SGU Mathematics Kickoff Meeting
March 9–10, 2015

PROGRAM

March 9th (Monday)	
14:30-14:40	Opening Address (Shigefumi Mori)
14:40-15:40	Manfred Lehn (Johannes Gutenberg-Universität Mainz)
	Rational curves and symplectic manifolds
16:10-17:10	Herbert Koch (Universität Bonn)
	The spaces U^p and V^p and global solutions to dispersive equations
18:00-	Reception
March 10th (Tuesday)	
10:00-11:00	Thomas Schick (Georg-August-Universität Göttingen)
	The topology of positive scalar curvature
11:20-12:20	Minhyong Kim (University of Oxford)
	Fundamental groups, reciprocity laws, and Diophantine equations
12:20-14:00	Lunch
14:00-15:00	Gilles Pisier (Texas A&M University)
	Quantum expanders, random matrices and geometry of operator spaces
15:30-16:30	Erwin Bolthausen (Universitt Zürich)
	Random polymers
16:30-	Closing Address (Atsushi Moriwaki)

Titles and Abstracts

Manfred Lehn (Johannes Gutenberg-Universität Mainz)

Rational curves and symplectic manifolds

A symplectic manifold in the context of this talk is a compact complex manifold with the additional structure that each tangent space is equipped with a skew-symmetric bilinear form which, when expressed in terms of local coordinates, varies holomorphically. Whereas there are plenty of examples for the real version of this notion, compact complex symplectic manifolds are rather difficult to construct. A famous and by now classical paper of Mukai shows how such manifolds can be obtained as moduli spaces of vector bundles on a certain complex surface. The key insight is that geometric properties of a point in such a parameter space can be intrinsically expressed in terms of the properties of the object that the point represents. Taking this idea further we consider moduli spaces (=parameter spaces) of rational curves that lie on a fourdimensional hypersurface of degree 3 and show how new symplectic manifolds can be obtained from these by a contraction process. This relates to work of Kuznetsov on the derived category of cubic fourfolds and connects with another strand of Mukai's early paper. In the talk I want to give an introduction to this circle of ideas and present some recent developments.

Herbert Koch (Universität Bonn)

The spaces U^p and V^p and global solutions to dispersive equations

Functions of finite p variation occur at various places in mathematics: Dispersive equations, harmonic analysis, and the rough path theory in probability. I will explain their definition and properties, the relation to Strichartz estimates and bilinear estimates, and the application to several dispersive equations.

Thomas Schick (Georg-August-Universität Göttingen)

The topology of positive scalar curvature

Given a smooth compact manifold M without boundary, does it admit a Riemannian metric with positive scalar curvature everywhere? This question reveals deep connections between geometry, topology, and analysis (through the modern method to answer it). The most classical answer is given for 2-dimensional surfaces by the Gauss-Bonnet theorem: if a manifold has positive curvature then its Euler characteristic is positive. In higher dimensions, the role of the Euler characterestic is taken by the index of the Dirac operator. To make efficient use of this, recent developments of operator algebras have to be used. The talk will mainly try to introduce the basics of these exciting methods, and explain in examples how it works.

Minhyong Kim (University of Oxford)

Fundamental groups, reciprocity laws, and Diophantine equations

In the late 19th century, Kurt Hensel introduced non-Archimedean completions of algebraic number fields, which was used soon afterwards by Minkowski and Hasse to study Diophantine equations via the so-called local-to-global principles. During the first world war, Takagi proved the existence theorem for class fields, leading up to his lecture at the ICM in Strasbourg, and laid the foundation for Artin's reciprocity law of class field theory. In this lecture, we will remark briefly on subsequent developments, concluding with a description of recent attempts to refine Hasse principles using arithmetic fundamental groups and non-abelian reciprocity laws.

Gilles Pisier (Texas A&M University)

Quantum expanders, random matrices and geometry of operator spaces

Using random unitary matrices, we show that there are well separated families of quantum expanders with asymptotically the maximal cardinality allowed by a known upper bound. This has applications of "geometric" nature, for the operator space analogue of Euclidean geometry. This allows us to provide sharp estimates for the growth of the multiplicity of M_N-spaces needed to represent (up to a constant $C > 1$) the M_N-version of the n-dimensional operator Hilbert space OH_n as a direct sum of copies of M_N. We show that, when C is close to 1, this multiplicity grows as $\exp \beta n N^2$ for some constant $\beta > 0$. The main idea is to identify quantum expanders with "smooth" points on the matricial analogue of the unit sphere, and to show that there are plenty of "uniformly smooth" points (more precisely as many as allowed by a soft metric entropy dimensional restriction). This generalizes to operator spaces a classical geometric result on n-dimensional Hilbert space (corresponding to $N = 1$). Our work strongly suggests to further study a certain class of operator spaces that we call matricially subGaussian.

Erwin Bolthausen (Universersitt Zürich)

Random polymers

A (directed) random polymer in $d + 1$ dimensions is a (standard) random walk in d dimensions where the time axis is considered as an additional dimension, which is transformed by a random potential in space and time. The best known and most famous case is the directed polymer in random environment which has a potential given by independent random variables in space and time. Some of the basic questions are open even in $1 + 1$ dimensions which is believed to be connected with the KPZ universality class. The only case of the $1 + 1$ dimensional directed polymer which has been fully analyzed is a very special one investigated by Kurt Johannson. Important partial results on the directed polymers have been obtained by Imbrie-Spencer, Bolthausen, Comets-Shiga-Yoshida, Carmona, Petermann, and many others.

The main focus of the talk will however be on the so-called copolymer, first discussed in the physics literature by Garel, Huse, Leibler and Orland in 1989 which models the behavior of a polymer at an interface. Important rigorous results have first been obtained by Sinai and Bolthausen-den Hollander, and have later further been developed by Bodineau, Giacomin, Caravenna and others. A basic object of interest is a critical line in the parameter space which separates a localized phase from a delocalized one. Particularly interesting is the behavior at the weak disorder limit where the phase transition is characterized by a universal critical tangent whose existence had first been proved in the Bolthausen-den Hollander paper, and whose exact value is still open. In a recent paper by Bolthausen-den Hollander-Opoku, a new lower bound has been derived, disproving a long-standing conjecture from the physics literature. The bound was obtained by an application of a sophisticated large deviation theory developed by Birkner-Greven-den Hollander. We present a sketch of an elementary version of this new bound.