# The 2nd KTGU Mathematics Workshop for Young Researchers

## Title and Abstract

**Plenary Talk:**

Yuji Odaka (Kyoto University, Japan)

Title: *Canonical metrics, Moduli-theoretic heights and Singularities*

Abstract: Thanks to the works of Klein, Poincaré and Koebe, done more than a century ago, compact Riemann surfaces are known to admit unique (Kahler) metric with constant Gaussian curvature. Calabi initiated a generalization of such "canonical" Kahler metrics to higher dimensional complex (Kahler) manifolds in his inspiring works in 50s and there has been much progress since then. In particular, nowadays a variety of equivalences between existence of such canonical Kahler metrics and certain purely algebra-geometric notions is one of the central topics of complex differential geometry. At the same time, we've been observing connections with other fields, including some unexpected. In the talk I will start with reviewing the story of such canonical metrics and then focus to show how it is actually related to *moduli theory*, (*birational geometry*,) and *arithmetics geometry*. We do so especially through analyzing the modular height I introduced at 1508.07716 which extends the Faltings height (1983) for arithmetic abelian varieties to general arithmetic varieties. Lastly I will also introduce some recent development of "local analogue" - i.e., for algebraic singularities or affine cones rather than projective varieties, mainly due to other people's works. It has motivations from "local canonical metric" around singularities, Sasakian geometry - a odd-dimensional analogue of Kahler geometry.

---

**Number Theory**

Kęstutis Česnavičius (UC Berkeley, USA)

Title: *The Manin constant in the semistable case*

Abstract: For an optimal modular parametrization $J_0(n) \twoheadrightarrow E$ of an elliptic curve $E$ over $\mathbb{Q}$ of conductor $n$, Manin conjectured the agreement of two natural $\mathbb{Z}$-lattices in the $\mathbb{Q}$-vector space $H^0(E, \Omega^1)$. Multiple authors generalized his conjecture to higher dimensional newform quotients. We will discuss the semistable cases of the Manin conjecture and of its generalizations using a technique that establishes general relations between the integral $p$-adic étale and de Rham cohomologies of abelian varieties over $p$-adic fields.

Yunqing Tang (Institute for Advanced Study, USA)

Title: *Cycles in the de Rham cohomology of abelian varieties over number fields*

Abstract: In his 1982 paper, Ogus defined a class of cycles in the de Rham cohomology of smooth proper varieties over number fields. In the case of abelian varieties, this class includes all the Hodge cycles by the work of Deligne, Ogus and Blasius. Ogus predicted that all such cycles are Hodge. In this talk, I will first introduce Ogus' conjecture as a crystalline analogue of Mumford–Tate conjecture and explain how a theorem of Bost (using methods *à la* Chudnovsky) on algebraic foliation is related. After this, I will discuss the proof of Ogus' conjecture for some families of abelian varieties under the assumption that the cycles lie in the Betti cohomology with real coefficients.

Adam Topaz (University of Oxford, UK)

Title: *Galois groups and automorphisms of fundamental groups*

Abstract: Following the spirit of Grothendieck's Esquisse d'un Programme, the Ihara/Oda-Matsumoto conjecture predicted a combinatorial description of the absolute Galois group of $\mathrm{Q}$ based on its action on geometric fundamental groups of varieties. This conjecture was resolved by Pop in the 90's using anabelian techniques. In this talk, I will discuss the proof of a stronger variant of this conjecture, which deals with mod-$\ell$ two-step nilpotent quotients of fundamental groups.

Shunsuke Yamana (Kyoto University, Japan)

Title: *Modular forms and representation numbers*

Abstract: It is one of classical problems in number theory to determine the number of representations of a symmetric matrix by a quadratic form. The main theorem of Siegel's analytic theory of quadratic form is a local-global relation between some average of representation numbers and a product of local data consisting of $\mathrm{p}$-adic representation densities. In this talk I will give a product formula for some different sum of representation numbers of a symmetric matrix of rank $2\mathrm{n}$ by quadratic forms of rank $4\mathrm{n}$. This formula is associated to a modular form of weight $2\mathrm{n}$ and can be considered as a generalization of the Siegel formula.

Hiraku Atobe (Kyoto University, Japan)

Title: *A conjecture of Gross-Prasad and Rallis for metaplectic groups*

Abstract: Let $\mathrm{G}$ be a quasi-split connected reductive group over a non-archimedean local field $\mathrm{F}$. We say that an irreducible smooth representation of $\mathrm{G}(\mathrm{F})$ is generic if it admits a

Whittaker model. The local Langlands conjecture (LLC) classifies irreducible smooth representations of $G(F)$ by $L$-parameters. Gross-Prasad and Rallis conjectured the $L$-parameters corresponding to generic representations are characterized by regularity of their adjoint $L$-functions (GPR). In this talk, we discuss (GPR) for metaplectic groups, which are double cover of symplectic groups, and are not algebraic groups.

---

**Representation Theory**

Sachin Gautum (Ohio State University, USA)

Title: *Quantum groups and difference equations*

Abstract: Infinite-dimensional quantum groups precede historically their finite-dimensional counterparts, and were discovered during 1970's in the study of exactly solved models of statistical mechanics. By now their structures and representation theories are quite well understood, while a lot of questions still remain open. In this talk, I will explain how the monodromy of difference equations can be used to answer a few of these questions. The use of difference equations in the theory of affine quantum groups is nothing new. However the family of equations we shall use seems to be. We will exploit this new technique to find explicit connections between various quantum groups, and relating their tensor structures. This talk is based on my joint research with V. Toledano Laredo.

Jiuzu Hong (University of North Carolina at Chapel Hill, USA)

Title: *Fusion ring, conformal blocks and diagram automorphisms*

Abstract: Let $G$ be a simply-connected algebraic group. By Verlinde formula, a fusion ring can be interpreted as the commutative ring of functions on certain finite subset of the maximal torus in $G$. Moreover the structure coefficients correspond to the dimension of conformal blocks associated to $G$. In this talk, I will define a variation of the fusion ring for each non simply-laced algebraic group $G$, and interpret the structure coefficients as the traces of the diagram automorphism action on the spaces conformal blocks associated to a simply-laced group $G'$ that is closely related to $G$.

Ivan Ip (Kyoto University, Japan)

Title: *Cluster realization of $U_q(g)$ and factorization of universal $R$ matrix*

Abstract: For each simple Lie algebra $g$g, I will talk about a new presentation of an embedding of $U_q(g)$ into certain quantum torus algebra, described by a quiver diagram, using the previous construction of positive representations of split real quantum groups.

Furthermore, with this realization we derive a factorization of the universal $R$ matrix which corresponds to a sequence of quiver mutations giving the half-Dehn twist of the triangulation of a twice-punctured disk with two marked points. This generalizes the well-known result of Faddeev for type $A_1$ and the recent work of Schrader-Shapiro for type $A_n$.

Tatsuyuki Hikita (RIMS, Kyoto University, Japan)

Title: *Tilting generators for hypertoric varieties*

Abstract: In this talk, I will give some formula for tilting generators of the derived category of coherent sheaves on smooth hypertoric varieties as direct sum of explicit line bundles. In particular, we construct certain family of t-structures which are analogous to the exotic t-structures on the Springer resolutions or Slodowy varieties constructed by Bezrukavnikov-Mirkovic. If I have time, I will explain some conjectures which might give a strategy to find tilting generators for conical symplectic resolutions with good Hamiltonian torus action.

---

**Differential Geometry**

Georgios Dimitroglou Rizell (Uppsala University, Sweden)

Title: *The nearby Lagrangian conjecture for the two-torus*

Abstract: In recent joint work with E. Goodman and A. Ivrii we establish several classification results for two-dimensional Lagrangian tori inside four-dimensional symplectic manifolds. Notably, we establish the nearby Lagrangian conjecture in this context, which can be formulated as follows: any Lagrangian torus inside the cotangent bundle of a torus which is homotopic to the zero section, is Hamiltonian isotopic to the graph of a closed one-form on the torus.

Yanli Song (Dartmouth College, USA)

Title: *Quantization of Hamiltonian $LG$-spaces*

Abstract: In this talk, I will discuss an approach to the quantization of an infinite dimensional Hamiltonian loop group space. We construct a Spin$^c$ structure on a finite-dimensional cross-section, and show that the corresponding Dirac operator has a well-defined index in the completion of the representation ring of the maximal torus. We study the multiplicities by deforming the operator with a suitable vector field. A quantization-commutes-with-reduction result follows from an interesting inequality just involving certain Lie-algebra data. This is a joint work with Yiannis Loizides and Eckhard Meinrenken.

Sho Hasui (Kyoto University, Japan)

Title: *On the classification of quasitoric manifolds*

Abstract: A quasitoric manifold is a $2n$--dimensional manifold with a good action of the compact torus $T^n=(S^1)^n$ of which the orbit space is naturally regarded as a simple polytope. Quasitoric manifolds are introduced by Davis and Januszkiewicz in 1991 as a topological counterpart of non-singular toric varieties. As the toric varieties are in one-to-one correspondence with the fans, the quasitoric manifolds are in one-to-one correspondence with a kind of combinatorial objects, called characteristic maps. Moreover, any projective non-singular toric variety is a quasitoric manifold. This talk focuses on the classification of quasitoric manifolds up to homeomorphism.

Takumi Yokota (RIMS, Kyoto University, Japan)

Title: *Barycenter of probability measures on $CAT(1)$-spaces of small radii*

Abstract: $\mathrm{CAT}$-spaces are metric spaces with upper curvature bounds in the sense of Alexandrov. Barycenter of probability measures on $\mathrm{CAT}(0)$-spaces, which are non-positively curved spaces, plays fundamental roles in the studies of $\mathrm{CAT}$-spaces in various research areas. In this talk we discuss the unique existence and properties of barycenter of probability measures on $\mathrm{CAT}(1)$-spaces. For this we employ Kendall's convex function instead of the convexity of distance functions.

---

**Probability Theory**

Erich Baur (École Normale Supérieure de Lyon, France)

Title: *Planar quadrangulations with a boundary and their limiting behavior*

Abstract: We discuss distributional limits of uniform random planar quadrangulations with a boundary when their size tends to infinity. Depending on the asymptotic behavior of the boundary length and of the scaling, we observe different limiting metric spaces, among them the Brownian half-plane with skewness, the infinite-volume Brownian disk and the infinite continuum random tree. Based on joint works with Grégory Miermont, Gourab Ray, and Loïc Richier.

Manuel Cabezas (Pontifical Catholic University of Chile)

Title: Scaling limit for the ant in a high-dimensional labyrinth

Abstract: It is believed that in high dimensions, a large critical percolation cluster should

scale to the so-called integrated super Brownian excursion (ISBE). Moreover, it is also believed that a simple random walk in the critical percolation cluster should scale to the Brownian motion on the ISBE. In this talk I will present a result that gives conditions for a general sequence of random subgraphs of $Z^d$ under which the random walk on these graphs scales to the Brownian motion on the ISBE. We will show how to apply this general theorem in the case where the graphs are obtained as the trace of critical branching random walks in $Z^d$, $d>12$. Joint work with Gerard Ben Arous and Alexander Fribergh.

Irmina Czarna (University of Wrocław, Poland)

Title: *Parisian ruin in risk theory*

Abstract: In the last few years, the idea of Parisian ruin has attracted a lot of attention. In Parisian-type ruin models, the insurance company is not immediately liquidated when it defaults: a grace period is granted before liquidation. In this talk I will formally define Parisian ruin, which generalizes the classic approach. Moreover, I will investigate Parisian ruin for a Lévy surplus process and a Lévy surplus process with an adaptive premium rate, namely a refracted Lévy process. More general Parisian boundary-crossing problems with a deterministic implementation delay are also considered. Examples are provided.

Kei Noba (Kyoto University, Japan)

Title: *Generalized refracted Lévy process and its application to exit problem*

Abstract: Generalizing Kyprianou-Loeffen's refracted Levy processes, we define a new refracted Lévy process which is a Markov process whose behaviors during the period of non-negative values and during that of negative ones are Lévy processes different from each other. To construct it we utilize the excursion theory. We study its exit problem, the potential measures of the killed processes and some approximation theorems. This study is the joint work with Professor Kouji Yano (Kyoto University).