The 4th KTGU Mathematics Workshop for Young Researchers

Title and Abstract

Plenary Talk

Hiroshi Iritani (Kyoto University)

Title: *Geometric quantization and Gromov-Witten invariants for local* P^2

Abstract: In this talk, I will explain geometric quantization appearing in the Gromov-Witten theory of local P^2. We construct a sheaf of "Fock spaces" over the moduli space of elliptic curves with $\Gamma_1(3)$-level structure and show that the Gromov-Witten potential of local P^2 defines a global section of this sheaf. This proves a conjecture of Aganagic-Bouchard-Klemm about the modularity of the Gromov-Witten potential of local P^2. This talk is based on joint work with Tom Coates.

Representation Theory Session

Igor Makhlin (Skolkovo Institute of Science and Technology)

Title: *FFLV degenerations and Gelfand-Tseltin degenerations*

Abstract: I will talk about two families of degenerate representations and flag varieties in type A. The first family (which I will call "FFLV degenerations") generalizes the so-called PBW degenerations and was considered in a joint paper with X. Fang, E. Feigin and G. Fourier written in 2017. These degenerations are compatible with the FFLV monomial basis and in a generic point provide the toric variety associated with the FFLV polytope. The second family ("Gelfand-Tsetlin degenerations") is a more recent construction which was introduced with the purpose of obtaining something similar for Gelfand-Tsetlin polytopes and the corresponding toric degenerations.

Naoki Genra (Kyoto University)

Title: *Screening operators and Parabolic inductions for* W*-algebras*

Abstract: (Affine) W-algebras are a family of vertex algebras defined by Drinfeld-Sokolov reductions. We introduce free field realizations of W-algebras by using Wakimoto representations of affine Lie algebras, which we call Wakimoto representations of W-algebras. Then W-algebras may be described as the intersections of kernels of screening operators. As applications,

parabolic inductions for W-algebras are obtained. This is motivated by results of Premet and Losev on finite W-algebras. In type A, this becomes a chiralization of coproducts by Brundan-Kleshchev. In type BCD, we also have analogs of the coproducts in special cases.

Ievgen Makedonskyi (Kyoto University)

Title: *Semi-infinite Plücker realtions and Weyl modules*

Abstract: First, we write down the semi-infinite Plücker relations, describing the Drinfeld-Plücker embedding of the (formal version of) semi-infinite flag varieties in type A. Second, we study the homogeneous coordinate ring, i.e. the quotient by the ideal generated by the semi-infinite Plücker relations. We establish the isomorphism with the algebra of dual global Weyl modules and derive a new character formula.

Ryo Fujita (Kyoto University)

Title: *Geometric realization of Dynkin quiver type quantum affine Schur-Weyl duality*

Abstract: Associated with a Dynkin quiver, Kang-Kashiwara-Kim constructed a bimodule over the quantum loop algebra and the quiver Hecke (KLR) algebra of the corresponding type as a generalized version of the quantum affine Schur-Weyl duality. They proved that the induced functor gives an isomorphism between Grothendiek rings of suitable monoidal categories of finite-dimensional modules. In this talk, we realize this bimodule via equivariant K-theory of a certain graded quiver variety. This is a Dynkin quiver version of Ginzburg-Reshetikhin-Vasserot's geometric realization of the usual quantum affine Schur-Weyl duality. As a result, we can see that Kang-Kashiwara-Kim's functor actually gives an equivalence of monoidal categories.

Tristan Bozec (Claude Bernard University Lyon 1)

Title: *Irreducible components of the global nilpotent cone*

Abstract: Given a curve X of genus g, the moduli stack of Higgs sheaves of rank r and degree d is known to be of dimension $2(g-1)r^2$. It can be viewed as the cotangent stack of the stack of coherent sheaves of type (r,d) over X, and Laumon proved that the substack of nilpotent Higgs pairs is Lagrangian. This substack is a global analog of the nilpotent cone, and is nothing but the 0-fiber of the Hitchin map. It is highly singular, and one first interesting step toward its comprehension is the study of its irreducible components. This is motivated by the fact that the number of these components which are stable (with respect to the usual slope) is given by the value at 1 of the Kac polynomial associated with the quiver with one vertex and g loops (conjectured by Hausel, Letellier and Rodriguez Villegas, proved by Mellit). I will give

a nice and natural combinatorial description of this set of components, in terms of lattice points of explicit polytopes.

Number Theory Session

Alexander Ivanov (University of Bonn)

Title: *Affine Deligne–Lusztig theory*

Abstract: The classical Deligne–Lusztig theory gives a geometric tool to construct representations of the finite group of rational points of a reductive group over a finite field. We develop an affine version of this, by constructing families of extended affine Deligne–Lusztig varieties attached to a reductive group G over a local field. The cohomology of (certain covers of) these varieties conjecturally allows to realize the automorphic induction from maximal tori in G. For $G=GL_2$ we show that almost all smooth representations of G arise in this way (also for G over a field of characteristic 2).

Piotr Achinger (Polish Academy of Sciences)

Title: *Serre–Tate theory for Calabi–Yau varieties*

Abstract: The classical Serre–Tate theory concerns the deformation theory of ordinary abelian varieties. It implies that their deformation spaces can be equipped with a group structure and a lifting of the Frobenius morphism, and consequently such varieties admit a canonical lifting to characteristic zero. In the talk, I will show how to obtain similar results mod p^2 for ordinary Calabi–Yau varieties of arbitrary dimension. The main tools will be Frobenius splittings and a new construction of relative Witt vectors of length two. This is joint work with Maciej Zdanowicz (EPFL).

Miyu Suzuki (Kyoto University)

Title: *Quaternion distinguished representations and base change for unitary groups*

Abstract: Base change lift is a map from the set of cuspidal automorphic representations of a unitary group to the set of automorphic representations of a general linear group. Flicker and Rallis conjectured that a cuspidal automorphic representation of GL is in the image of the base change lift if and only if it is distinguished with respect to certain subgroup. Considering quaternion distinguished representations, we will propose a slight generalization of this conjecture and prove it for $GL(2)$ by using a relative trace formula.

Daniel Fiorilli (University of Ottawa)

Title: *Chebyshev's bias in Galois groups*

Abstract: This work is joint with Florent Jouve. In this talk we will discuss Chebyshev's bias in the distribution of primes according to Chebotarev conditions. For example we will compare the number of primes pp congruent to 1 modulo 3 for which 2 is a cube modulo p with those for which this condition does not hold. One of our goals will be to study extreme biases, that is we will give conditions on the implied Galois groups which guarantee significant asymmetries. We will see that those questions are strongly linked with the representation theory of this group. For example, in the S_n case we will take advantage of the rich and combinatorial representation theory of the symmetric group — in particular bounds of Roichman, Féray, Sniady, Larsen et Shalev. We will also apply inverse Galois type results.

Kenji Sakugawa (Kyoto University)

Title: *On relative fundamental groupoids of modular curves*

Abstract: The relative unipotent fundamental group of a modular curve is an affine group scheme over the field of rational numbers classifying relatively unipotent Q-local systems over the modular curve. Its ring of regular functions has a natural structure of a system of realizations in the sense of Deligne and the detailed study of its Hodge realization has been started recently by Richard Hain and Francis Brown. In this talk, I explain its motivic construction and a result about its mixed elliptic quotients.

Geometry Session

Morimichi Kawasaki (Kyoto University)

Title: *Function theoretical applications of Lagrangian spectral invariants*

Abstract: Entov and Polterovich et al. gave many applications of Hamiltonian (Oh-Schwarz) spectral invariants. For example, they proved some non-displaceability, gave an upper bound of Poisson bracket invariants. In this talk, I give some Lagrangian versions of their results.

Sobhan Seyfaddini (CNRS, Mathematics Institute of Jussieu-Paris Rive Gauche)

Title: *The Arnold conjecture in continuous settings*

Abstract: One of the several versions of the Arnold conjecture states that a Hamiltonian diffeomorphism of a closed and connected symplectic manifold must have at least as many fixed points as the cuplength of the manifold. In this talk, we will discuss this conjecture for Hamiltonian homeomorphisms.

Kyle Hayden (Columbia University)

Title: *Morse theory for complex curves*

Abstract: Every four-dimensional Stein domain has a Morse function whose regular level sets are contact three-manifolds. This allows us to study complex curves via their intersection with these contact level sets, where we can apply three-dimensional tools. As an application of these ideas, we give a braid-theoretic characterization of the links in Stein-fillable contact manifolds that bound complex curves in their Stein fillings. We will focus on possible future directions and related open questions. (Some of this is joint work in progress with Baykur, Etnyre, Hedden, Kawamuro, and Van Horn-Morris.)

Takahiro Oba (Kyoto University)

Title: *Lefschetz-Bott fibrations on complex line bundles over symplectic manifolds*

Abstract: A Lefschetz-Bott fibration on a symplectic manifold is a smooth map to a surface with only Lefschetz-Bott critical points, which are modeled on Morse-Bott critical points. As Lefschetz fibrations have played an important role in the study of Stein fillings of contact manifolds, we expect Lefscehtz-Bott fibrations to help us understand symplectic fillings. However, little is known about symplectic manifolds admitting such fibrations. In this talk, we show that a good class of complex line bundles over symplectic manifolds admits Lefschetz-Bott fibrations over $\mathbb{C}$. As an application, we give a geometric interpretation of symplectic mapping class group relations of Milnor fibers given by Acu and Avdek.

Wenyuan Yang (Peking University)

Title: *Counting conjugacy classes in groups*

Abstract: In this talk, we introduce a class of statistically convex-cocompact groups and count conjugacy classes when a contracting element is present. Our main result is an asymptotic formula for the number of conjugacy classes of all elements and primitive elements. As corollaries, our results hold for relatively hyperbolic groups, $\mathrm{CAT}(0)$ groups with a rank-1 element, and certain subgroups of mapping class groups. Another consequence of this formula is that the generating function for conjugacy classes is transcendental. This is joint work with Ilya Gekhtman (U. Toronto).

PDE Theory Session

Zihua Guo (Monash University)

Title: *Generalized Strichartz estimates for Schrödinger type equations and applications*

Abstract: In this talk we give a survey on the recent studies for the generalized Strichartz estimates for Schrödinger type equations and their applications to the nonlinear dispersive equations/systems. The generalized Strichartz estimates include: almost sharp estimates in the

radial case or spherically averaged case, and for Schrödinger equations with potential; the applications include: Klein–Gordon equation, Zakharov system, Gross–Pitaevskii equation.

Sung-Jin Oh (Korea Institute for Advanced Study)

Title: *The Threshold Theorem for the hyperbolic Yang–Mills equation*

Abstract: In this lecture, I will present the recent proof (joint with D. Tataru) of the Threshold Theorem for the energy critical hyperbolic Yang–Mills equation in $(4+1)$ dimensions. This theorem provides a sharp criterion for global existence and scattering in terms of the energy of the initial data. Moreover, we prove that failure of global existence/scattering is characterized by "bubbling" of a solution to the harmonic Yang–Mills equation. Our proof lies at the intersection of many recent developments, such as null form estimates and function spaces; parametrix construction via pseudodifferential gauge renormalization; induction on energy; monotonicity formulae arising from the normalized scaling vector field etc. Also of note is the use of the associated parabolic flow, namely the Yang–Mills heat flow, to construct a high quality global gauge (called the caloric gauge), extending the idea of Tao for the harmonic map heat flow.

Ikkei Shimizu (Kyoto University)

Title: *Remarks on local theory for Schrödinger maps near harmonic maps*

Abstract: In this talk, we consider the Schrödinger map equation (Landau–Lifshitz equation). In the study of the equation, one of the major method is to reduce it to some nonlinear Schrödinger equation, which is called the modified Schrödinger map equation. Especially, we focus on the reduction using the Coulomb gauge. We show that the derivation of modified equation can be justified even from rough solutions in the equivariant case. As a consequence, we improve a result on the uniqueness of solutions near harmonic maps.

Helge Dietert (CNRS, Mathematics Institute of Jussieu–Paris Rive Gauche, from Oct. 2018)

Title: *Well-posedness of the Prandtl equation*

Abstract: The Prandtl equation was derived in a formal analysis by Prandtl in 1904 in order to describe the boundary layer behaviour of a viscous flow at high Reynolds number. In this talk, I will present the well-posedness issue and explain a recent result with David Gérard-Varet on the well-posedness in Gevrey regularity without any structural assumption (like the monotonicity of the velocity profile).

Mitsuo Higaki (Kyoto University)

Title: *On the stability of planar stationary flows in a non-symmetric exterior domain*

Abstract: We consider the asymptotic stability of two-dimensional stationary flows in an exterior

domain without symmetry. Especially, we prove the local L^2-stability of a flow whose leading profile is the rotating flow decaying in the scale-critical order $O(|x|^{-1})$.

Probability Theory Session

Toru Sera (Kyoto University)

Title: *Functional limit theorem for sojourns near indifferent fixed points*

Abstract: Interval maps with indifferent fixed points are typical examples of infinite ergodic transformations. They have been studied as models of intermittent phenomena, such as an intermittent transition to turbulence in convective fluids. In this talk, we will present a functional limit theorem for their occupation measures, where the limit processes are concentrated to indifferent fixed points.

Takuya Murayama (Kyoto University)

Title: *Characterization of the explosion time for the Komatu-Loewner evolution*

Abstract: The chordal Komatu-Loewner equation extends the chordal Loewner equation in the upper half-plane H, which is well known in the SLE theory, to a standard slit domain D. Through a certain procedure, the chordal Loewner equation produces a family of bounded sets $\{F_t\}$ called hulls. In the SLE theory, such a family represents the scaling limits of two-dimensional stastical physical models. The Komatu-Loewner equation also produces such a family of growing hulls, which we call the Komatu-Loewner evolution. In this talk, I will characterize the explosion time for the Komatu-Loewner equation for the slits (in the range D_t of the canonical map $g_t:D\setminus F_t\to D_t$ in a natural fashion with relation to the exit time of the evolution F_t from the domain D. My result is a refinement of a part of the study by Bauer and Friedrich (2008).

Eviatar Procaccia (Texas A&M University)

Title: *Stabilization of Diffusion Limited Aggregation in a Wedge*

Abstract: We prove a discrete Beurling estimate for the harmonic measure in a wedge in Z^2, and use it to show that Diffusion Limited Aggregation (DLA) in a wedge of angle smaller than $\pi/4$ stabilizes. This allows to consider the infinite DLA and questions about the number of arms, growth and dimension. I will present some conjectures and open problems.

Yuki Tokushige (Kyoto University)

Title: *The construction of reflecting random walks on a hyperbolic group and the associated jump processes on the Gromov boundary*

Abstract: In this talk, we will discuss the construction of reflecting random walks on a hyperbolic group and the associated jump processes on the Gromov boundary under an assumption that the Gromov boundary satisfies a certain low-dimensional property. We will also discuss some potential theoretic properties of harmonic measures of random walks. (based on a joint work with Pierre Mathieu)

Sebastian Andres (University of Cambridge)

Title: *Green kernel asymptotics for two-dimensional random walks under random conductances*

Abstract: The random conductance model is a well-established model for a random walk in random environment. In recent years the behaviour of the associated heat kernel and Green function has been intensively studied, and in dimension $d \geq 3$ the asymptotics of the Green kernel are meanwhile quite well-understood. In this talk we present precise asymptotics of the potential kernel and the Green function of the walk killed upon exiting balls in dimension $d=2$. This result holds, for instance, in the case of uniformly elliptic conductances, random walks on supercritical percolation clusters or ergodic degenerate conductances satisfying a moment condition. At the end we discuss an extension to the time-dynamic random conductance model and an application to the Ginzburg-Landau $\nabla\phi$-interface model. This talk is based on a joint work with Jean-Dominique Deuschel and Martin Slowik (TU Berlin).

Poster Session

Daiki Hayashida	*The characteristic polynomials of abelian varieties over finite fields*
Affonso Henrique	*Bow varieties for the symplectic group*
Takahiro Murotani	*A p-adic analytic approach to the absolute Grothendieck conjecture*
Shuta Nakajima	*Ergodic property of infinite geodesics in First-passage percolation*
Kenshiro Tashiro	*A sufficient condition for the rapid convergence to the asymptotic cone for nilpotent groups*
Takuto Yamamoto	*Randomized Final-Date Problem for Systems of Nonlinear Schrödinger Equations*
Kosuke Yamato	*A limit theorem for inverse local times of jumping-in diffusion processes*