The 9th KTGU Mathematics Workshop for Young Researchers

Title and Abstract

Plenary Talk

Seiichiro Kusuoka (Kyoto University)

Title: *Construction of a non-Gaussian and rotation-invariant $¥Phi ^4$-measure and associated flow on ${¥mathbb R}^3$ through stochastic quantization*

Abstract: In this talk, I explain the motivation and open problems of the $¥Phi ^4$-quantum field theory, and a result of my jointwork with Sergio Albeverio on the construction of a rotation-invariant $¥Phi ^4$-measure on ${¥mathbb R}^3$. The construction is based on the method of singular SPDEs, and the main idea is that we construct the $¥Phi ^4$-measure, not by discrete approximations, but by a continuous approximation (i.e. regularization and localization).

Algebraic geometry, Number theory Session

Swaraj Pande (University of Michigan)

Title: *A Frobenius version of Tian's Alpha invariant, and the F-signature of Fano varieties*

Abstract: The Alpha invariant of a complex Fano manifold was introduced by Tian to detect its K-stability, an algebraic condition that implies the existence of a Kähler-Einstein metric. Demailly later reinterpreted the Alpha invariant algebraically in terms of a singularity invariant called the log canonical threshold. In this talk, we will present an analog of the Alpha invariant for Fano varieties in positive characteristics, called the Frobenius-Alpha invariant. This analog is obtained by replacing "log canonical threshold" with "F-pure threshold", a singularity invariant defined using the Frobenius map. We will review the definition of these invariants and the relations between them. The main theorem proves some interesting properties of the Frobenius-Alpha invariant; namely, we will show that its value is always at most 1/2 and make connections to a version of local volume called the F-signature. Time permitting, we will also discuss the semicontinuity properties of the Frobenius-Alpha invariant.

Ruijie Yang (Max Planck Institute for Mathematics, Germany)

Title: *Higher multiplier ideal sheaves and the Hodge theory of the singularities of pairs*

Abstract: The multiplier ideal sheaves associated to a complex manifold X and a Q-effective divisor D is a family of ideal sheaves indexed by rational numbers, pioneered in the work of

Nadel, Siu and Kawamata in 1980s. They are important tools in birational geometry (especially Mori's minimal model program), algebraic geometry and commutative algebra. In this lecture, I will introduce a new family of ideal sheaves associated to (X,D), called higher multiplier ideals, indexed by a rational number and an integer. When the integer index is zero, they recover the usual multiplier ideal sheaves. These ideals are closely related, but different from, the Hodge ideals of Mustata and Popa. We investigate their local and global properties systematically. The construction of these ideals relies on the Hodge theory of the Kashiwara-Malgrange V-filtration along the effective divisor D and one of the new idea is to exploit the global structure of the V-filtration using the notion of twisted Hodge modules, which generalizes M.Saito's theory of Hodge modules and Sabbah-Schnell's theory of complex Hodge modules. Compared to the usual multiplier ideals, the higher multiplier ideals provide more refined information of the singularities of pairs and I will discuss new applications to conjectures of Casalaina-Martin, Gruschevsky and Debarre on singularities of theta divisors on principally polarized abelian varieties and the geometric Riemann-Schottky problem. This is based on the joint work with Christian Schnell.

Tatsuki Kinjo (RIMS, Kyoto University)

Title: *Cohomological study of the Hitchin moduli space via DT theory*

Abstract: Donaldson-Thomas (DT) theory is a counting theory of coherent sheaves on Calabi--Yau threefolds. In this talk, I will explain that an interesting symmetry of the cohomology of the moduli space of Higgs bundles on a Rieman surface, generalizing a version of topological mirror symmetry, can be proved using an idea from DT theory. This talk is based on a joint work with Naoki Koseki (arXiv:2112.10053).

Teppei Takamatsu (Kyoto University)

Title: *On criteria for quasi-F-splitting*

Abstract: In algebraic geometry of positive characteristic, singularities defined by the Frobenius map, including the notion of F-splitting, have played a crucial role. Yobuko introduced the notions of quasi-Frobenius-splitting and F-split heights, which generalize and quantify the notion of F-splitting. In this talk, I will present several criteria for quasi-Frobenius-splitting, along with their applications. This talk is based on a joint paper with Tatsuro Kawakami, Hiromu Tanaka, Jakub Witaszek, Fuetaro Yobuko, and Shou Yoshikawa.

Miyu Suzuki (Kyoto University)

Title: *Existence of linear periods: the archimedean case*

Abstract: We say that a representation has a period when it has a non-zero invariant linear form with respect to a certain subgroup. Prasad and Takloo-Bighash formulated an epsilon dichotomy

conjecture for linear periods in the p-adic case. In this talk, we prove the archimedean analogue of their conjecture using the Schwartz homology. This is joint work with Hiroyoshi Tamori.

Differential Geometry Session

Jianfeng Lin (Tsinghua University)

Title: *On moduli spaces of smooth 4-manifolds*

Abstract: The moduli space of a smooth manifold is defined as the classifying space of its diffeomorphism group. Understanding the homotopy type of this space helps us to classify families of manifolds. In this talk, I will discuss some new properties of the moduli spaces of 4-manifolds. Some of them are special in dimension 4 (e.g. the homological instability phenomena), while some of them also appear in higher dimensions (e.g. a discrepancy between the smooth moduli space and the topological moduli space). The talk is based on a joint work with Hokuto Konno and a joint work with Yi Xie.

Mike Miller (Columbia University)

Title: *Filtered instanton Floer homology and cosmetic surgery*

Abstract: If Y is a closed oriented 3-manifold, its Chern-Simons function is a function on a certain infinite-dimensional space, and the instanton Floer homology I_*(Y) is constructed as the Morse homology of this function. What's special about the Chern-Simons function is that it depends only on the topology of Y, not any other geometric input or auxiliary data. As a result, we can define filtered Floer homologies F_r I_*(Y) which are roughly the Morse homology of the sublevel set cs^{-1}(-infty, r], and these give topological invariants of Y with good structural properties. There is a long history of using the Chern-Simons function to prove results about homology cobordism of 3-manifolds. It was used in Furuta's 1990 proof that the homology cobordism group is infinitely generated; recently, Nozaki-Sato-Taniguchi used the CS filtration to give examples of integer homology spheres Y so that any 4-manifold bounding Y must be indefinite (there must be some essential surface Sigma with self-intersection number zero). I will discuss applications of a different sort: distinguishing the diffeomorphism types of two 3-manifolds using their filtered Floer homology, with applications to cosmetic surgery problems. This work is joint with Tye Lidman.

Hayato Imori (Kyoto University)

Title: *Equivariant instanton homology and knot concordance invariants*

Abstract: Floer theory has provided powerful tools in the study of the knot concordance group. In the context of Yang-Mills gauge theory, the S^1-equivariant version of instanton homology

theory provides a categorification of the classical knot signature and refined numerical knot concordance invariants. In this talk, the speaker will introduce constructions of knot concordance invariants from equivariant instanton Floer theory and some applications related to the existence of non-abelian $SU(2)$-representations of fundamental groups and the structure of the knot concordance group. This talk is based on a joint work with Aliakbar Daemi, Kouki Sato, Christopher Scaduto, and Masaki Taniguchi.

Masaki Taniguchi (Kyoto University)

Title: *Involutive instanton Floer theory and its applications*

Abstract: We present a variant of involutive instanton Floer theory that serves as an obstruction to the existence of diffeomorphisms on certain 4-manifolds with boundaries. From this framework, we provide several new phenomena in 4-dimensional topology. Our research yields a new collection of corks, exotic contractible 4-manifolds which survive after stabilization by CP^2 or -CP^2, and establishes topological constraints concerning non-orientable surfaces with boundaries. This is joint work with Abhishek Mallick, Irving Dai, and Antonio Alfieri.

Partial Differential Equation Session

Younghun Hong (Chung-Ang University)

Title: *Quantum-classical correspondence from an analytic point of view*

Abstract: In physics, Bohr's correspondence principle asserts that the theory of quantum mechanics can be reduced to that of classical mechanics in the limit of large quantum numbers. This rather vague statement can be formulated explicitly in an analytic point of view. Starting from coherent states, we discuss the correspondence between basic inequalities and that between measures. Then, we present the convergence from quantum to kinetic white dwarfs. This talk is based on joint work with Jinmyoung Seok and Sangdon Jin.

Sameer Iyer (University of California, Davis)

Title: *Reversal in the Stationary Prandtl Equations*

Abstract: We discuss a recent result in which we investigate reversal and recirculation for the stationary Prandtl equations. Reversal describes the solution after the Goldstein singularity, and is characterized by spatio-temporal regions in which $u > 0$ and $u < 0$. The classical point of view of regarding the Prandtl equations as an evolution x completely breaks down. Instead, we view the problem as a quasilinear, mixed-type, free-boundary problem. Joint work with Nader Masmoudi.

Kelei Wang (Wuhan University)

Title: *Refined blow up analysis of supercritical concentration phenomena*

Abstract: Concentration phenomena can be observed in many PDE problems, such as the bubbling phenomena in harmonic maps and many other geometric variational problems. In the supercritical case of these problems, i.e. when the spatial dimension is above the critical one, the concentration set is usually a high dimensional set. The lower order information about this concentration set (e.g. rectifiability, energy identity) has been explored for more than three decades. In this talk, I would like to discuss some problems concerning higher order information of this supercritical concentration phenomena, through two typical examples: Allen–Cahn equation and nonlinear heat equation.

Sonae Hadama (RIMS, Kyoto University)

Title: *Asymptotic stability of a wide class of stationary solutions for the Hartree and Schr¥"{o}dinger equations for infinitely many particles*

Abstract: We consider the Hartree and Schr¥"{o}dinger equations describing the time evolution of wave functions of infinitely many interacting fermions in three–dimensional space. These equations can be formulated using density operators and have infinitely many stationary solutions. In this talk, we give a result and rough idea to deal with a wide class of stationary solutions. We emphasize that our result includes Fermi gas at zero temperature. This is one of the most important steady states from the physics point of view; however, its asymptotic stability has been left open after the seminal work by Lewin and Sabin which first formulated this stability problem and gave significant results.

Junpei Kawakami (RIMS, Kyoto University)

Title: *Global approximation for the cubic nonlinear Schrödinger equation with strong magnetic confinement*

Abstract: We consider nonlinear Schrödinger equation with strong magnetic fields in 3D. This model was derived by R L. Frank, F. Méhats, C. Sparber in 2017. To describe asymptotic behavior of the NLS for small data, we use the time–averaged model which was derived by the same authors as "the strong magnetic confinement limit" of the NLS. We construct asymptotic solutions which satisfy both asymptotic in time evolution and convergence in the strong magnetic confinement limit. We also analyze the error between the solution to the NLS and the time–averaged model for the same initial data and obtain global estimates.

Probability theory Session

William Hide (Durham University)

Title: *Spectral gaps for random hyperbolic surfaces*

Abstract: We study the low-energy spectrum of the Laplacian on finite-area hyperbolic surfaces. A quantity of particular interest is the spectral gap which provides information about the connectivity of the surface, the rate of mixing of the geodesic flow and error terms in geodesic counting. We shall look at the size of the spectral gap for random hyperbolic surfaces. I will discuss some different constructions of random surfaces and explain recent developments in this area. Based on joint works with Michael Magee and with Joe Thomas.

Thomas Leblé (University of Paris-Cité)

Title: *Counting Points In Boxes: The Riesz Family & Friends*

Abstract: Members of the "Riesz family" are statistical physics systems that include Coulomb and log-gases. They give rise to certain point processes, for which a basic (and seemingly naive) question is to study the law of the number of points in large "boxes". I will present several old/recent results around this theme, the keywords are: "number-rigidity", "hyperuniformity" and more generally "cancellation of fluctuations" – all reflecting unusual properties of those systems due to the specificities of the long-range interaction potential.

Liu Enhao (Kyoto University)

Title: *Curse of Dimensionality on Persistence Diagrams*

Abstract: Topological data analysis (TDA) brings a topological perspective to data analysis, exploring the shape of data. In recent decades, high-dimension, low-sample-size (HDLSS) data have been encountered frequently in many fields. Particularly when HDLSS data comes to statistical analysis, it gives rise to some severe issues. For example, applying principal component analysis (PCA) to the HDLSS data performs poorly due to inconsistencies of eigenvalue and eigenvector estimates. These inconsistencies in the statistical analysis of HDLSS data are called the curse of dimensionality. On the other hand, persistent homology is a computationally efficient tool in TDA, and it has been proven to be stable even in the presence of noise. Therefore, it is of interest to investigate the applicability of persistent homology in the HDLSS data setting. In this talk, I will first present the asymptotic behavior of persistence diagrams, the visualization of persistent homology, of high-dimension random data. Based on the asymptotic behavior, I claim the unreliability of using observed persistence diagrams due to the high-dimension noise, indicating that there also exists the curse of dimensionality on persistence diagrams. Finally, I will show an attempt to mitigate the curse of dimensionality by applying the

normalized PCA to the observed data. Our proof involves estimating the minimum eigengap of a real Wishart matrix and computing moments by utilizing the Weingarten calculus.

Shunsuke Usuki (Kyoto University)

Title: *On a lower bound of the number of integers in Littlewood's conjecture*

Abstract: Littlewood's conjecture is the long-standing open problem from 1930's in simultaneous Diophantine approximation, saying that $¥liminf_{n¥to¥infty}n¥|n¥alpha¥|¥|n¥beta¥|=0$ holds for every pair $(¥alpha,¥beta)$ of real numbers. Actually, this number-theoretic problem is closely related to some dynamical system: the action of diagonal matrices on the homogeneous space ${¥rm SL}(n,¥mathbb{R})/{¥rm SL}(n,¥mathbb{Z}), and, in 2000's, M. Einsiedler, A. Katok and E. Lindenstrauss showed that the exceptional set for Littlewood's conjecture has Hausdorff dimension zero as a corollary of some rigidity of invariant measures under this diagonal action. In this talk, I will explain that we can obtain a further result: a lower bound of the number of $n¥in¥mathbb{N}$ which makes $n¥|n¥alpha¥|¥|n¥beta¥|$ small for every $(¥alpha,¥beta)$ except for a set of Hausdorff dimension zero, by studying the behavior of empirical measures w.r.t. the diagonal action.

Poster Presentation

Tomohiro Aya (Kyoto UUniversity)

Title: *Convergence rates in stochastic homogenization of elliptic and parabolic equations with unbounded coefficients*

Abstract: Quantitative stochastic homogenization is a field that aims to obtain a rate of the convergence in stochastic homogenization under the assumption of quantitative mixing conditions in ergodicity. In this talk, we consider stochastic homogenization of elliptic and parabolic equations with unbounded and non-uniformly elliptic coefficients. Extending subadditive arguments, we get an estimate for the rate of the convergence of the solution of the Cauchy-Dirichlet problem under the condition that coefficients in the unit cube have a certain exponential integrability.

Masahisa Ebina (Kyoto University)

Title: *Ergodicity and central limit theorems for stochastic wave equations in high dimensions*

Abstract: This presentation considers a stochastic wave equation in spatial dimensions higher than three. The driving noise is assumed to be a Gaussian noise that is white in time and has some spatial correlation. I will discuss the difficulties in considering high-dimensional stochastic

wave equations and explain how one can use the Malliavin calculus tools to study the solution's ergodicity and fluctuations.

Kenji Fukushi (Kyoto University)

Title: *An extension of Lusternik-Schnirelmann category of closed 1-form to non compact manifolds*

Abstract: The extention of Morse theory to S^1 valued Morse functions, more generally, to closed 1-form is called Morse-Novikov theory. From this theory, we can obtain the powerful invariants of manifolds with local system coefficient. Based on this, Michael Farber extended Lusternik-Schnirelmann category which associated with closed 1-form. This category no longer gives the lower bound of the number of zeros of closed 1-form. However, if the number of zeros of closed 1-form is less than this category, the dynamics of the gradient flow of closed 1-form must have Homoclinic cycle. In this talk, I will explain the LS category of closed 1-form, and I will introduce the extension of the Farber`s result to non compact manifolds.

Kentaro Inoue (Kyoto University)

Title: *Degeneration of p-divisible groups in terms of logarithmic geometry*

Abstract: The moduli space of abelian varieties are compactified by Faltings-Chai. Recently, Kajiwara-Kato-Nakayama realized this compactification as the moduli space of log abelian varieties which are degenerated objects in the world of logarithmic geometry. In the study of the mod p-fiber of this compactified moduli space, log p-divisible groups occur as degenerated objects of p-divisible groups. In this poster session, we explain about our result on the slope filtration of log p-divisible groups.

Hirotatsu Nagoji (Kyoto University)

Title: *Normalizability of the Gibbs measures associated with multivariate version of $P(¥Phi)_2$ model*

Abstract: We consider the Gibbs measures associated with multivariate version of $P(¥Phi)_2$ quantum field model on the torus. We observe the (non-)normalizability of the measures by the variational method introduced by Barashkov and Gubinelli.

Kei Noda (Kyoto University)

Title: *Analyticity in space-time of solutions to evolution equations with multilinear operators based on maximal regulatrity*

Abstract: Space-time analyticity for evolution equations with nonlinear term represented by multilinear operators is considered via the parameter trick. By extending the case of bilinear

operators in the previous work, we obtain space–time analyticity for the equations of Fujita type. In this poster session, we will introduce previous studies in the bilinear case and outline the proof of the main result by extending the result to the multilinear case.