

The 5th KTGU Mathematics Workshop for Young Researchers

Title and Abstract

Plenary Talk

Yasunori Maekawa (Kyoto University)

Title: *On stability of scale-critical two-dimensional flows*

Abstract: In this talk we survey a recent progress on the stability problem of time-global solutions to the two-dimensional Navier–Stokes equations for viscous incompressible flows. Our main interest is the stability of flows in a scale-critical regime, where the linearity and the nonlinearity are considered to be balanced with each other and the difficulty appears particularly in the two-dimensional case.

Number Theory Session

Bin Xu (Tsinghua University)

Title: *Structure of A-packets for p-adic symplectic and orthogonal groups*

Abstract: The global A-packets appear in Arthur’s conjectural description of the discrete automorphic spectrum of a reductive group over the number field. Their local components, also called local A-packets, are conjectured to be finite sets of irreducible admissible representations of the reductive group over the local field. On the other hand, one has the L-packets from the conjectural local Langlands correspondence. So it is a natural problem to understand the relation between the two. In this talk, I will describe Vogan’s conjectural answer to this question using microlocal geometry. I will also present some results for the p-adic symplectic and orthogonal groups.

Yuanqing Cai (Kyoto University)

Title: *Twisted doubling integrals for classical groups and their non-linear covers*

Abstract: In the 1980s, Piatetski–Shapiro and Rallis discovered a family of Rankin–Selberg integrals for the classical groups that did not rely on Whittaker models. This is the so-called doubling method. It grew out of Rallis’ work on the inner products of theta lifts — the Rallis inner product formula. Recently, a family of global integrals that represent the tensor product L-functions for classical groups (joint with Friedberg, Ginzburg, and Kaplan) and the tensor product

L-functions for covers of symplectic groups (Kaplan) was discovered. These integrals are called twisted doubling integrals and can be viewed as generalizations of the doubling method. In this talk, we explain how to develop the twisted doubling integrals for all classical groups and their non-linear extensions in a more conceptual manner.

Benjamin Collas (Kyoto University)

Title: *Arithmetic homotopic geometry, towards higher symmetries*

Abstract: Following Grothendieck's first insight, such as developed by the seminal work of Nakamura, Tamagawa, Mochizuki et al., anabelian arithmetic geometry deals with first homotopy and scheme properties. This approach mostly ignores the higher homotopy and symmetries formalism that was later envisioned in "Pursuing stacks". The goal of this talk is to provide a modest introduction to these questions for the working arithmetic geometer; We will explain how the Artin-Mazur étale homotopy type arranges geometric structures, arithmetic, and motivic theories in a perfect triangle with respect to higher dimensions and symmetries.

Valentijn Karemaker (Utrecht University and Stockholm University)

Title: *Arboreal Galois representations*

Abstract: Arboreal Galois representations are useful tools to describe the Galois theory of iterates of rational maps. These representations were introduced in the 1980s to study the prime divisors of non-linear recurrences (also called dynamical sequences). We study so-called dynamical Belyi maps; for these rational maps, we fully determine their arboreal Galois representations, and prove a result about the density of prime divisors of the corresponding dynamical sequences. This is joint work with Irene Bouw and Özlem Ejder.

Yu Yang (Kyoto University)

Title: *Raynaud-Tamagawa theta divisors and fundamental groups of curves in positive characteristic*

Abstract: In this talk, I will discuss the theory of Raynaud-Tamagawa theta divisors and its application to the anabelian geometry of curves in characteristic $p > 0$. In particular, I will explain some results concerning the anabelian phenomena for the topological and combinatorial structures of pointed stable curves which were proved by A. Tamagawa and the speaker.

Algebraic Geometry Session

Takahiro Nagaoka (Kyoto University)

Title: *The universal covers of hypertoric varieties and Bogomolov's decomposition*

Abstract: Projective toric varieties have been extensively studied by using the associated polytopes. Similarly, hypertoric varieties can be studied by looking at the associated hyperplane arrangements. In this talk, we consider the (singular) universal covers of affine hypertoric varieties, and we show that taking the universal cover corresponds to taking the “simplification” of the associated hyperplane arrangements. As an application, we give a necessary and sufficient condition for the uniqueness of (holomorphic) symplectic structures on hypertoric varieties, and we establish the analogue of Bogomolov’s decomposition for affine hypertoric varieties. Moreover, as a byproduct of this application, we can show that any (\mathbb{C}^*) -equivariant isomorphism between two smooth hypertoric varieties is automatically maximal hamiltonian torus \mathbb{T} -equivariant. This implies that the $\mathbb{C}^* \times \mathbb{T}$ -equivariant classification is equivalent to the \mathbb{C}^* -equivariant classification.

Ching-Jui Lai (National Cheng Kung University)

Title: *Birational geometry of some Calabi–Yau threefolds*

Abstract: Even with recent great advances in birational geometry (after the work of Mori, Kollar, Birkar–Cascini–Hacon McKernan, Birkar, and many others), the geometry of Calabi–Yau threefolds remains mysterious, for examples, the finiteness problem of CY3’s and the wide open Morrison–Kawamata cone conjecture. In this talk, we focus on yet another open conjecture on the moduli of CY3, known as Reid’s fantasy. We will explain Reid’s fantasy, some of its recent progress, and make it clear how our recent result on birational geometry of some CY3’s plays the role. This is a joint work in progress with Dr. Sz–Sheng Wang (Southern–East University in China).

Akihiro Kanemitsu (Kyoto University)

Title: *Fano manifolds and stability of tangent bundles*

Abstract: A Fano manifold X is defined as a smooth projective variety with ample anticanonical divisor. It was conjectured that, if the Picard rank of a Fano manifold X is one, then the tangent bundle of X is stable or, at least, semistable. In this talk, I will study this conjecture for a certain class of Fano manifolds, which are classified by Pasquier. As a consequence, we will provide counter-examples for the above conjecture.

Yuki Hirano (Kyoto University)

Title: *Stability conditions for 3-fold flops*

Abstract: For a triangulated category, Bridgeland introduced the notion of stability conditions on the category, and he showed that the space of stability conditions has a natural structure of a complex manifold. In this talk, we consider a flopping contraction from a Gorenstein terminal 3–

fold X to the affine scheme associated to a complete local ring with Gorenstein terminal singularity, and we describe the space of Bridgeland stability conditions on a certain triangulated subcategory of the derived category of coherent sheaves on X . As an application, we determine the associated autoequivalence group which acts on the space of stability conditions, and for a simple class of 3-fold flops we describe the double quotient space, called the stringy Kähler moduli space, of the space of stability conditions by the autoequivalence group action and a natural \mathbb{C} -action. This is a joint work with M. Wemyss.

Pierrick Bousseau (Swiss Federal Institute of Technology in Zurich)

Title: *Quasimodular forms from Betti numbers*

Abstract: This talk will be about refined curve counting on local \mathbb{P}^2 , the noncompact Calabi–Yau 3-fold total space of the canonical line bundle of the projective plane. I will explain how to construct quasimodular forms starting from Betti numbers of moduli spaces of one-dimensional coherent sheaves on \mathbb{P}^2 . This gives a proof of some stringy predictions about the refined topological string theory of local \mathbb{P}^2 in the Nekrasov–Shatashvili limit. Partly based on work in progress with Honglu Fan, Shuai Guo, and Longting Wu.

Geometry Session

Hülya Argüz (Versailles Saint–Quentin–en–Yvelines University)

Title: *A tropical and log geometric approach to A-infinity structures*

Abstract: We discuss A-infinity structures in the ring of theta functions, constructed by Gross–Hacking–Siebert as the homogeneous coordinate ring of mirrors to varieties with effective anti-canonical class. Our approach uses tropical geometry and log Gromov–Witten theory. We investigate tropical analogues of holomorphic discs, introduced by Abouzaid–Gross–Siebert as tropical Morse trees, and show a correspondence between tropical Morse trees and certain log Gromov–Witten invariants. Our main object of study for this is the Tate curve, which is a degeneration of elliptic curves to a cycle of projective lines.

Hideo Takioka (Kyoto University)

Title: *On the Γ -polynomial and its cabling for knots*

Abstract: The Γ -polynomial is an invariant for oriented knots in the three sphere, which is the common zeroth coefficient polynomial of the HOMFLYPT and Kauffman polynomials.

Let $p(>0)$ and q be coprime integers. For a knot invariant I , we consider the (p,q) -cabling of I , that is, the map sending a knot K to the value $I(K(p,q))$, where $K(p,q)$ is the (p,q) -cable knot

of K . In this talk, we focus on the (p,q) -cabling of the Γ -polynomial and show some results on the cases $p=1,2,3$.

Ruizhi Huang (Chinese Academy of Sciences)

Title: *String structures, modular invariants and non-abelian group actions*

Abstract: Spin structure and its higher analogies play important roles in index theory and mathematical physics. In particular, Witten genera for String manifolds have nice geometric implications. As a generalization of the work of Chen–Han–Zhang (2011), we introduce the general String^c structures based on the algebraic topology of Spin^c groups. It turns out that there are infinitely many distinct universal String^c structures indexed by the infinite cyclic group. We then construct a family of the so-called generalized Witten genera for Spin^c manifolds, the geometric implications of which can be exploited in the presence of String^c structures. As in the un-twisted case studied by Witten, Liu, etc, in our context there are also integrality, modularity, and vanishing theorems for effective non-abelian group actions. We will give some applications of our vanishing theorem. This is a joint work with Haibao Duan and Fei Han.

Shunsuke Tsuji (Kyoto University)

Title: *The Goldman Lie algebra and the total Johnson homomorphism on a homology cylinder*

Abstract: Using the Goldman Lie algebra of a surface, we introduce a new method of computing the automorphism of the fundamental group induced by a homology cylinder of the surface. Using the method, we can compute the total Johnson homomorphism of the homology cylinder. As an application, we give a formula by Kuno and Massuyeau.

Delphine Moussard (Aix-Marseille University)

Title: *Three-dimensional characterization of the slice genus of knots and links*

Abstract: A slice knot is a knot in the 3-sphere that bounds a disk smoothly embedded in the 4-ball. More generally, the slice genus of a knot is the minimal genus of a surface smoothly embedded in the 4-ball and bounded by the knot. We will prove that the slice genus of a knot can be characterized as the minimal genus of a surface immersed in the 3-sphere except at a finite number of branched points, with no clasp intersection and no triple point of a certain type, and bounded by the knot. This generalizes a characterization of slice knots by Kawauchi, Shibuya and Suzuki. Moreover, we will extend this characterization to the slice genus of a colored link.

PDE Theory Session

Atsushi Nakayasu (Kyoto University)

Title: *Convexity preserving properties for Hamilton–Jacobi equations in geodesic spaces*

Abstract: We study convexity preserving properties for a class of time–dependent Hamilton–Jacobi equations in complete geodesic spaces. Convexity preserving properties for nonlinear evolution equations are well known in the Euclidean space. We extend the classical results for first order equations to the Busemann spaces such as a junction by using a recently developed theory of viscosity solutions on geodesic spaces. This talk is based on a joint work with Qing Liu (Fukuoka University).

Daisuke Kawagoe (Kyoto University)

Title: *$W^{1,p}$ estimate of solutions to the stationary transport equation with the incoming boundary condition*

Abstract: We consider a boundary value problem of the stationary transport equation in a two–dimensional bounded convex domain with the incoming boundary condition. In this talk, we obtain a $W^{1,p}$ estimate of the solution for $1 \leq p < p_m$, where p_m is a real number depending only on the shape of the domain.

Nicholas Edelen (University of Notre Dame)

Title: *Regularity of minimal surfaces near quadratic cones*

Abstract: Hardt–Simon proved that every area–minimizing hypercone C having only an isolated singularity fits into a foliation of \mathbb{R}^{n+1} by smooth, area–minimizing hypersurfaces asymptotic to C . In this talk we prove that if a minimal surface M in the unit ball $B_1 \subset \mathbb{R}^{n+1}$ lies sufficiently close to a minimizing quadratic cone (for example, the Simons’ cone), then $M \cap B_{1/2}$ is a $C^{1,\alpha}$ perturbation of either the cone itself, or some leaf of its associated foliation. In particular, we show that singularities modeled on these cones determine the local structure not only of M , but of any nearby minimal surface. Our result also implies the Bernstein–type result of Simon–Solomon, which characterizes area–minimizing hypersurfaces asymptotic to a quadratic cone as either the cone itself, or some leaf of the foliation. This is joint work with L. Spolaor.

Isao Kato (Kyoto University)

Title: *On the Cauchy problem for the Zakharov system in three spatial dimensions with radial initial data*

Abstract: We study the Cauchy problem for the 3D Zakharov system at the critical space.

Bejenaru and Herr (2011) proved local well-posedness for 3D Zakharov system in subcritical space by the Fourier restriction norm method with convolution estimate. We prove the global well-posedness at the critical space with small and radial initial data by U^2, V^2 method introduced by Koch and Tataru. We follow the argument by Kato and Tsugawa (2017), which was proved small data global well-posedness at the critical space for four and higher spatial dimensions by constructing the solution in U^2, V^2 type spaces. However, in 3D we need spherically symmetric data to obtain well-posedness at the critical space since the critical exponent is negative (resp. 0) for the wave equation (resp. the Schrödinger equation) for 3D case and the Zakharov system has the quadratic nonlinearity, so it seems hard to control some nonlinear interaction between the Schrödinger equation and the wave equation. This is a joint work with Dr. S. Kinoshita (Univ. Bielefeld).

Probability Theory Session

Baris Evren Ugurcan (University of Wuppertal)

Title: *Singular SPDEs and renormalization of Anderson Hamiltonian*

Abstract: In this talk, I will recall and describe the renormalization of the Anderson Hamiltonian in the context of paracontrolled distributions and show the well-posedness theory for stochastic NLS and NLW equations with multiplicative noise on 2 and 3 dimensional torus (joint with Gubinelli-Zachhuber); and then also talk about our recent work on the full space case and explain the renormalization of the Anderson Hamiltonian and associated SPDEs in the full space.

Changji Xu (University of Chicago)

Title: *Random walk among Bernoulli obstacles*

Abstract: Consider a discrete time simple random walk on \mathbb{Z}^d , $d \geq 2$ with random Bernoulli obstacles, where the random walk will be killed when it hits an obstacle. We show that the following holds for a typical environment: conditioned on survival up to time n , the random walk will be localized in a single island. In addition, the limiting shape of the island is a ball which is free of obstacles and the asymptotic volume is also determined. This talk is based on joint works with Jian Ding, Ryoki Fukushima, and Rongfeng Sun.

Takahiro Mori (Kyoto University)

Title: *Asymptotic behaviors of intersection measures*

Abstract: For several independent (possibly different) stochastic processes on the same state space, consider the intersection measure of the processes, which is intuitively regarded as the occupation measure of intersections of the processes. W. König and C. Mukherjee (2013) showed

a Donsker–Varadhan type large deviation principle for the normalized intersection measure when the processes are Brownian motions before exiting some bounded open set. X. Chen and J. Rosen (2005) showed exponential asymptotics of the intersection measure when the processes are Brownian motions or stable processes. In this talk, we discuss extensions of these facts via Dirichlet form techniques, when the state space is compact in some sense. If time allows, we will also talk about recent progress in the case of non-compact ones.

Stefan Junk (Kyoto University)

Title: *Large deviations for directed polymers in the whole weak disorder phase*

Abstract: In the directed polymer model we study a random process affected by a space-time random environment. The process is known to satisfy a large deviation principle, but not much is known about its rate function. In weak disorder, the rate function is known to agree with the rate function on simple random walk only under a sub-optimal L^2 -boundedness assumption. We show that if the discrete-time model is replaced by a natural continuous-time model, then this result holds without assuming L^2 -boundedness. Joint work with Ryoki Fukushima.

Yushi Hamaguchi (Kyoto University)

Title: *Time-inconsistent consumption-investment problems in incomplete markets*

Abstract: We study a time-inconsistent consumption-investment problem in an incomplete market under general discount functions. Time-inconsistency for a dynamic control problem means that the so-called Bellman's principle of optimality does not hold. In such a problem, instead of finding a global optimal control (which does not exist), we seek for a time-consistent Nash equilibrium control, which is a game theoretic concept. We provide a necessary condition and a verification theorem for a Nash equilibrium control in terms of a coupled forward-backward stochastic differential equation. Moreover, we prove the uniqueness of the Nash equilibrium control by showing that the original time-inconsistent problem is equivalent to an associated time-consistent one.

Poster Session

Masayuki Hayashi	<i>Instability of algebraic standing waves for nonlinear Schrödinger equations with double power nonlinearities</i>
Shuji Horinaga	<i>Algebraicity of special values of automorphic L-functions attached to Siegel cusp forms</i>
Wenbin Luo	<i>The continuity of χ-volume functions over adelic curves</i>

Tatsu-Hiko Miura	<i>Navier-Stokes equations in a curved thin domain</i>
Hiroshi Noguchi	<i>Genuine characters on the metaplectic group of $SL_2(\mathbb{Z}_p)$ and the application</i>
Tomoya Ohnishi	<i>Arakelov geometry over a trivially valued field</i>
Kohei Sasaya	<i>Ahlfors regular conformal dimension of metrics on infinite graphs and spectral dimension of the associated random walks</i>
Masahiro Takeda	<i>Cohomology of the classifying spaces of $U(n)$-gauge groups over the 2-sphere</i>