The 6th KTGU Mathematics Workshop for Young Researchers

Title and Abstract

Plenary Talk

Narutaka Ozawa (Kyoto University)

Title: *Kazhdan's property (T) and semidefinite programming*

Abstract: Kazhdan's property (T) for groups has a number of applications in pure and applied mathematics. It has long been thought that groups with property (T) are rare among the "naturally-occurring" groups, but it may not be so and it may be possible to observe this by extensive computer calculations. After an introduction, I will present a computer assisted (but mathematically rigorous) method of confirming property (T) based on semidefinite programming with some operator algebraic input. I will report the recent results by several hands that confirmed property (T) of $\mathrm{Aut}(\mathrm{Fd})$ for $\mathrm{d}>3$. This solves a well-known problem in geometric group theory.

Representation Theory Session

Mamoru Ueda (Kyoto University)

Title: *Affine super Yangians and rectangular* W*-superalgebras*

Abstract: Motivated by the generalized AGT conjecture in this talk I will construct surjective homomorphisms from the affine super Yangians to the universal enveloping algebras of rectangular W-superalgebras. This result is a super affine analogue of a result of Ragoucy and Sorba, which gave surjective homomorphisms from finite Yangians of type A to rectangular finite W-algebras of type A.

Huanchen Bao (National University of Singapore)

Title: *Flag manifolds over semifields*

Abstract: The study of totally positive matrices, i.e., matrices with positive minors, dates back to 1930s. The theory was generalised by Lustig to arbitrary split reductive groups using canonical bases, and has significant impacts on the theory of cluster algebras, higher teichmüller theory, etc. In this talk, we survey basics of total positivity and explain its generalization to general semifields. This is based on joint work with Xuhua He.

Joshua Ciappara (University of Sydney)

Title: *Algebraic and geometric actions of the Hecke category*

Abstract: In the past 50 years, a guiding problem in the study of reductive groups G over fields of positive characteristic p has been the determination of character formulas for important classes of modules. Lusztig's conjecture for characters of simple modules was established for p sufficiently large, but was shown by Williamson not to hold under the originally stated hypothesis $p>h$, where h is the Coxeter number associated to G. Subsequently, Riche and Williamson proposed new character formulas for simple and indecomposable tilting modules, which would follow from a conjecture of a more categorical nature. Specifically, they showed that the existence of an action of the Hecke category on the principal block of G by wall-crossing functors would imply their character formulas. The goal of the talk will be to introduce these objects, state the categorical conjecture precisely, and give a flavour for how it may be proved using the geometric tools of Smith-Treumann theory.

Shoma Sugimoto (Kyoto University)

Title: *On the logarithmic* W-*algebras*

Abstract: For a finite-dimensional simply-laced simple Lie algebra gg and an integer $p≥2$, we can attach the logarithmic W-algebra $W(p)\_Q$. When $g=sl\_2$, $W(p)\_Q$ is called the triplet W-algebra, and studied by many people as one of the most famous examples of $C2$-cofinite but irrational vertex operator algebra. However, apart from the triplet W-algebra, not much is known about the representation theory of $W(p)\_Q$. In this talk, I will explain some results on the representation theory of $W(p)\_Q$. After we construct $W(p)\_Q$ and $W(p)\_Q$-modules $W(p,λ)\_Q$ geometrically along the preprint of Feigin-Tipunin, we first prove the simplicity, $W\_k(g)$-module structure, and character formula of $W(p,λ)\_Q$ in the cases where ¥sqrt{p}¥bar{ λ} is in the closure of the fundamental alcove. In particular, for $p≥h-1$, $W(p)\_Q$ is simple and decomposed into a direct sum of simple $W\_k(g)$-modules with a natural G-action. Second, we give a purely W-algebraic algorithm to calculate nilpotent elements in the Zhu's C_2-algebra of $W(p)\_Q$ much easier than direct calculation. Applying this algorithm to the cases of $p=2,3$ for $g=sl\_3$, we show that $W(p)\_Q$ is C_2-cofinite in these cases.

Robert McRae (Tsinghua University)

Title: *Structure of some non-semisimple tensor categories of modules for the Virasoro Lie algebra*

Abstract: The Virasoro algebra is the unique non-trivial one-dimensional central extension of the Lie algebra of polynomial vector fields on the circle, and it is foundational in algebraic

approaches to the mathematical construction of two-dimensional conformal quantum field theories. In applications to physics, one typically focuses on representations of the Virasoro algebra at a fixed central charge, which is the scalar by which the canonical central element acts. Recently, it has been shown that for any central charge cc, a certain category of finite-length modules for the Virasoro algebra at central charge cc has braided tensor category structure. These categories are non-semisimple for certain rational central charges, and I will discuss the structure of these tensor categories at central charge $c=13-6p-6p-1$ for integers $p>1$. These central charges are important in logarithmic conformal field theory. At these central charges, I will discuss the classification of irreducible modules, the construction of projective covers, and explicit formulas for the tensor products of irreducible modules. I will show that these tensor categories are also rigid, which means that every module has a dual in a strong sense. Conjecturally, these categories for the Virasoro algebra are tensor equivalent to certain non-semisimple module categories for quantum sl_2 at suitable roots of unity. Although this is still a conjecture, I will explain a relationship between a suitable semisimplification of the Virasoro category and semisimple categories associated to quantum sl_2. This is joint work with Jinwei Yang.

Topology Session

David Leturcq (Kyoto University)

Title: *Computation of an high-dimensional configuration space knot invariant*

Abstract: In this talk, we study some invariants for long embeddings of R^n in R^{n+2} (or in spaces obtained after replacing the unit ball of R^{n+2} with a homology ball). These invariants are defined as combinations of configuration space integrals, and generalize a construction of Bott, Cattaneo, and Rossi. We present a more general and flexible definition of these invariants. These new definitions allow to interpret the invariant as a combination of counting of diagrams with some vertices on the knot, such that any edge has its ends in some particular chain of the two-point configuration space. With specific chains associated to hypersurfaces whose boundary is the knot, this yields explicit formulas of our invariants and allows us to rephrase them in terms of Alexander polynomials.

Wataru Yuasa (Kyoto University)

Title: *Skein and cluster algebras of marked surfaces without punctures for* sl_3

Abstract: A skein algebra is a space of knots and links on a surface with skein relations. For a surface Σ with marked points in the boundary, we introduce a skein algebra S_{sl_3,Σ} consisting of sl_3-webs on Σ with the boundary skein relations. We realize a subalgebra CA^q_{sl_3,Σ} of the quantum cluster algebra associated with a moduli space $A_{SL_3,\Sigma}$ of the decorated SL_3-local systems on Σ inside the skein algebra $S_{sl_3,\Sigma}$. We also prove that any sl_3-web in the skein algebra is expressed as a Laurent polynomial of a cluster. Moreover, we show that elevation-preserving webs in Σ have positive coefficients in a cluster associated with an ideal triangulation of Σ. This talk is based on joint work with Tsukasa Ishibashi.

Katsumi Ishikawa (Kyoto University)

Title: *Generalizations of Galkin quandles and covering invariants of knots*

Abstract: A series of quandles, Galkin quandles, was introduced as nontrivial extensions of the dihedral quandle of order 3 associated with pointed abelian groups. In this talk, we generalize the definition to any Alexander quandles and give a topological interpretation to their colorings. In particular, some metabelian covering invariants can be obtained as quandle invariants.

Victoria Lebed (University of Caen Normandy)

Title: *Unexpected applications of homotopical algebra to knot theory*

Abstract: Interactions between knot theory and homotopical algebra are numerous and natural. But the connections unveiled in this talk are rather unexpected. Following a recent preprint with Markus Szymik, I will explain how homotopy can help one to compute the full homology of racks and quandles. These are certain algebraic structures, useful in knot theory and other areas of mathematics. Their homology plays a key role in applications. Although very easy to define, it is extremely difficult to compute. Complete computations have been done only for a few families of racks. Our methods add a new family to this list, the family of permutation racks. The necessary background on racks and quandles, and their role in braid and knot theories, will be given in detail.

Benjamin Bode (Institute of Mathematical Sciences)

Title: *Stable knots and links in electromagnetic fields*

Abstract: An electromagnetic field consists of two time-dependent vector fields on R^3, namely the electric and the magnetic field, which together satisfy Maxwell's equations. Sets of closed flow lines of a vector field form a link. We show that for every link L there is an electromagnetic field, whose magnetic field has a set of closed flow lines ambient isotopic to L for all time. Since these closed flow lines turn out to be (projections of) real analytic Legendrian links with respect to the standard contact structure on the 3-sphere, the proof is entirely based on contact topology and the geometry of complex plane curves.

Operator Algebra Session

Ian Charlesworth (University of California)

Title: *Free Stein Dimension and Regularity in Free Probability*

Abstract: Regularity in free probability is the result of building analogues in the non-commutative (free) setting of results from information theory and entropy theory in probability. The goal is to learn properties of a von Neumann algebra, viewed as a non-commutative probability space, from information-theoretic properties of its generators. I will give an overview of this approach, and then focus on some recent joint work with Brent Nelson involving the free Stein dimension. This is a quantity which measures how well behaved a set of variables is under free differential calculus, and depends only on the polynomial algebra of the variables measured. I will conclude by discussing some recent developments, such as restrictions on the Stein dimension in the presence of polynomial relations.

Félix Parraud (Kyoto University)

Title: *Interpolation between random matrices and their free limit with the help of free stochastic calculus*

Abstract: It has been known for a long time that as their size grow to infinity, many models of random matrices behave as free operators. This link was first explicited by Voiculescu in 1991 in a paper in which he proved that the trace of a polynomial in independent GUE matrices converges towards the trace of the same polynomial evaluated in free semicircular variables. In 2005, Haagerup and Thorbjørnsen proved the convergence of the norm instead of the trace. The main difficulty of their proof was to prove a sharp enough upper bound of the difference between the trace of random matrices and their free limit. They managed to do so with the help of the so-called linearization trick which allows to relate the spectrum of a polynomial of any degree with scalar coefficients with a polynomial of degree 1 with matrix coefficients. A drawback of this method is that it does not give easily good quantitative estimates. In arXiv:1912.04588, we introduced a new strategy to approach those questions which does not rely on the linearization trick and instead is based on free stochastic calculus. In this talk I will first introduce necessary notion in Random Matrix Theory and Free Probability. Then I will explain the method that we used and finally I will give the results that we obtained in this paper as well as in other related paper such as arXiv:2005.13834 and arXiv:2011.04146.

Sheng Yin (University of Toulouse)

Title: *Atoms of freely independent random variables with atomic distributions*

Abstract: It is well-known in free probability that for the sum of two freely independent random variables X,Y, the atoms of its distribution can only appear at the sum $\alpha+\beta$, where α, β are atoms for X,Y. In this talk, we address the question that for a given polynomial in multiple freely independent random variables, where are its atoms and what are the weights of these atoms. We focus on the case that freely independent random variables have atomic distributions, in which situation the calculation of atoms can be turned to a matrix version of the atom calculation by the linearization. Therefore we can apply a recent result by Mai, Speicher and Yin to find the atoms by some purely algebraic notions and tools. In particular, we will show that for a polynomial p in freely independent atomic random variables $X\_1,\dots,X\_d$, its atoms can only appear at $p(\lambda\_1,\dots,\lambda\_d)$, where $\lambda\_1,\dots,\lambda\_d$ are atoms of $X\_1,\dots,X\_d$. This talk is based on a ongoing joint-project with O. Arizmendi, G. Cébron and R. Speicher.

Akihiro Miyagawa (Kyoto University)

Title: *Convergence for non-commutative rational functions evaluated in random matrices*

Abstract: The Gaussian Unitary Ensemble (GUE) is a basic model of random matrices, which is the only probability distribution on selfadjoint matrices that is both invariant under unitary conjugation and whose entries are independent. To study convergence the emipirical eigenvalue distribution of models costructed from GUE's in the large limit of the size of matrices is one of interests for Random Matrix theory and free probability theory. Based on a joint work with Benoit Collins and Felix Parraud, I will explain a result of convergence for models obtained from non-commutative rational functions which involve polynomials and quotients of GUE's.

Jonathan Novak (University of California)

Title: *HCIZ, BGW, and other capital letters*

Abstract: The Harish-Chandra/Itzykson-Zuber (HCIZ) and Brezin-Gross-Witten (BGW) integrals are a pair of related matrix integrals which play a prominent role in random matrix theory, representation theory, and free probability. Although neither integral can be exactly evaluated, a longstanding conjecture asserts that they admit asymptotic expansions whose coefficients are generating function for combinatorial invariants of compact Riemann surfaces. I will present a proof of this conjecture, and discuss some of its ramifications.

Applied Mathematics Session

Théo Lacombe (INRIA)

Title: *An Optimal Transport perspective on Topological Data Analysis*

Abstract: Topological Data Analysis (TDA) is a field of data sciences that aims at encoding the structure of complex objects appearing in challenging machine learning tasks (think of graphs, time series, and shapes for instance) through topological descriptors called *persistence diagrams* (PDs). PDs come with specific *optimal partial transport* metrics that are supported by strong theoretical properties, but the resulting *space of persistence diagrams* has a complex structure. Understanding the geometry and the topology of this space is crucial as one targets statistical and learning applications. This presentation will start with a general introduction to TDA and Optimal Transport (OT), the latter being an independently well-developed field in applied mathematics. The second part will be related to research I have done during my Ph.D., in particular we will showcase an important connection between TDA and OT and how this connection allows us to establish new theoretical and computational results in TDA.

Jun Miyanaga (Kyoto University)

Title: *Limit theorems for random cubical complex processes*

Abstract: Recently, multiscale topological features of weighted higher-dimensional cubes have been studied via persistence diagrams in various application fields such as digital image processing. Therefore, it is important to understand their stochastic structure. For that purpose, firstly we extend the previously obtained results on a law of large numbers and a central limit theorem for Betti numbers to persistent Betti numbers. Secondly, we show a law of large numbers for persistence diagrams using the theory of vague convergence for random measures that was developed by Hiraoka, Shirai, and Trinh in 2018. Thirdly, we establish a large deviation principle for histograms of persistence diagrams which represent the average frequency densities on subdivision rectangles. Furthermore, constructing a suitable projective limit using linear maps that average among the values on subdivision rectangles, we show the sequence of the histograms ordered by size of rectangle also satisfies a large deviation principle as elemens in the projective limit. This is a joint work with Yasuaki Hiraoka, Shu Kanazawa, and Kenkichi Tsunoda.

Elisa Davoli (TU Wien)

Title: *Nonlocal-to-local convergence of Cahn-Hilliard equations*

Abstract: In this talk, we will consider a class of nonlocal viscous Cahn-Hilliard equations with Neumann boundary conditions for the chemical potential. The double-well potential will be allowed to be singular (e.g. of logarithmic type), while the singularity of the convolution kernel will not fall in any available existence theory under Neumann boundary conditions. We will prove well-posedness for the nonlocal equation in a suitable variational sense. Secondly, we will show that the solutions to the nonlocal equation converge to the corresponding solutions to the local equation, as the convolution kernels approximate a Dirac delta. The asymptotic behavior will be analyzed by means of monotone analysis and Gamma-convergence results, both when the

limiting local Cahn–Hilliard equation is of viscous type and of pure type. This is based on a series of joint works with H. Ranetbauer, L. Scarpa, and L. Trussardi.

Yuuki Shimizu (Kyoto University)

Title: *A current-valued solution of the Euler equation and its application*

Abstract: There are two mathematical models that describe two-dimensional incompressible and inviscid fluid flows: the Euler flow and the point vortex dynamics. The point vortex dynamics is formally derived from the Euler flow. Since it is given as a finite-dimensional Hamiltonian system, it is easier to deal with mathematically than the Euler flow. In fact, it is adopted as a simple model of two-dimensional turbulence. Then, a natural question arises; How can we interpret the point vortex dynamics as an Euler flow in an appropriate mathematical sense? In other words, we need to determine a space of solutions of the Euler equation to which the point vortex dynamics belongs. This is one of problems in the analysis of the 2D Euler equation. In this talk, we discuss the problem is solved by a geometric method rather than analytic one.

Vikas S. Krishnamurthy (University of Vienna)

Title: *Liouville links and chains: new steady solutions of the 2D Euler equation*

Abstract: We present a large class of new exact solutions to the steady, incompressible 2D Euler equation. The hybrid solutions consist of a set of stationary point vortices embedded in a background sea of Liouville-type vorticity that is exponentially related to the stream function. By considering limiting values (0 and ∞) of a real non-negative parameter A in the solutions, the background vorticity can be turned off, leaving behind point vortices in otherwise irrotational flow. Different point vortex equilibria can appear at the two limits $A=0$ and $A=\infty$. We refer to the family of hybrid equilibria continuously parametrised by A as a "Liouville link". Several such families can be glued together to form a "Liouville chain", which can be of finite or infinite length. We will present the required background material first, followed by a discussion of several examples illustrating the main results. This is joint work with Miles Wheeler, Darren Crowdy and Adrian Constantin.