cohen-Macaulay. This argument is the same as the proof of Theorem 1 in [K2]. Since T' is a simple normal crossing divisor, T' has only Du Bois singularities. The quasi-isomorphism $\mathcal{O}_T \to Rf_*\mathcal{O}_{T'} \to \mathcal{O}_T$ implies that T has only Du Bois singularities (cf. [K1, Corollary 2.4]). Since the composition $\omega_T \to f_*\omega_{T'} \to \omega_T$ is an isomorphism, we obtain $f_*\omega_{T'} \simeq \omega_T$. By the Grothendieck duality, $Rf_*\mathcal{O}_{T'} \simeq R\mathcal{H}om_T(Rf_*\omega_{T'}^\bullet, \omega_T^\bullet) \simeq R\mathcal{H}om_T(\omega_T^\bullet, \omega_T^\bullet) \simeq \mathcal{O}_T$. So, $R^if_*\mathcal{O}_{T'} = 0$ for all $i > 0$. □

We obtained the following vanishing theorem in the proof of Proposition 2.4. It plays a crucial role in Section 4.

Corollary 2.5. *Under the notation in the proof of* Proposition 2.4*,* $R^if_*\mathcal{O}_{T'} = 0$ *for any* $i > 0$ *and* $f_*\mathcal{O}_{T'} \simeq \mathcal{O}_T$*.*

Lemma 2.6. *Let* (X, D) *be a dlt pair. Assume that* $f : Y \to X$ *is a small* $\mathbb{Q}$*-factorialization. Then* $(Y, \widetilde{D})$ *is dlt, where* $\widetilde{D}$ *is the strict transform of* D *on* Y*.*

Proof. By the definition of dlt pair, every generic point of lc center of (X, D) is contained in the smooth locus of X. Thus, f is an isomorphism over every generic point of lc center of (X, D). Therefore, $(Y, \widetilde{D})$ is dlt. □

3. Dlt pairs with trivial log canonical divisors

This section is a supplement to [F2, Section 2]. See also [F1, Section 2]. We introduce a new invariant for dlt pairs with trivial log canonical divisors.

Definition 3.1. Let (X, D) be a dlt pair such that $K_X + D \sim 0$. We put

$$\widetilde{\mu} = \widetilde{\mu}(X, D) = \min\{\dim W \mid W \text{ is an lc center of } (X, D)\}.$$

It is related to the invariant μ, which was defined in [F2]. See 4.11 below.

We use the MMP with scaling as in the proof of Lemma 3.4 below. Then we can prove [F2, Proposition 2.4] for $\mathbb{Q}$-factorial dlt pairs. Therefore, we obtain the following proposition.

Proposition 3.2. *Let* (X, D) *be a* $\mathbb{Q}$*-factorial dlt pair such that* $K_X + D \sim 0$*. Let* W *be any minimal lc center of* (X, D)*. Then* $\dim W = \widetilde{\mu}(X, D)$*. Moreover, all the minimal lc centers of* (X, D) *are birational each other.*

Remark 3.3. In Proposition 3.2, let C be an lc center of (X, D). Then there exists an effective $\mathbb{Q}$-divisor Δ on C such that (C, Δ) is klt. So, we can take a small $\mathbb{Q}$-factorialization $\widetilde{C} \to C$. By combining Lemma 2.6 with the above fact, non-$\mathbb{Q}$-factoriality of C causes no troubles when we investigate C.

The next lemma is new. We will use it in Section 4.

Lemma 3.4. *Let (X, D) be a $\mathbb{Q}$-factorial dlt n-fold such that $K_X + D \sim 0$. Assume that $D \neq 0$. Then there exists an irreducible component D_0 of D such that $h^i(X, \mathcal{O}_X) \leq h^i(D_0, \mathcal{O}_{D_0})$ for any i.*

Proof. By the assumption, $K_X + (1-\varepsilon)D$ is not pseudo-effective for $0 < \varepsilon \ll 1$. Let H be an ample divisor such that $K_X + (1-\varepsilon)D + \delta H$ is not pseudo-effective for $0 < \delta \ll \varepsilon$ and $K_X + (1-\varepsilon)D + H$ is nef and klt. Apply the MMP with scaling. Then we obtain a sequence of divisorial contraction and log flips:

$$X = X_0 \dashrightarrow X_1 \dashrightarrow \cdots \dashrightarrow X_k,$$

and an extremal Fano contraction $\varphi : X_k \to Z$. By the construction, there is an irreducible component D_0 of D such that the strict transform D'_0 of D_0 on X_k dominates Z. Since X and X_k have only rational singularities, we have $h^i(X, \mathcal{O}_X) = h^i(X_k, \mathcal{O}_{X_k})$ for any i. Since $R^i\varphi_*\mathcal{O}_{X_k} = 0$ for any $i > 0$, we have $h^i(X_k, \mathcal{O}_{X_k}) = h^i(Z, \mathcal{O}_Z)$ for any i. Since D_0 and Z have only rational singularities, $h^i(Z, \mathcal{O}_Z) \leq h^i(D_0, \mathcal{O}_{D_0})$ for any i. Therefore, we have the desired inequalities $h^i(Z, \mathcal{O}_Z) \leq h^i(D_0, \mathcal{O}_{D_0})$ for any i. □

Example 3.5. Let $X = \mathbb{P}^2$ and D be an elliptic curve on $X = \mathbb{P}^2$. Then (X, D) is dlt and $K_X + D \sim 0$. In this case, $h^1(X, \mathcal{O}_X) = 0 < h^1(D, \mathcal{O}_X) = 1$.

By Proposition 3.2, the proof of Lemma 3.4, and [GHS], we obtain the next proposition.

Proposition 3.6. *Let (X, D) be a $\mathbb{Q}$-factorial dlt pair such that $K_X + D \sim 0$. Assume that $\widetilde{\mu}(X, D) = 0$. Then $h^i(X, \mathcal{O}_X) = 0$ for any $i > 0$. Moreover, X is rationally connected.*

4. Log canonical singularities

In this section, we consider when an isolated log canonical singularity with the index one is Cohen-Macaulay or not.

4.1. Let $P \in X$ be an n-dimensional isolated lc singularity with the index one. We assume that $n \geq 3$ since normal surfaces are Cohen-Macaulay by the definition. By the algebraization theorem, we always assume that X is an algebraic variety in this paper. Assume that $P \in X$ is not lt. We consider a resolution $f : Y \to X$ such that (i) f is an isomorphism outside $P \in X$, and (ii) $f^{-1}(P)$ is a simple normal crossing divisor on Y. In this setting, we can write $K_Y = f^*K_X + F - E$, where F and E are both effective Cartier divisors without common irreducible components. In particular, E is a reduced simple normal crossing divisor on Y.

Lemma 4.2. *The cohomology group $H^i(E, \mathcal{O}_E)$ is independent of f for any i.*

Proof. Let $f' : Y' \to X$ be another resolution with $K_{Y'} = f'^*K_X + F' - E'$ as in 4.1. By the weak factorization theorem (see [M, Theorem 5-4-1] or [AKMW, Theorem 0.3.1(6)]), we can assume that $\varphi : Y' \to Y$ is a blow-up whose center $C \subset \mathrm{Supp}(E + F)$ is smooth, irreducible, and transversal to $\mathrm{Supp}(E + F)$. Thus, we can directly check that $H^i(E, \mathcal{O}_E) \simeq H^i(E', \mathcal{O}_{E'})$ for any i. □

4.3. Let Γ be the dual graph of E and $|\Gamma|$ the topological realization of Γ. Note that the vertices of Γ correspond to the components E_i, the edges correspond to $E_i \cap E_j$, and so on, where $E = \sum_i E_i$ is the irreducible decomposition of E. More precisely, E defines a conical polyhedral complex Δ (see [KKMS, Chapter II, Definition 5]). By [KKMS, p.70 Remark], we get a compact polyhedral complex Δ_0 from Δ. The dual graph Γ of E is nothing but this compact polyhedral complex Δ_0. Therefore, we obtain the following lemma.

Lemma 4.4. *The dual graph Γ is well defined and $|\Gamma|$ is independent of f.*

Proof. As we explained above, the well-definedness of Γ is in [KKMS, Chapter II]. By the weak factorization theorem (see [M, Theorem 5-4-1] or [AKMW, Theorem 0.3.1(6)]), we can easily check that the topological realization $|\Gamma|$ does not depend on f. □

4.5. The next conjecture follows from the special termination theorem in dimension n. So, it is a consequence of the LMMP in dimension $< n - 1$.

Conjecture 4.6 ($\mathbb{Q}$-factorial dlt modification)**.** *Let $P \in X$ be an isolated n-dimensional lc singularity with the index one. Then there exists a proper birational morphism $f : Y \to X$ such that $K_Y + D = f^*K_X$ and (Y, D) is a $\mathbb{Q}$-factorial dlt pair.*

4.7. From now on, we assume that Conjecture 4.6 holds true. Let $f : Y \to X$ be a proper birational morphism as in Conjecture 4.6. Then we have $0 \to \mathcal{O}_Y(-D) \to \mathcal{O}_Y \to \mathcal{O}_D \to 0$. By the vanishing theorem, we obtain $R^if_*\mathcal{O}_Y(K_Y) = 0$ for any $i > 0$. Therefore, $R^if_*\mathcal{O}_Y \simeq R^if_*\mathcal{O}_D \simeq H^i(D, \mathcal{O}_D)$ for any $i > 0$. By applying Corollary 2.5, we can construct a resolution $g : V \to Y$ such that $K_V + E - F = g^*(K_Y + D) = h^*K_X$, where F and E are both effective Cartier divisors without common irreducible components, $h = f \circ g$, g is an isomorphism outside D, $R^ig_*\mathcal{O}_E = 0$ for any $i > 0$, and $g_*\mathcal{O}_E \simeq \mathcal{O}_D$. Therefore, $H^i(D, \mathcal{O}_D) \simeq H^i(E, \mathcal{O}_E)$ for any i. So, we obtain the next proposition.

Proposition 4.8. *Assume that* Conjecture 4.6 *holds. Let* $f : Y \to X$ *be a resolution as in* 4.1. *Then* $R^if_*\mathcal{O}_Y \simeq H^i(E, \mathcal{O}_E)$ *for any* $i > 0$. *Therefore,* $P \in X$ *is Cohen-Macaulay, equivalently,* $P \in X$ *is Gorenstein, if and only if* $H^i(E, \mathcal{O}_E) = 0$ *for* $0 < i < n - 1$.

Proof. It is a direct consequence of Lemma 4.2 and Corollary 2.3. □

Remark 4.9. In 4.7, $(K_Y + D)|_D = K_D \sim 0$. Therefore, $H^{n-1}(D, \mathcal{O}_D)$ is dual to $H^0(D, \mathcal{O}_D)$, where $n = \dim X$. So, $R^{n-1}f_*\mathcal{O}_Y \simeq \mathbb{C}(P)$. Thus, $P \in X$ is not a rational singularities.

Remark 4.10. The isomorphisms $R^if_*\mathcal{O}_Y \simeq H^i(E, \mathcal{O}_E)$ for any $i > 0$ and $R^{n-1}f_*\mathcal{O}_Y \simeq \mathbb{C}(P)$, where $f : Y \to X$ is a resolution as in 4.1, should be proved without assuming Conjecture 4.6.

4.11. Let $P \in X$ be an isolated lc singularity that is not lt. Assume that there is a proper birational morphism $f : Y \to X$ such that $K_Y + E = f^*K_X$ and (Y, E) is $\mathbb{Q}$-factorial dlt. We define

$$\mu = \mu(P \in X) = \min\{\dim W \mid W \text{ is an lc center of } (Y, E)\}.$$

This invariant μ was first introduced in [F2]. By Proposition 3.2, any minimal lc center of (Y, E) is μ-dimensional and all the minimal centers are birational each other.

Now, the following theorem is not difficult to prove.

Theorem 4.12. *Assume that* Conjecture 4.6 *holds. We assume* $\mu(P \in X) = 0$. *Then* $H^i(E, \mathcal{O}_E) \simeq H^i(|\Gamma|, \mathbb{C})$. *Therefore,* $P \in X$ *is Cohen-Macaulay, equivalently,* $P \in X$ *is Gorenstein, if and only if*

$$H^i(|\Gamma|, \mathbb{C}) = \begin{cases} \mathbb{C} & \text{for } i = 0, n-1, \\ 0 & \text{otherwise.} \end{cases}$$

Note that $|\Gamma|$ *is oriented and* $|\Gamma|$ *has no boundaries.*

Proof. We use the spectral sequence in 4.13 to calculate $H^i(E, \mathcal{O}_E)$. By Lemma 3.4 and Proposition 3.6, $H^q(E^{[p]}, \mathcal{O}_{E^{[p]}}) = 0$ for any $q > 0$. Therefore, we obtain $H^i(E, \mathcal{O}_E) \simeq H^i(|\Gamma|, \mathbb{C})$ for any i. □

4.13. Let E be a simple normal crossing variety and $E=\sum_i E_i$ the irreducible decomposition. We put $E^{[0]}=\coprod_i E_i$, $E^{[1]}=\coprod_{i,j}(E_i\cap E_j)$, $\cdots$, $E^{[p]}=\coprod_{i_0,\cdots,i_p}(E_{i_0}\cap\cdots\cap E_{i_p})$, $\cdots$. Let $a_p: E^{[p]}\to E$ be the obvious map. Then it is well known that $(a_0)_*\mathcal{O}_{E^{[0]}}\to(a_1)_*\mathcal{O}_{E^{[1]}}\to\cdots\to(a_p)_*\mathcal{O}_{E^{[p]}}\to\cdots$ is a resolution of $\mathcal{O}_E$. By taking the associated hypercohomology, we obtain a spectral sequence $E_1^{p,q}=H^q(E^{[p]},\mathcal{O}_{E^{[p]}})\Rightarrow H^{p+q}(E,\mathcal{O}_E)$.

We close this section with the following obvious two propositions.

Proposition 4.14. *By the above spectral sequence, $P\in X$ is Cohen-Macaulay implies that $H^1(|\Gamma|,\mathbb{C})=0$.*

Proof. By the spectral sequence in 4.13, it is easy to see that $H^1(|\Gamma|,\mathbb{C})\neq 0$ implies $H^1(E,\mathcal{O}_E)\neq 0$. □

Proposition 4.15. *Let $P\in X$ be an n-dimensional isolated lc singularity with the index one. If $P\in X$ is Cohen-Macaulay, then $\chi(\mathcal{O}_E):=\sum_i(-1)^i h^i(E,\mathcal{O}_E)=1+(-1)^{n-1}=\sum_{p,q}(-1)^{p+q}\dim E_1^{p,q}$.*

Remark 4.16. Tsuchihashi's cusp singularities give us many examples of three dimensional index one isolated lc singularities with $\mu=0$ that are not Cohen-Macaulay.

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