KTGU Mathematics Workshop for Young Researchers

Title and Abstract

Plenary Talks:

Jamie Mingo (Queen's University, Canada)

Title**:** *Cumulants: Free and Classical*

Abstract**:** In probability theory the method of moments is frequently used to test for the convergence of a sequence of random variables. Another sequence which serves the same purpose and is frequently easier to work with is the sequence of cumulants. A crucial feature of cumulants is that they work very well with independence. For non-commuting random variables there are other kinds of independence, in particular Voiculescu's free independence. For this theory there are analogues of cumulants called the free cumulants. In the free case the theory of cumulants is much richer and there are higher order versions for which no classical analogue exists. I will explain the construction and use of these higher order versions.

Hiraku Nakajima (Kyoto University, Japan)

Title**:** *Coulomb branches of gauge theories*

Abstract**:** I will explain the underlying idea of my recent construction of certain symplectic varieties called Coulomb branches, for non-experts. A key is to use hypothetical 3D-topological quantum field theories as building blocks.

Algebra

Alexander Ellis (University of Oregon, USA)

Title**:** *Quantum $gl(1|1)$ and tangle Floer homology*

Abstract**:** The Reshetikhin-Turaev construction associates a polynomial link invariant to a quantum Kac-Moody algebra and a choice of representation. The work of Khovanov, Lauda, Rouquier, and Webster on the 2-representation theory of these algebras has given us link homology theories for all these Kac-Moody types. By contrast, the knot Floer homology of Ozsváth-Szab• Ewhich categorifies the Alexander polynomial, arises from pseudoholomorphic curve counting. We start to bridge the gap between Lie-theoretic and Floer-theorietic link homology theories by showing that the recent combinatorial tangle

Floer homology of Petkova-Vértesi categorifies the construction of the Alexander polynomial as the Reshetikhin-Turaev construction for quantum $\mathrm{gl}(1|1)$'s vector representation. This is joint work with Ina Petkova and Vera Vértesi.

Andrew Macpherson (IHÉS, France)

Title**:** *A "non-commutative" approach to non-Archimedean geometry*

Abstract**:** That Grothendieck's algebraic geometry can be defined in terms of categories of modules over commutative rings and descent conditions has been understood since the time of FGA. In recent times, there has been some interest in getting this kind of framework to run for non-Archimedean geometry. In this talk, I'll explain how certain module categories over "topological" rings can be used to characterise Raynaud's definition of analytic geometry via "admissible" blow-ups.

Ren He Su (Kyoto University, Japan)

Title**:** *The Kohnen plus space and Jacobi form*

Abstract**:** The Kohnen plus space is a subspace of the space of modular forms of half-integral weight. It was initially introduced by Kohnen in 1980 and is characterized by the Fourier coefficients of the forms in it. For example, in the classic case, the Kohnen plus space of weight $k+1/2$ consists of forms whose Fourier coefficients $c(n)$ vanish unless $(-1)^{kn}$ is congruent to 0 or 1 mod 4. It was later shown by Eichler and Zagier in 1985 that the plus space is, in the classic case, isomorphic to the space of Jacobi forms. In this talk, we want to generalize the case to Hilbert-Siegel modular forms. We will see how the isomorphism works and that it is actually deeply related to the Weil representation.

You Qi (Yale University, USA)

Title**:** *Categorification at prime roots of unity*

Abstract**:** We sketch an algebraic approach to categorification of quantum groups at a prime root of unity, with the scope of eventually categorifying Witten-Reshetikhin-Turaev three-manifold invariants. This is based on joint work of the speaker with B. Elias, M. Khovanov and J. Sussan.

Hang Xue (Max Planck Institute, Germany)

Title**:** *The Gan-Gross-Prasad conjecture for Fourier-Jacobi periods*

Abstract**:** We will explain the Gan-Gross-Prasad conjecture for Fourier-Jacobi periods and its refinements. We will also explain its relation with the Ichino-Ikeda's conjecture.

Geometry

Francesca Iezzi (University of Warwick, UK)

Title**:** *Graphs of curves, arcs, and spheres, and connections between all these objects*

Abstract**:** Given a surface S, the curve graph of S is defined as the graph whose vertices are simple closed curves on S up to isotopy, where two vertices are adjacent if the two corresponding curves can be realised as disjoint curves. This object was defined by Harvey in the 70's, and has been an extremely useful tool in the study of surface mapping class groups. Similarly one can define the arc graph of a surface with boundary, and the sphere graph of a 3manifold. In this talk I will introduce all these objects, describe some of their properties and some maps between these objects. Time permitting, I will describe some joint work with Brian Bowditch, where we prove that, under particular hypothesis, there exists a retraction of the sphere graph of a 3manifold onto the arc graph of a surface.

Ailsa Keating (Columbia University, USA)

Title**:** *Higher-dimensional Dehn twists and symplectic mapping class groups*

Abstract**:** Given a Lagrangian sphere S in a symplectic manifold M of any dimension, one can associate to it a symplectomorphism of M, the Dehn twist about S. This generalises the classical two-dimensional notion. These higher-dimensional Dehn twists naturally give elements of the symplectic mapping class group of M, i.e. $\pi_0(\mathrm{Symp}(M))$. The goal of the talk is to present parallels between properties of Dehn twists in dimension 2 and in higher dimensions, with an emphasis on relations in the mapping class group.

Jing Mao (Harbin Institute of Technology, Weihai, China)

Title**:** *A Cheng-type eigenvalue comparison theorem and its applications*

Abstract**:** Given a manifold M, we build two spherically symmetric model manifolds based on the maximum and the minimum of its curvatures. We then show that the first Dirichlet eigenvalue of the Laplace-Beltrami operator on a geodesic disk of the original manifold can be bounded from above and below by the first eigenvalue on geodesic disks with the same radius on the model manifolds. These results (which we call *Cheng-type eigenvalue comparison theorem*) may be seen as extensions of classical Cheng's eigenvalue comparison theorems, where the model constant curvature manifolds have been replaced by more general spherically symmetric manifolds. To prove this, we extend Rauch's and Bishop's comparison theorems to this setting. Besides, some interesting examples will be discussed to show intuitively the advantage of our results. This talk is based on a joint-work

with Prof. Pedro Freitas and Prof. Isabel Salavessa in CVPDE.

Tapio Rajala (University of Jyväskylä, Finland)

Title**:** *Planar Sobolev extensions*

Abstract**:** I will review results on Euclidean domains that admit a bounded extension operator from the first order Sobolev space defined on the domain to the corresponding Sobolev space defined on the whole space. The main result is a geometric characterization of bounded simply connected W^{1,p}-extension domains in the planar case for $1<p<2$ which implies an interesting duality result for extension domains. This is joint work with Pekka Koskela and Yi Zhang.

Hang Wang (University of Adelaide, Australia)

Title**:** *A fixed-point theorem on noncompact manifolds*

Abstract**:** The Lefschetz number of an isometry of a compact manifold measures of the “size" of the fixed-point set. This is incorporated in the Atiyah-Segal-Singer fixed point theorem, by computing the equivariant index of an elliptic operator on a compact manifold, equipped with a compact Lie group action. In this talk the Atiyah-Segal-Singer fixed point formula is generalized to noncompact manifolds. We use tools from K-theory and noncommutative geometry to deal with elliptic operators having infinitely dimensional kernels and explore applications in representation theory of some noncompact Lie groups and positive scalar problems in differential geometry. This talk represents joint work with Peter Hochs.

Analysis

Yoshihiro Abe (RIMS, Kyoto University, Japan)

Title**:** *Extreme value statistics of random models on trees*

Abstract**:** I will consider random models on trees such as branching Brownian motions, branching random walks, and local times for simple random walks on binary trees. I will describe known and new results on extrema of these models and show that the laws of the maxima have similar asymptotic behavior.

Antonio Auffinger (Northwestern University, USA)

Title**:** *The Parisi Formula: duality and equivalence of ensembles*

Abstract**:** In 1979, G. Parisi predicted a variational formula for the thermodynamic limit of the free energy in the Sherrington-Kirkpatrick model and described the role played by its minimizer, called the Parisi measure. This remarkable formula was proven by Talagrand in 2006. In this talk I will explain a new representation of the Parisi functional that finally connects the temperature parameter and the Parisi measure as dual parameters. Based on joint-works with Wei-Kuo Chen.

Diego Ayala (University of Michigan, USA)

Title**:** *Extreme Vortex States in Hydrodynamic Systems*

Abstract**:** By numerically solving suitable constrained optimization problems, we assess the sharpness of analytic estimates for the instantaneous rate of growth and the finite-time growth of certain norms of solutions to the Navier-Stokes equation in 2 and 3 dimensions. Connections with the problem of finite-time singularity formation in the three-dimensional case are addressed.

Stephen Gustafson (University of British Columbia, Canada)

Title**:** *Ground states and dynamics for perturbed critical nonlinear Schrödinger equations*

Abstract**:** The “energy critical" nonlinear Schrödinger equation has explicit static “ground state" solutions. For perturbed versions of this equation, standing-wave solutions can be found by either a variational method, or as (degenerate) perturbations of critical static solutions. We show these two constructions agree, and use the variational characterization to classify the dynamics “below" these perturbed ground states. This is joint work with Matt Coles and Tai-Peng Tsai.

Kohei Suzuki (Kyoto University, Japan)

Title**:** *Equivalence between convergence of Brownian motions and convergence of metric measure spaces satisfying $RCD^*(K,N)$ conditions*

Abstract**:** By recent developments of geometric analysis, one can define Brownian motions as diffusion processes associated with the so-called Cheeger energies on non-smooth spaces, which are no more manifolds, but have metric measure structures and synthesized Ricci curvature bounds. Since Brownian motions are determined only by information of the underlying metric measure structures, behaviour of Brownian motions should be related to properties of the underlying geometry. In this talk, we focus on two notions of convergences, one is the measured Gromov-Hausdorff convergence of the underlying

metric measure spaces, and the other is the convergence in law of Brownian motions. We show that these two convergences are equivalent under Riemannian curvature-dimension conditions $\mathrm{RCD}^*(K,N)$ with uniform diameter bounds.