The 8th KTGU Mathematics Workshop for Young Researchers

Title and Abstract

Plenary Talk

Masaki Tsukamoto (Kyoto University)

Title: *Introduction to mean dimension*

Abstract: Mean dimension is a topological invariant of dynamical systems introduced by Gromov in 1999. It counts the number of parameters per unit time for describing the given dynamical system. The study of mean dimension has two main streams: One is the original direction due to Gromov, and another is due to Elon Lindenstrauss and Benjamin Weiss. The original motivation of Gromov was to explore a new frontier of geometric analysis by using this notion. Independently of this motivation, Lindenstrauss and Weiss found deep applications of mean dimension in topological dynamics. In this talk I will survey some highlights of both the directions.

Representation Theory Session

Alexis Bouthier (Sorbonne University Paris)

Title: *Singular support for ind-schemes*

Abstract: We give a construction of a singular support for ind-schemes that generalizes the one of Beilinson and Saito. Then we explain how it can be used to compute the singular support of some affine Springer sheaves and obtain an affine version of results of Mirkovic-Vilonen.

Alexei Latyntsev (University of Southern Denmark)

Title: *Quantum vertex algebras and cohomological Hall algebras*

Abstract: There is an extremely rich history of interaction between string theory and the mathematics of moduli spaces, for instance cohomological Hall algebras/algebras of BPS states, or vertex/chiral algebras. In this talk, I will explain a link between two of these: Joyce's vertex algebras attached to the moduli stack of objects in an abelian category, and one dimensional CoHAs. This is based on my recent paper 2110.14356, whose main result says that the cohomologies of such stacks are quantum vertex algebras": the factorisation/vertex analogues of quasitriangular bialgebras. The main technical tool is a bivariant" Euler class which makes torus localisation work in this context. I will discuss applications of these techniques to CoHAs of coherent sheaves on a curve and future directions.

Wille Liu (Academia Sinica)

Title: *Translation functors for trigonometric double affine Hecke algebras*

Abstract: Double affine Hecke algebras were introduced by Cherednik around 1995 as a tool to study the Macdonald polynomials. The trigonometric double affine Hecke algebras (TDAHA), degenerate version of the former, have also been found related to several other areas, notably representation theory of p-adic groups. In this talk, I will be focusing on specific aspects of the representation theory of the TDAHA. Given a root system and two families of complex parameters c and c' such that c—c' takes values in Z, there is an equivalence of the derived categories of modules of the resulting TDAHA: D^b(H_c-mod)≅D^b(H_c'-mod), called translation functor. After a brief introduction to the TDAHA, I will talk about a construction of translation functors.

Yuchen Fu (RIMS, Kyoto University)

Title: *Factorization Modules and Quantum Category* $\mathcal{O}$

Abstract: Given a braided Hopf algebra A in some braided monoidal category C, we explain how to establish an equivalence between the category of left modules over its Majid double bosonization $DBos(A)$ and the category of factorization modules over a factorization algebra $Fact(A)$. For C an abelian category, this was a celebrated result by Bezrukavnikov, Finkelberg and Schechtman; our construction, which uses different methods, generalizes it to the derived setting. We will also explain how this allows us to express objects in the quantum BGG category $\mathcal{O}$ in terms of factorization modules.

Ana Kontrec (RIMS, Kyoto University)

Title: *Representation theory and duality properties of some minimal affine* W*-algebras*

Abstract: One of the most important families of vertex algebras are affine vertex algebras and their associated W-algebras, which are connected to various aspects of geometry and physics. Among the simplest examples of W-algebras is the Bershadsky-Polyakov vertex algebra $W_k(g,f_{min})$, associated to $g=sl(3)$ and the minimal nilpotent element f_{min}. In this talk we are particularly interested in the Bershadsky-Polyakov algebra W_k at positive integer levels, for which we obtain a complete classification of irreducible modules. In the case $k=1$, we show that this vertex algebra has a Kazama-Suzuki-type dual isomorphic to the simple affine vertex superalgebra $L_{k'}(osp(1|2))$ for $k'=-5/4$. This is joint work with D. Adamovic.

Kota Murakami (Kyoto University)

Title: *Categorifications of deformed symmetrizable generalized Cartan matrices*

Abstract: Motivated from studies of the representation theory of quantum loop algebras, Geiss-

Leclerc–Schröer introduced the notion of the generalized preprojective algebra associated with a symmetrizable generalized Cartan matrix and its symmetrizer. We study a several parameter deformation of a symmetrizable generalized Cartan matrix as a numerical aspect of the multi-graded module category of the generalized preprojective algebra. In particular, we will interpret some numerical formula about this matrix in terms of braid group symmetries of our graded module category. This is a joint work with Ryo Fujita (RIMS).

Topology Session

Francesco Fournier-Facio (Eidgenössische Technische Hochschule Zürich)

Title: *Aut-invariant quasimorphisms on groups*

Abstract: For every group G, there is a natural action of $\mathrm{Aut}(G)$ on the space of homogeneous quasimorphisms of G. This action is very poorly understood, in particular it is hard to produce fixpoints, i.e. Aut-invariant quasimorphisms, which can be used to estimate Aut-invariant norms on groups. I will report on joint work with Ric Wade (Oxford) where we construct Aut-invariant quasimorphisms on all Gromov-hyperbolic groups, and more.

Kohei Kikuta (Osaka University)

Title: *Autoequivalence groups of K3 surfaces and Mapping class groups*

Abstract: Autoequivalence groups of derived categories of K3 (complex) surfaces are interesting objects in group theory. Via homological mirror symmetry, one can see an analogy between autoequivalence groups and mapping class groups of (real) surfaces. In this talk, we explain this analogy, especially focus on spherical twists, spaces of stability conditions and complexes of spherical objects, which are analogue of Dehn twists, Teichmuller spaces and curve complexes, respectively. This talk is partly based on a joint work with Federico Barbacovi.

Harry Petyt (University of Oxford)

Title: *Large-scale geometry of mapping class groups*

Abstract: Mapping class groups are classical objects in topology and group theory. In recent years there has been a lot of interest in understanding their geometry, and in particular in the question of which features of nonpositive curvature they display. In this talk I'll discuss some recent results in this direction. Partly based on joint work with Thomas Haettel and Nima Hoda.

Sheng Bai (Kyoto University)

Title: *Equivalence of state surfaces*

Abstract: State surfaces of a link are special spanning surfaces for the link corresponding to

Kauffman states of link diagrams. We follow D. Bar-Nartan, J. Fulman and L. H. Kauffman's method to show that any two connected state surfaces of the same link are related up to isotopy by addition of small half-twisted bands. We further show that every state surface is isotopic to a checkerboard surface. We find that the self-linking number of state surface is the first grading in Khovanov homology. Using our main result, we prove that if two spanning surfaces for the same link have the same boundary slope on each component of the link, then they are tube-equivalent. Finally we recover some classical results from this theorem. This is a joint work with Louis H. Kauffman.

Yichen Tong (Kyoto University)

Title: *Homotopy Commutativity in Hermitian symmetric spaces*

Abstract: A fundamental problem on H-spaces is to find whether or not a given H-space is homotopy commutative. It is proved that the loop spaces of some homogeneous spaces are homotopy nilpotent, but we do not even know they are homotopy commutative or not. In this talk we investigate the homotopy commtativity of loop spaces of irreducible Hermitian spaces case-by-case. The method also applies to compute the homotopy nilpotency of flag manifolds. This is a joint work with Daisuke Kishimoto and Masahiro Takeda.

Operator Algebra Session

Srivatsav Kunnawalkam Elayavalli (University of California, Los Angeles)

Title: *The small at infinity boundary for von Neumann algebras*

Abstract: In this talk I will describe a generalized notion of a small at infinity compactification a la Ozawa of a finite von Neumann algebra introudced by the speaker, Ding and Peterson. The key technical novelty here was to bypass an old well known obstruction, involving the absence of interesting derivations from $B(L2M)$ into the compact operators, by considering the closure of the compacts in a topology of Magajna from the theory of strong operator bimodules. By effectively working in this topological framework we clarify old problems of Anantharaman-Delaroche involving various viewpoints of the Haagerup property for $II1$ factors, and also the notion of mixingness for bimodules. This work additionally allows for several kinds of rigidity results via a generalized notion of proper proximality (a la Boutonnet-Ioana-Peterson) for von Neumann algebras. Some of the applications I will talk about from this work include: absense of weakly compact cartan subalgebars for a wide class of II_1 factors; a structure result for subfactors of $L(G)$ where G is non amenable bi-exact, settling a problem of Popa for the case of $F2$ and solid ergodicity for various Gaussian actions, extending works of Boutonnet and Chifan-Ioana. By developing an abstract upgrading result for the notion of relative proper

proximality, the speaker and Ding also recently obtained a new application of this framework, involving the structure of free products. I will discuss these results and touch on some key new ideas.

Koichi Oyakawa (Vanderbilt University)

Title: *Bi-exactness of relatively hyperbolic groups*

Abstract: Bi-exactness is an analytic property of groups defined by Ozawa and of fundamental importance to the study of operator algebras. In this talk, I will show that finitely generated relatively hyperbolic groups are bi-exact if and only if all peripheral subgroups are bi-exact. This is a generalization of Ozawa's result which claims that finitely generated relatively hyperbolic groups are bi-exact if all peripheral subgroups are amenable.

Pieter Spaas (University of Copenhagen)

Title: *Obstructions to stability and lifting properties for groups with property (T)*

Abstract: We will start with discussing the notion of Hilbert-Schmidt stability for countable discrete groups. We will motivate its definition, discuss some examples, and establish a cohomological obstruction to it for certain groups with property (T). This will allow us to provide examples of groups that are not Hilbert-Schmidt stable. We will then further discuss some related lifting properties for groups and their operator algebras. This is based on a joint work with Adrian Ioana and Matthew Wiersma.

Ryoya Arimoto (RIMS, Kyoto University)

Title: *On the type of the von Neumann algebra of an open subgroup of the Neretin group*

Abstract: The Neretin group is the totally disconnected locally compact group consisting of almost automorphisms on the rooted tree. In 2021, P.-E. Caprace, A. Le Boudec, and N. Matte Bon proved that the Neretin group is not of type I and conjectured that a distinguished open subgroup of this group is not of type I either. In this talk, I will show that the group von Neumann algebra of this open subgroup is of type II and answer their question.

Junichiro Matsuda (Kyoto University)

Title: *Algebraic connectedness and bipartiteness of quantum graphs*

Abstract: The notion of quantum graphs was introduced as a non-commutative analogue of classical graphs in quantum information theory, and it has been developed in the interactions between theories of operator algebras, quantum information, quantum groups, tensor categories, non-commutative geometry, etc. Similarly to the classical case, the degree of a regular quantum graph is shown to be the spectral radius of the adjacency matrix. Thus it makes sense to

consider the behavior of the spectrum in $[-d,d]$ for dd-regular undirected quantum graphs. We generalize the well-known fact that the spectrum of the adjacency matrix can characterize the connectedness and bipartiteness of a regular graph. We also obtained the equivalence between bipartiteness, quantum and classical two-colorability, and symmetry of the spectrum of connected regular quantum graphs.

Akihiro Miyagawa (Kyoto University)

Title: *The conjugate system for the q-Gaussians*

Abstract: The q-Canonical Commutation Relation (q-CCR) is an interpolation between the CCR and the CAR with a parameter q. In the 1990s, M. Bożejko and R. Speicher found that the q-CCR is represented on the q-Fock space. The q-Gaussians are realized as the field operators with the vacuum state, which forms a non-commutative distribution. The von Neumann algebra generated by q-Gaussians has been studied for many years, and it is known that this algebra shares several properties with the free group factor. In terms of applications, the q-Gaussians are also related to a random matrix model of quantum holography, the so-called SYK model. On the other hand, a conjugate system is a notion of free probability introduced by D. Voiculescu. This carries important information about a non-commutative distribution of given operators and has many implications for the generated von Neumann algebra. In this talk, I will start to give an overview of the q-Gaussians and related topics. In the sequel, I will explain the existence of a conjugate system for the q-Gaussians. This talk is based on the joint work with R. Speicher.

Applied Mathematics Session

Daniel Ginsberg (Princeton University)

Title: *The stability of model shocks and the Landau law of decay*

Abstract: It is well-known that in three space dimensions, smooth solutions to the equations describing a compressible gas can break down in finite time. One type of singularity which can arise is known as a shock, which is a hypersurface of discontinuity across which the integral forms of conservation of mass and momentum hold and through which there is nonzero mass flux. One can find approximate solutions to the equations of motion which describe expanding spherical shocks. We use these model solutions to construct global-in-time solutions to the irrotational compressible Euler equations with shocks. This is joint work with Igor Rodnianski.

Naoki Sato (The University of Tokyo)

Title: *Nested invariant tori foliating a vector field and its curl: toward steady Euler flows in toroidal domains without Euclidean isometries*

Abstract: This work studies the problem of finding a three-dimensional solenoidal vector field such that both the vector field and its curl are tangential to a given family of toroidal surfaces. We show that this question can be translated into the problem of determining a periodic solution with periodic derivatives of a two-dimensional linear elliptic second-order partial differential equation on each toroidal surface, and prove the existence of smooth solutions. An example of smooth solution foliated by toroidal surfaces that are not invariant under Euclidean isometries is also constructed explicitly, and it is identified as an equilibrium of anisotropic magnetohydrodynamics. The problem examined here represents a weaker version of a fundamental mathematical problem that arises in the context of fluid mechanics and magnetohydrodynamics concerning the existence of regular steady Euler flows and equilibrium magnetic fields in bounded domains without Euclidean isometries. The existence of such configurations represents a key theoretical issue for the design of the confining magnetic field in nuclear fusion reactors known as stellarators. The connection with the study of vortex flows over curved surfaces is also discussed.

Taichi Uemura (Stockholm University)

Title: *Normalization and coherence for ∞-type theories*

Abstract: ∞-type theories are a higher dimensional generalization of type theories introduced by Nguyen and the speaker to tackle coherence problems in the $(\infty,1)$-categorical semantics of type theories. There, the coherence problem for an ∞-type theory is whether the initial model of the ∞-type theory is 0-truncated. Normalization for a type theory is the property that every type or term in the type theory has a unique normal form. An application is calculation of equality of types and terms. In this talk, I present a technique for normalization for ∞-type theories to solve the coherence problems for ∞-type theories at some level of generality. Normalization allows us to calculate path spaces of types and terms and to determine the truncation level of the initial model.

Yuki Amano (RIMS, Kyoto University)

Title: *Locally Defined Independence Systems on Graphs*

Abstract: The maximization for the independence systems defined on graphs is a generalization of combinatorial optimization problems such as the maximum b-matching, unweighted MAX-SAT, matchoid problem, and maximum timed matching. In this paper, we consider the problem under the local oracle model to investigate the global approximability of the problem by using the local approximability. We first analyze two simple algorithms FixedOrder and Greedy for the maximization under the model, which shows that they have no constant approximation ratio. Here algorithms FixedOrder and Greedy apply local oracles with fixed and greedy orders of

vertices, respectively. These results can be generalized to the hypergraph setting. We then propose two approximation algorithms for the k-degenerate graphs, whose approximation ratios are $\alpha+2k-2$ and αk, where α is the approximation ratio of local oracles. We also propose an $(\alpha+k)$-approximation algorithm for bipartite graphs, in which the local independence systems in the one-side of vertices are k-systems with independence oracles.

Jean-Simon Pacaud Lemay (RIMS, Kyoto University)

Title: *Tangent Categories: A Bridge between Differential Geometry and Algebraic Geometry*

Abstract: Differential geometry and algebraic geometry share many similarities, with one common aspect being that differentiation is a fundamental notion for both. The theory of tangent categories uses category theory to provide the foundations of differential calculus over smooth manifolds. As such, tangent categories have been able to formalize numerous aspects of differential geometry such as: tangent bundles, vector fields, vector bundles, differential equations, Lie brackets, connections, etc. Due to their generality, tangent categories have also found applications in synthetic differential geometry and computer science. Recently, tangent categories have also found surprising connections and applications to algebraic geometry. In this talk, I will give an introduction to tangent categories and discuss its relation to algebraic geometry. This will highlight how tangent categories provide a connecting bridge between differential geometry and algebraic geometry.

Kento Yasuda (RIMS, Kyoto University)

Title: *Most probable path of an active Brownian particle*

Abstract: In this talk, I show the latest research on the transition path of a free active Brownian particle (ABP) on a two-dimensional plane between two given states. The extremum conditions for the most probable path connecting the two states are derived using the Onsager-Machlup integral and its variational principle. We provide explicit solutions to these extremum conditions and demonstrate their nonuniqueness through an analogy with the pendulum equation indicating possible multiple paths. The pendulum analogy is also employed to characterize the shape of the globally most probable path obtained by explicitly calculating the path probability for multiple solutions. We comprehensively examine a translation process of an ABP to the front as a prototypical example. Interestingly, the numerical and theoretical analyses reveal that the shape of the most probable path changes from an I to a U shape and to the ℓ shape with an increase in the transition process time. The Langevin simulation also confirms this shape transition. If I have enough presentation time, I will explain an additional topic, an enzyme molecule characterized by odd elasticity.