Investigations of moduli of stab. bundles on surf.
j.w Nicole Mestro.

X smooth proj surf

fix r, c_1, c_2 $c_1 \in NS(X)$

$M = M(r, c_1, c_2)$ = moduli sp of (semi) stab. vect bundles on X with rank $= r$ c_1, c_2. given.

General properties

r, c_1 fix. $c_2 \gg 0 \Rightarrow M$ is good i.e. generically smooth of expected dim. and irreducible.

Bogomolov-Gieseker inequality $c_2 - 2\frac{r-1}{r}c_1^2 < 0$

$\Rightarrow M = \phi$.

Quest. What happens intermidiates value c_2?
Can we understand in some case?

Mestrano 1997.

$\exists$ examples $X^2 \subset \mathbb{P}^3$ deg ≥ 27 where

$M_X(2,1,c_2)$ is not irreducible.

(at least one good and one not good comp)

~~Our subject~~:

Recall if X is Calabi-Yau (K3 or Abelian)

$\Rightarrow$ (Mukai, ...) M is $\left| \begin{array}{l} \text{a smooth symplectic variety.} \\ \text{irreducible (Yoshioka).} \end{array} \right.$

This applies to deg 4 hypersurf in $\mathbb{P}^3$.

Our Situation, $X \subseteq \mathbb{P}^3$. deg $X = 5$. smooth, very general.

$\Rightarrow K_X = \mathcal{O}_X(1)$

$\mathrm{Pic}(X) = \mathbb{Z} \cdot \mathcal{O}_X(1) \rightarrow$ "Stability" is well-def

~~rank~~

rank $r=2$. $c_1 = \mathcal{O}_X(1)$ i.e. $\det(E) \cong \mathcal{O}_X(1)$

slope Semistability implies slope stability.

$\Rightarrow$ all points of $M_X(2,1,c_2)$ are stable.

$\Rightarrow H^0(\mathrm{End}(E)) = \mathbb{C}$.

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1^{st} quest. When is the moduli good?

2^{nd} quest. What about irreduciblity?

Plan: 1. Smoothnesss of M,
disscussion of $c_2 \le 9$.

2. Deformation to the boundary, irreducibility.

3. The case $c_2 = 10$.

$X \subset \mathbb{P}^3$ $\deg X = 5$. $K_X = \mathcal{O}_X(1)$ $h^1(\mathcal{O}_X(n)) = 0$ $(\forall n)$
$\mathrm{Pic}(X) \simeq \mathbb{Z} \cdot \mathcal{O}_X(1)$

$$\chi(\mathcal{O}_X(n)) = \frac{5}{2}(n^2 - n) + 5.$$

fix $c_2 \in \mathbb{N}$.

$M =$ the moduli space ~~of stab~~ bundle E
with rank $= 2$, $\det E = \mathcal{O}_X(1)$, $c_2(E) = c_2$.
$\Rightarrow E^{\vee} = E(1)$

$\chi(E(n)) = 5n^2 + 10 - c_2$ (! if $c_2 = 0 \Rightarrow \chi = 5n^2$)

$\boxed{\deg E(n) = 2n+1}$

$H^0(\mathbb{P}^3, \mathcal{O}(n)) \twoheadrightarrow H^0(X, \mathcal{O}(n))$ $\forall n$.
$\xrightarrow{\sim}$ $(n \le 4)$

Expected dimensions:

$$E^\vee \otimes E = \mathrm{End}(E) = \underset{\text{trace free end.}}{\mathrm{End}^0(E)} \oplus \mathcal{O}_X 1_E$$

deformation & obstruction are given by.

$$H^1(X, \mathrm{End}^0(E)), \quad \& \quad H^2(X, \mathrm{End}^0(E)).$$

Kuranishi theory. the moduli sp can be locally viewed as the set of the map.

$$H^1(\mathrm{End}^0(E)) \to H^2(\mathrm{End}^0(E)).$$

$$\dim_E(M) \geq h^1(\mathrm{End}^0(E)) - h^2(\mathrm{End}^0(E)) = \underbrace{-\chi(\mathrm{End}^0(E))}_{\text{expected dim.}}$$

<u>Def</u> M is good at E if $\dim_E(M) = -\chi(\mathrm{End}^0(E))$

Notice if $h^2(\mathrm{End}^0(E)) = 0 \Rightarrow M$ is good and smooth at E.

An irr comp of M is good $\Leftrightarrow$ $h^2(\mathrm{End}^0(E)) = 0$ at general pt.

In K-theory
$$E \sim \mathcal{O} \oplus \mathcal{O}_X(1) - c_2(pt)$$
$$E^\vee \sim \mathcal{O} \oplus \mathcal{O}(-1) - c_2(pt)$$
$$E \otimes E^\vee \sim \mathcal{O} \oplus \mathcal{O}(-1) \oplus \mathcal{O} \oplus \mathcal{O}(1) - 4c_2(pt)$$
$$\mathrm{End}^0(E) \sim \mathcal{O} \oplus \mathcal{O}(-1) \oplus \mathcal{O}(1) - 4c_2(pt)$$
$$\chi(\mathrm{End}^0(E)) = \frac{5}{2}((-1)^2 - (-1)) + 5 + 5 + \frac{5}{2}((1)^2 - 1) + 5 - 4c_2$$
$$= 20 - 4c_2.$$

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$E^\vee \otimes K = E \otimes \mathcal{O}(-1) \otimes \mathcal{O}(1) = E.$

$\Rightarrow$ Expected dim $= 4c_2 - 20$.

Serre duality : use $K_X \sim \mathcal{O}_X(1)$

$$H^i(E(u)) \simeq H^{2-i}(E(-u))^\vee.$$

for $\mathrm{End}^0(E) \sim \mathrm{End}^0(E)^\vee$

$$H^i(\mathrm{End}^0(E)) \simeq H^{2-i}(\mathrm{End}^0(E) \otimes K_X)^\vee$$
$$\simeq H^{2-i}(\mathrm{End}^0(E)(1))^\vee.$$

in any case $H^2(\mathrm{End}^0(E))$ is dual to $\mathrm{Hom}^{\varphi=0}(E, \underset{E(1)}{\overset{\shortparallel}{E \otimes K_X}})$.

$\Rightarrow$ So an element of the dual of the space of obstructions may be seen as a Higgs field $\varphi : E \to E \otimes K_X$, $\mathrm{Tr}\,\varphi = 0$

(E, φ) is called a "Hitchin pair".

in particular.

$\mathrm{Tr}(\varphi) = 0 \Rightarrow \varphi$ has a spectral surf.

In our case $K_X = \mathcal{O}_X(1)$

$$\begin{array}{ccc} Z & \subseteq & \mathrm{Tot}(K_X) \\ & \underset{\text{2:1 covering}}{\searrow} & \downarrow \\ & & X \end{array}$$

Z is given by $T^2 - \det(\varphi)$.

$$\det(\varphi) \in H^0(K_X^{\otimes 2}) = H^0(\mathcal{O}_X(2)).$$

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Strategy to understand if M is good.

to try to bound the dimensions

the space of co-obstructed ~~[illegible]~~ bundles $\{(E,\varphi)\}$.

if we can show $\dim\{(E,\varphi)\} < 4c_2 - 20$.

$\Rightarrow$ all irr comp are good.

What can φ look like ?

(1) $\det(\varphi)=0$, φ is nilpotent ($\varphi^2=0$). will see

then φ comes from an exact seq.

$$(*) \qquad 0 \to \mathcal{O}_X \to E \to \mathcal{I}_P(1) \to 0 \qquad |P| = c_2.$$

via $E \to \mathcal{I}_P(1) \xrightarrow{\psi} \mathcal{O}_X(1) \to E(1)$

(2) $\det\varphi = \alpha^2 \in H^0(\mathcal{O}_X(2))$ for $\alpha \in H^0(\mathcal{O}_X(1))$.

The spectral cover Z is reducible a union of $(T-\alpha)(T+\alpha)$

In this case one can see $\exists$ exact seq(*)

(3)

$\det(\varphi)$ is not a square, it defines $R \subset X$.

reduced div

in $|\mathcal{O}_X(2)|$.

$Z \to X$ is ramified cover

ramified along R.

<u>Thm</u> M is good for $c_2 \geq 10$. not good for $c_2 \leq 9$.

<u>proof</u> by knowing the dimensions of the case (i) (ii) and (iii)

Yesterday

$\{ E \mid {}^{\exists}\varphi : E \to E(1) \}$

(i) φ is nilpotent

(ii) $\underset{\substack{\Uparrow \\ H^0(\mathcal{O}_X(2))}}{\det(\varphi)} = \alpha^2 \qquad \alpha \in H^0(X, \mathcal{O}_X(1))$

(iii) $\det(\varphi)$ is not square $\leftrightarrow$ $R \in |\mathcal{O}_X(2)|$ reduced div.

$\mathrm{Tot}(K_X) \supset \underset{\sim}{Z} =$ 2:1 covering of X branched over R defined by $T^2 - \det(\varphi)$

spectral covering of φ

Z is irreducible.

$\tilde{Z} \to Z$ resolution. ($\mathbb{Z}/2$ equiv)

out side of codim 2 subset $(E, \varphi) \leftrightarrow a$.

line bundle $\mathcal{L}$ on Z and $E = \pi_* \mathcal{L}$.

$\hat{\pi} : \tilde{Z} \to X$, extend $\mathcal{L}$ to $\tilde{Z}$,

$E = \hat{\pi}_*(\tilde{\mathcal{L}})^{**}$

(E, φ) is determined by $\hat{\mathcal{L}} \in \mathrm{Pic}(\tilde{Z})$

<u>dim count</u> $\dim\{ E \in \text{case (iii)} \}$

$\leq \{ \underbrace{\text{Space of } R \in |\mathcal{O}_X(2)|}_{9} \} + \max\dim \mathrm{Pic}^0(\tilde{Z})$

for general R, $\mathrm{Pic}^0(\tilde{Z}) = 0$

Take a curve $C = U \cap X$ U = general plane in $\mathbb{P}^3$.

$$\begin{array}{l} Z_C \\ \downarrow 2:1 \\ R \cap C \subset C \end{array} \qquad \mathrm{Pic}^0(\tilde{Z}) \hookrightarrow \mathrm{Prym}(Z_C/C)$$

10 points genus 6

Z_C has genus 16.
$\mathrm{Prym}(Z_C/C)$ dim 10.

total dim (case (iii)) ≤ 18

$c_2 \geq 10$ exp. dim ≥ 20. this case doesn't contribute to a general pt.
$4c_2 - 20$.

Case (i) φ has factor as.

$$E \to \mathcal{I}_p(1) \to \mathcal{O}(1) \to E(1)$$

$$(*) \qquad 0 \to \mathcal{O}_X \to E \to \mathcal{I}_p(1) \to 0.$$

Case (ii) でも (*) の exact seq があることがわかる.

$$M \supset V = \{ E \mid h^0(E) \geq 1. \}$$

For $V \ni E$, we have at least one exact seq (*).

This figures in O'Grady's approach:
Nisse for bundles on quintic (1995)

Understanding bundles in V

This fundamental point about the Serre constr.
is that P is a l.c.i. subscheme which satisfies
the Cayley- conditions $CB(n)$
for $CB(2)$

$$\xi \in \mathrm{Ext}^1(\mathcal{I}_{P/X}(1), \mathcal{O}_X) = H^1(\mathcal{I}_P(2))^\times$$
ext class of (*).

$$0 \to \mathcal{I}_P(2) \to \mathcal{O}_X(2) \to \mathcal{O}_P(2) \to 0.$$

$$0 \to H^0(\mathcal{I}_P(21)) \to H^0(\mathcal{O}_X(21)) \to H^0(\mathcal{O}_P(21)) \to H^1(\mathcal{I}_P(2)) \to 0.$$

$H^1(\mathcal{I}_P(1))^*$ may be seen as the space of funct:

$$H^0(\mathcal{O}_P(21)) \xrightarrow{\xi} \mathbb{C} \quad \text{s.t. } \xi|_{H^0(\mathcal{O}_X(21))} = 0.$$

in K-theory $E \sim \mathcal{O}_X + \mathcal{O}_X(1) - |P| \cdot pt$.

$$\Rightarrow c_2(E) = P.$$

Cayley-B_____ the conditions to insure that:
E defined by ξ is loc. free;
$\forall P' \subset P$ colength 1 subscheme $\xi I_{P'/P} \neq 0$.

$$H^0(\mathcal{O}_P(21)) \xrightarrow{\xi} \mathbb{C}$$
$$\downarrow$$
$$H^0(\mathcal{O}_{P'}(21)) \cdot \mathsf{X}^?$$

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<u>Def</u> P satisfies $CB(n)$ if

$\forall\ P' \subset P$ colength 1 subsch

$$\begin{tikzcd} \mathrm{Ker}' \arrow[d, equal] \arrow[dr] & & \\ \mathrm{Ker} \arrow[r] & H^0(\mathcal{O}_X(n)) \arrow[r] \arrow[dr] & H^0(\mathcal{O}_P(n)) \arrow[d] \\ & & H^0(\mathcal{O}_{P'}(n)) \end{tikzcd}$$

P' impose the same cond. on P on $H^0(\mathcal{O}_X(n))$

$(\exists\ \xi$ s.t $CB_\xi \Leftrightarrow CB(2))$.

<u>1^{st} example</u> P consists of 11 pts in general posit on X.
以上.

$h^0(\mathcal{O}_X(2)) = 10$. any $P' \subset P$ imposes vanishing
same on P.

so P satisfies $CB(2)$

for $c_2 \geq 11$. $V \underset{\text{open}}{\subset} \{\text{all } P\}$

For a given choice of subscheme P, the space of ext.
is $\mathbb{P}(H^1(\mathcal{J}_P(2))^*)$

$\dim V(c_2) = 3c_2 - 11.$

$$\underset{h^0(E)=1}{\{E\}} \leftrightarrow \{(E,s)\} = \{(P,\xi)\}\underbrace{\{ \to \{P\}}_{\dim\ c_2-10-1}{}_{2c_2}.$$

$c_2 \geq 10 \Rightarrow 3c_2 - 11 <$ exp. dim.

($c_2 = 10 \Rightarrow \chi(E(u)) = 5u^2$ ← 重要ぽい)

$\Rightarrow$ For $c_2 \geq 11$ the sing pts form a strict subset of the moduli sp $\Rightarrow$ good.

$c_2 = 10 \Rightarrow$ 10 general pts don't satisfy $CB(2)$.

vanish at 9 pts leave ~~at least~~ an element of $H^0(\mathcal{O}_X(2))$.

which is non zero at 10 point.

To construct a $CB(2)$ collection of 10 pts,

choose a sect. of $H^0(\mathcal{O}_X(2))$ defining $Y \subseteq X$, $Y \in |\mathcal{O}_X(2)|$.

Then choose $P = 10$ general pt on Y

if we start with 9 general pts on Y, Y is the only sect allowed. It vanishes at 10 pts.

$\{P\} = \{(Y, P)\}$. $\{Y\}$ dim 9. $h^0(\mathcal{O}_X(2)) = 10$.

$\{P\}_Y$ has dim 10 $\quad V(10)$ has dim $19 = 3c_2 - 11$.

$\leq$ ex. dim $= 20$

$\Rightarrow M(c_2)$ is good.

<u>Thm</u> For $c_2 \geq 10$, $M(c_2)$ is good. (Nisse 1995 $c_2 \geq 13$)

Next quest: What do the $M(c_2)$ look like $c_2 \leq 9$. ?,

$$\chi(E) = \underbrace{h^0(E) - h^1(E) + h^2(E)}_{h^0(E) = h^2(E) \text{ by Serre dual.}} = 10 - c_2.$$

$$\Rightarrow 2h^0(E) - h^1(E) = 10 - c_2.$$

ということは…

$c_2 \leq 9 \Rightarrow h^0(E) > 0 \longrightarrow \exists$ exact seq $(*)$

$c_2 \leq 7 \Rightarrow h^0(E) \geq 2 \rightarrow h^0(\mathcal{I}_P(1)) \geq 1$ $P \subset$ plane

$c_2 \leq 5 \Rightarrow h^0(E) \geq 3 \rightarrow$ $P \subset (\text{plane}) \cap (\text{Plane}) = \text{line}$

$$M(c_2) \subset V(c_2) \text{ for } c_2 \leq 9.$$

$$0 \to \mathcal{O}_X \to E \to \mathcal{I}_P(1) \to 0$$

$c_2 \leq 5 \quad P \subseteq \ell$ in $\mathbb{P}^3$

$$\begin{tikzcd} H^0(\mathcal{O}_{\mathbb{P}^3}(2)) \arrow[r, "\cong"] \arrow[dr] & H^0(\mathcal{O}_X(2)) \arrow[dr] & \\ & H^0(\mathcal{O}_\ell(2)) \arrow[r] & H^0(\mathcal{O}_P(2)) \end{tikzcd}$$

$H^0(\mathcal{O}_\ell(2)) = \mathbb{C}^3$

If $P' \subset P$ $\quad |P'| = 3$ vanishing of $s \in H^0(\mathcal{O}_{\mathbb{P}^3}(2))$ on P' $\Rightarrow$ vanishing on ℓ.
$\Rightarrow$ vanishing on P.

not true in case $|P'| \leq 2$

$\Rightarrow \quad P \subset \ell$ satisfies $CB(2) \Leftrightarrow |P| \geq 4$

$|\ell \cap X| = 5 \Rightarrow \exists$ 2 cases here

$c_2 = 4$, $P =$ 4 out of the 5 pts in $X \cap \ell$

$c_2 = 5$, $P = X \cap \ell$.

Today's conditions
to understand $CB(2)$ or $CB(4)$ ...
we need to understand curves in $\mathbb{P}^3$.

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It turns out in fact ;

$0 \to R_q \to \mathcal{O}(1)^{\oplus 3} \to \mathcal{I}_{q/\mathbb{P}^3} \to 0$

$c_2 = 4$ $\quad\sim$ reflexive

rank $= 2$.

$\dim M(4) = 2 \quad M(4) \simeq X = \{ q \in X \}$

$$\begin{array}{ccc} \Psi & & \Psi \\ R_q^{\vee\vee} & \longleftarrow & q \end{array}$$

$c_2 = 5$. $\quad M(5) \leftarrow \dim = 3$.

$$\begin{array}{ccc} M(5) & & R_q \\ \| & & \updownarrow \\ \supset \{ q \in \mathbb{P}^3 \setminus X \} . & \ni & q \end{array}$$

These moduli sp are non cpt.
(except the 1st one $M(4 1)$)

We can ~~compactify~~ consider the compactification.
by torsion free sheaves
(Uhlenbeck compact.)

$\overline{M} = \overline{M}(2.1.c_2)$ be the moduli sp of (semi)stable Tor free sheaves E on X.

$r=2$ $\det = \mathcal{O}(1)$, $c_2(E) = c_2$.

This has what O'Grady the double dual stratification; if $E \in \overline{M(c_2)}$

$$0 \to E \to \underset{\substack{\Uparrow \\ \text{loc. free}}}{E^{w}} \to \underset{\text{length } d}{\overset{\leftarrow \text{0-dim}}{Q}} \to 0$$

if $d=0 \Rightarrow E = E^{w}$.
if $E \in \partial M := \{E : \text{non locally free}\}$

$$c_2(E^{w}) = c_2(E) - d$$
E^{w} is stable, $E^{w} \in M(c_2-d)$

The invariant d defines stratification of $\bar{M}(c_2)$ with strata defined $\bar{M}(c_2, c_2-d)$, we have:

$$\begin{tikzcd} & \bar{M}(c_2, c_2-d) \ni E \arrow[dl] \arrow[dr, hook] & \\ \bar{M}(c_2-d) \ni E^W & & \bar{M}(c_2) \end{tikzcd}$$

the fiber of this map have dim $= 3$.
and are irreducible with dense open subset consisting of points E s.t.

$$A = E^W / E = \bigoplus_{i=1}^{d} R_i \quad R_i = \mathbb{C}_{y_i}$$

$$E \leftrightarrow \text{quot } E^W \to S \to 0$$
$$S = \bigoplus_{i=1} S_i$$

points $y_1 \cdots y_d \in X$
at each pt
rk 1 quotient:
$(E^W)_{y_i} \to \mathbb{C}_{y_i} \to 0$
$\in \mathbb{P}^1$

Thm (Li, Ellin...? -Lehn

The closure of this open set $= \mathrm{Quot}(E^W, d)$

$\Downarrow$

$\bar{M}(c_2)$ is projective.

c_2	4	5	6	7	8	9	10	11	12 ...
expec dim $= 4c_2 - 20$	-4	0	4	8	12	16	20	24	28 ...
dim obs at gen pt	6	3	3	1	1	1 non red!	0	0	0
dim	2	3	7	9	13	16	20	24	28
$d=1$ $M(c_2, c_2-d)$		5	6	10	12	16	19	23	27
$d=2$			8	9	13	15	19	22	26
$d=3$				11	12	16	18	22	25
$d=4$					14	15	19	21	25
$d=5$						17	18	22	24
$d=6$							20	21	25
$d=7$								23	24
$d=8$									26
$\dim H^1(E(1))$ generic	0	1	0	0	0	0	0	0	0

Note $\overline{M(c_2)}$ = closure of $M(c_2) \subseteq \overline{M}(c_2)$

$$\overline{M}(10) = \overline{M(10)} \cup M(10,4)$$

<u>Thm 1</u> $M(c_2)$ is irreducible for $c_2 \leq 9$

Thm 2 define $M^{sm}(10) \subseteq M(10)$ components generically seminatural

$$H^1(E) = H^1(E(1)) = \cdots = H^1(E(4)) = 0.$$

$$\Rightarrow M^{sm}(10) \text{ is irr.}$$

Thm 3 $M(c_2)$ is irr for $c_2 \geq 10$.

$c_2 \leq 9 \Rightarrow \exists$ seq: $0 \to \mathcal{O}_X \to E \to \mathcal{I}_P(1) \to 0$

(discuss Thm 1) $c_2 = 4, 5$ we saw yesterday $P \subset$ line

$c_2 = 6, 7$. $P \subset$ Plane (exercise)

$c_2 = 8, 9$.

First expected you think of let $N \subset \mathbb{P}^3$ rat normal curve.

$N \cong \mathbb{P}^1$ $\mathcal{O}_N(1)$ has deg 3

Recall we are looking for $P \subseteq X \subseteq \mathbb{P}^3$ satisfying $CB(2)$ i.e. C.B. for $H^0(\mathcal{O}_X(2))$ $= H^0(\mathcal{O}_{\mathbb{P}^3}(2))$

$P =$ any 8 or 9 pts in $N \cong \mathbb{P}^1$

$\mathcal{O}_{\mathbb{P}^3}(2)|_N =$ a line bdl. of deg 6 on $\mathbb{P}^1$.

$P' \subset P$ contains 7 pts $f \in H^0(\mathcal{O}_N(2))$, $f(P') = 0$

$\Rightarrow f|_N = 0 \Rightarrow f(P) = 0$

$P \subset X \cap N$ choose 8 or 9 pts.

The full families

$c_2 = 8$ "Cayley - octed"

choose $W^3 \subseteq \mathbb{C}^{10} = H^0(\mathcal{O}_{\mathbb{P}^3}(2))$

let $Z := \bigcap_{w \in W} \mathrm{zero}(w)$ = int of 3 quad.

for W general it has 8 pts call that P

$H^0(\mathcal{O}_{\mathbb{P}^3}(2)(-P)) = W = \mathbb{C}^3 \subseteq \mathbb{C}^{10}$

all $\in W$ vanish on P.

If $P \subseteq X \Rightarrow$ set $(P, \xi) = (E, \Lambda)$

$\{P, \xi\} = \{E, \Lambda\}$ $\quad P \notin \text{Plane} \Rightarrow h^0(E) = 1$.

$\overset{(=)}{\swarrow} \qquad \searrow$

$\{P\} \quad h^1(\mathcal{I}_P(1)) = 1. \qquad \{E\}$

When is $\{P\}_X$ irr of dim 13.

$$
\begin{tikzcd}
 & \{(X,P)\} \arrow[dl, "\text{fib} = \mathbb{P}^{47}"'] \arrow[dr, "\text{fib} = \text{smooth dim } 13"] & \\
\{P \subset \mathbb{P}^3\} & & \{X\}
\end{tikzcd}
$$

$\{(X,P)\}$ = dim = 47+21 = 68

$\{P \subset \mathbb{P}^3\}$ dim = 21

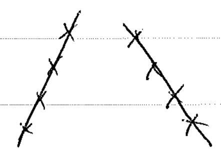
$\{X\}$ dim 55 $\quad h^0(\mathcal{O}_{\mathbb{P}^3}(5)) = 56$.

Argument ~~of~~ for irreducibility of the plane

$\{(X,P)\}$ irrd $\Rightarrow$ Galois act transitive.

it suffice to find a canonical comp of fiber ~~for~~ X

for ex using $P \subseteq N$ or better using

$P =$ 

is a (2,2) complete intersect.

O'Grady's method allows one to show that $\partial Z \neq \phi$. for an irrd comp $Z \subset M(c_2)$

Lemma (O'Grady) if $\partial Z \neq \phi \Rightarrow \operatorname{codim} \partial Z = 1$

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Thm (Nijsse)

for $c_2 \geq 10$, any irr comp $Z \subset \overline{M(c_2)}$ has $\partial Z \neq \phi$.

Thm 1 we have seen the families

pant ; dimension count to show there are no strange component, which might come from $P \subseteq \underset{\sim}{Z}$.

(2.2) complete int in $\mathbb{P}^3$

but Z reducible $Z = Z' \cup Z'' \cup \ldots$

Thm 2 similar quest for (3.3) compl. int.

Thm 1 & 2 + Thm (Nijsse) $\Rightarrow$ Thm 3.

$c_2 = 10$ we can check that all points

$\overline{M}(10)$ corresponds to $H^1(E(1)) = 0 \Rightarrow$ sm.

$c_2 = 11$ check that general pt on

$\overline{M(11,10)} \cap \overline{M(11,4)}$ is smooth in $\overline{M}(11)$

$\Rightarrow \overline{M}(11)$ irrd.

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$c_2 \geq 12$ only one boundary comp.