The 7th KTGU Mathematics Workshop for Young Researchers

Title and Abstract

Plenary Talk

Atsushi Ichino (Kyoto University)

Title: *Automorphic forms of half-integral weight*

Abstract: We discuss recent progress in the theory of automorphic forms of half-integral weight, which was initiated by Shimura in 1973.

Algebraic Geometry Session

Shou Yoshikawa (The University of Tokyo)

Title: *Fedder's type criterion for quasi-Frobenius splitting*

Abstract: In characteristic zero, the analytic structure on the complex varieties plays the essential role for studying the geometry. In positive characteristic, we sometimes use the Frobenius morphism and its properties instead of analytic structure. Mehta and Ramanathan introduced the notion of Frobenius splitting and proved the Kodaira vanishing for Frobenius splitting varieties. After that, Yobuko introduced the weaker notion, called quasi-Frobenius splitting, and he proved that quasi-Frobenius splitting also gives the Kodaira vanishing. In this talk, I will introduce a criterion for quasi-Frobenius splitting of hypersurfaces in a projective space or a weighted projective space. It is a generalization of the criterion for Frobenius splitting proved by Fedder. Moreover, I will introduce the interesting examples and applications of the criterion. This talk is based on joint work with Tatsuro Kawakami, Teppei Takamastu, Hiromu Tanaka, Jakub Witaszuk and Fuetaro Yobuko.

Stefano Filipazzi (École Polytechnique Fédérale de Lausanne)

Title: *On the boundedness of elliptic Calabi-Yau threefolds*

Abstract: In this talk, we will discuss the boundedness of Calabi-Yau threefold admitting an elliptic fibration. First, we will survey some ideas to address weak forms of boundedness of varieties admitting an elliptic fibration. Then, we will switch focus to Calabi-Yau varieties and discuss how the Kawamata-Morrison cone conjecture comes in the picture when studying boundedness properties for this class of varieties. To conclude, we will see how this circle of

ideas applies to the case of elliptic Calabi–Yau threefolds. This talk is based on work joint with C.D. Hacon and R. Svaldi.

Joaquín Moraga (Princeton University)

Title: *Reductive quotients of klt singularities*

Abstract: The study of quotients by reductive groups is an important topic in algebraic geometry. It manifests when studying moduli spaces, orbit spaces, and G–varieties. Many important classes of singularities, as rational singularities, are preserved under quotients by reductive groups. In this talk, we will show that the singularities of the MMP are preserved under reductive quotients. As an application, we show that many good moduli spaces,as the moduli of smoothable K–polystable varieties, have klt type singularities.

Philip Engel (University of Georgia)

Title: *Compact moduli of K3 surfaces*

Abstract: By the Torelli theorem, the moduli space F_g of polarized K3 surfaces is a non–compact quotient of a symmetric space by the action of an arithmetic group. In this capacity, F_g has an infinite class of semitoric compactifications, defined by combinatorial data called a semifan. I will discuss joint work with Alexeev that certain semitoric compactifications of F_g are moduli spaces of stable K3 surface pairs. Just as stable graphs are cartoon depictions of stable curves, polygonal tilings of the two–sphere are cartoon depictions of the surfaces appearing at the boundary of F_g.

Masafumi Hattori (Kyoto University)

Title: *A decomposition formula for J–stability*

Abstract: J–stability plays an important role in K–stability and deeply related to the existence of a stationary solution of J–flow. Strikingly, G.Chen, Datar–Pingali and J.Song proved Lejmi–Szekelyhidi conjecture, uniform J–stability and J–positivity are equivalent, by differential geometric arguments recently. However, this fact has not been proved in algebro–geometric way before. In this talk, I would like to explain a decomposition formula of non–Archimedean J–functional, the $(n+1)$–dimensional intersection number into n–dimensional intersection numbers and its applications to prove the conjecture for singular algebraic surfaces and to show that there exists a J–stable but not uniformly J–stable variety. (arXiv:2103.04603)

Number Theory Session

Hiroshi Noguchi (Kyoto University)

Title: *Multiplier systems and Hilbert modular forms of half-integral weight for* $\mathrm{SL}_2(\mathfrak{o})$

Abstract: Let F be a totally real number field and $\mathfrak{o}$ the ring of integers of F. We study theta functions which are Hilbert modular forms of half-integral weight for $\mathrm{SL}_2(\mathfrak{o})$. First, we obtain an equivalent condition that there exists a multiplier system of half-integral weight for $\mathrm{SL}_2(\mathfrak{o})$. Second, we determine the condition of F that there exists a theta function which is a Hilbert modular form of half-integral weight for $\mathrm{SL}_2(\mathfrak{o})$. The theta function is defined by a sum on a fractional ideal of F.

Masao Oi (Kyoto University)

Title: *A characterizing result on supercuspidal representations*

Abstract: It is known that any discrete series representation of a real reductive group has a simple description of its Harish-Chandra character on elliptic maximal tori and is characterized by it. For p-adic reductive groups, there exists a further smaller class of representations inside discrete series representations, which are called supercuspidal representations. In this talk, I will explain that an analogous result to the real case can be established for some specific supercuspidal representations of p-adic reductive groups. This is joint work with Charlotte Chan (University of Michigan).

Rui Chen (Zhejiang University)
Jialiang Zou (National University of Singapore)

Title: *Arthur's multiplicity formula and local Langlands correspondence for (special) orthogonal and unitary groups via theta lifts* (Speaker: Jialiang Zou)

Abstract: It is expected that, in some cases, theta lifts realize Langlands functoriality lifts (cf. Adams conjecture and Prasad conjecture). Motivated by this, one can transfer the local Langlands correspondence and Arthur's multiplicity formula from one group to another group via theta lifts. This provides an efficient way to establish new cases of local Langlands correspondence and Arthur's multiplicity formula. In this talk, I will describe how to use this idea to establish the local Langlands correspondence and Arthur's multiplicity formula for (special) orthogonal and unitary groups, which extend the works of Arthur and Mok to non-quasi-split cases. This talk is based on joint work with Rui Chen.

Chen Wan (Rutgers University)

Title: *Period integrals and multiplicities for some strongly tempered spherical varieties*

Abstract: In this talk I will discuss the local and global conjectures for some strongly tempered spherical varieties. Both conjectures are very similar to the Gan–Gross–Prasad models. More specifically, globally the square of the period integrals should be related to the central value of some L–functions of symplectic type. Locally each tempered L–packet should contain a unique distinguished element with multiplicity one and the unique distinguished element should be determined by certain epsilon factors. I will also discuss the proof of the local conjecture in many cases. This is a joint work with Lei Zhang and part of it is still an ongoing work.

Tony Feng (Massachusetts Institute of Technology)

Title: *Higher theta series*

Abstract: Theta series are special complex–analytic functions, which arose originally in the problem of counting lattice vectors of a given length. They enjoy remarkable symmetry properties, nowadays known as modularity, and special value formulas, nowadays known as Siegel–Weil formulas. Kudla discovered an analog of theta series in arithmetic geometry, arising from the enumerative geometry of arithmetic moduli spaces. Much research has gone towards investigating analogous modularity properties and analogous Siegel–Weil formulas for these so–called arithmetic theta series. I will talk about recent work with Zhiwei Yun and Wei Zhang that proposes in the function–field context a common generalization of theta series and arithmetic theta series, which we call "higher" theta series, and what we can prove towards "higher modularity" and "higher Siegel–Weil formulas".

Differential Geometry Session

Tadashi Fujioka (Kyoto University)

Title: *Noncritical maps on geodesically complete spaces with curvature bounded above*

Abstract: Curvature bounds of metric spaces were introduced by Alexandrov in terms of triangle comparison. Spaces with curvature bounded below and above are called CBB and CBA spaces, respectively. In general, the structure of CBA spaces is much more complicated than that of CBB spaces. To obtain some control, we will consider geodesically complete CBA spaces, or GCBA spaces. Lytchak and Nagano recently published the fundamental theory of GCBA spaces. Their main technical tool is a strainer map, which is a distance map whose differential satisfies a kind of orthogonality condition. In this talk, I will introduce a new type of regularity of distance maps which generalizes strainer maps. This regularity can be thought of as a dual concept of noncritical maps in CBB geometry defined by Perelman.

Tianyi Zheng (University of California San Diego)

Title: *Random walks and harmonic functions on groups of intermediate growth*

Abstract: The question about existence of groups of intermediate growth (super-polynomial but sub-exponential) was raised by Milnor in the 60s. First examples of such groups were constructed by Grigorchuk in the early 1980s. We will discuss probabilistic and algebraic aspects in studying such groups, and some open questions.

Aliakbar Daemi (Washington University in St. Louis)

Title: *Lagrangians, $\mathrm{SO}(3)$-instantons and the Atiyah-Floer Conjecture*

Abstract: A useful tool to study a 3-manifold is the space of the representations of its fundamental group, a.k.a. the 3-manifold group, into a Lie group. Any 3-manifold can be decomposed as the union of two handlebodies. Thus, representations of the 3-manifold group into a Lie group can be obtained by intersecting representation varieties of the two handlebodies. Casson utilized this observation to define his celebrated invariant. Later Taubes introduced an alternative approach to define Casson invariant using more geometric objects. By building on Taubes' work, Floer refined Casson invariant into a graded vector space whose Euler characteristic is twice the Casson invariant. The Atiyah-Floer conjecture states that Casson's original approach can be also used to define a graded vector space and the resulting invariant of 3-manifolds is isomorphic to Floer's theory. In this talk, after giving some background, I will discuss some recent progress on the Atiyah-Floer conjecture for $\mathrm{SO}(3)$ bundles, which is based on a joint work with Kenji Fukaya and Maksim Lipyanskyi. I will only assume a basic background in algebraic topology and geometry.

Yuta Kusakabe (Kyoto University)

Title: *Oka manifolds and ellipticity*

Abstract: A complex manifold is an Oka manifold if continuous maps from Stein manifolds can be deformed into holomorphic maps with approximation and interpolation. Dual to Stein manifolds, which are the most natural sources of holomorphic maps, Oka manifolds are the most natural targets. On the other hand, ellipticity is opposite to Kobayashi-Eisenman-Brody hyperbolicity in complex geometry, and it means the existence of many dominating holomorphic maps from complex Euclidean spaces. In this talk, we discuss the relationship between these notions, mentioning some recent results.

Nicolas Tholozan (École Normale Supérieure)

Title: *Sharpness of compact Clifford--Klein forms*

Abstract: Let G/H be a reductive symmetric space. In 2016, Kobayashi and Kassel conjectured

that the discrete subgroups of G acting properly discontinuously and cocompactly on G/H are ≪sharply≫ embedded in G. Here, I will present a proof of this conjecture, obtained jointly with Fanny Kassel, and discuss its many consequences (spectral theory of these compact quotients, stability under deformation and, in some cases, non-existence of such quotients)

Partial Differential Equation Session

Hiroyuki Tsurumi (Kyoto University)

Title: *Existence of the 2D stationary Navier-Stokes flow on the whole plane around a radial flow*

Abstract: In this talk, we consider the stationary Navier-Stokes equations on the two-dimensional whole space, and show the existence of a classical solution for a given small and smooth external force around a radial flow. The advantage of our result is that there is no restriction on external forces regarding to its structure such as symmetry, while the previous studies require such structural assumptions. For construction of a solution, we analyze the vorticity-streamfunction system using the polar coordinates. This talk is based on a joint work with Yasunori Maekawa (Kyoto University).

Christophe Prange (Cergy Paris Université)

Title: *Quantitative regularity for the Navier-Stokes equations: concentration of critical norms and slight criticality breaking*

Abstract: In this talk we will explain a strategy to obtain quantitative regularity estimates in a critical regime for the three-dimensional Navier-Stokes equations. Such results enable to slightly break the criticality of this problem. We will in particular give a partial answer to a conjecture of Terence Tao about the blow-up of slightly supercritical Orlicz norms. This talk is based on joint works with Tobias Barker (University of Bath, UK).

Chun-Yen Shen (National Taiwan University)

Title: *Additive Combinatorics and Geometric Measure Theory*

Abstract: In this talk, we will briefly introduce the history of sum-product problems in finite fields and their connections to other problems in discrete geometry and analytic number theory including our recent results about exponential sums over finite fields. We will also talk about its continuous version in reals and our recent results in geometric measure theory. Future direction and works will be discussed at the end of my talk.

Yifei Wu (Tianjin University)

Title: *Low regularity Fourier integrators for some nonlinear dispersive equations*

Abstract: In recent years, more and more attention has been paid to the low regularity numerical study based on the practical needs. In this talk, some Fourier integrators are proposed for solving the KdV equation and the nonlinear Schrodinger equation. The designation of the scheme is based on the exponential-type integration and the Phase-Space analysis of the nonlinear dynamics. By the rigorous analysis, the new schemes provide the first-order or second-order accuracy in Sobolev spaces for rough data, and reduce the regularity requirement of existing methods so far for optimal convergence. Moreover, the conservation laws of the numerical solutions are considered.

Kenjiro Ishizuka (Kyoto University)

Title: *Global dynamics around* 2 *solitons for the nonlinear damped Klein-Gordon equations*

Abstract: In this talk, we consider the nonlinear damped Klein-Gordon equations. In the one dimensional case, Cote, Martel and Yuan (2021) proved the full soliton resolution. We give a complete classification of solutions into 5 types of global behavior for all initial data in a small neighborhood of each superposition of two ground state (2-solitons) with the opposite signs and sufficient spatial distance. This talk is based on a joint work with Kenji Nakanishi (Kyoto University).

Probability Theory Session

Ismaël Bailleul (Université de Rennes 1)

Title: *Analysis of the Anderson operator*

Abstract: This talk will be about the Anderson operator in a closed Riemann surface: A perturbation of the Laplace-Beltrami operator by a (random) white noise potential. Despite the very low regularity of this potential the operator can be defined as an unbounded symmetric operator on L^2 with a compact resolvent and explicit geometric lower bounds on its spectral gap. I will explain how a fine description of its heat kernel can give control on its eigenvalues and eigenfunctions and provides a key ingredient in the construction of a (doubly) random field called Anderson Gaussian free field, closely linked to the polymer measure.

Kohei Sasaya (Kyoto University)

Title: *Some relation between spectral dimension and Ahlfors regular conformal dimension of resistanse metrics*

Abstract: The spectral dimension $\mathrm{d_S}$ is a stochastic or analytic dimension, which is related to the asymptotic behavior of the Markov process and/or spectrum of the Laplacian of the metric space. The Ahlfors regular conformal dimension dimAR is a geometric dimension of the metric

space, defined by a type of homeomorphisms, called quasisymmetry. In Kigami(2020), in the cases of the Brownian motions on generalized Sierpinski carpets and gaskets, he showed that either $\dim_{AR} \leq d_s < 2$ or $\dim_{AR} \geq d_s \geq 2$ holds. In this lecture, we extend this result to the framework of resistance forms.

Francesco De Vecchi (Universität Bonn)

Title: *Stochastic quantization of exponential-type quantum field theories*

Abstract: Stochastic quantization is a method, proposed by Parisi and Wu, of constructive Euclidean quantum field theory for building the Schwinger functions of a quantum model from the invariant solutions of suitable (parabolic, hyperbolic or elliptic) stochastic partial differential equations (SPDEs). In the talk we provide an introduction of the topic and of some recent developments in the field, focusing on the analytic and probabilistic aspects of the problem. We propose a more detailed analysis of the SPDEs related to the two-dimensional exponential-type models such as the Hoegh-Krohn, or Liouville quantum gravity, quantum field theory and the massive sinh interaction. The talk is mainly based on a joint work with Sergio Albeverio and Massimiliano Gubinelli, and on a joint work with Nikolay Barashkov.

Kosuke Yamato (Kyoto University)

Title: *A unifying approach to non-minimal quasi-stationary distributions for one-dimensional diffusions*

Abstract: I talk about non-minimal quasi-stationary distributions for one-dimensional diffusions. Distributional convergence of the process conditioned to survive to non-minimal quasi-stationary distributions is discussed. I give a method of reducing the convergence to the tail behavior of the lifetime via a property which I call the first hitting uniqueness. As an application, for Kummer diffusions with negative drifts, I present a class of initial distributions converging to each non-minimal quasi-stationary distribution.

Baris Evren Ugurcan (Bergische Universität Wuppertal)

Title: *Renormalization of Anderson Hamiltonian and Stochastic PDEs*

Abstract: I will describe the renormalization of the Anderson Hamiltonian by using paracontrolled distributions and then go into the the well-posedness theory for stochastic NLS and NLW equations with multiplicative noise on two and three dimensional torus (joint with Gubinelli-Zachhuber). Time permitting, I will also describe recent work about the full space case.