

# The 1st MMS Workshop for Young Researchers

Date: November 20(Wed) - 21(Thu), 2024

Venue: Department of Mathematics, Science Bldg. No.3, North Campus, Kyoto University

| Time                      | Representation Theory<br>(Room 127)           | Topology<br>(Room 251)                      | Operator Algebra<br>(Room 152)            | Applied Mathematics<br>(Room 110)            |
|---------------------------|-----------------------------------------------|---------------------------------------------|-------------------------------------------|----------------------------------------------|
| 20th November (Wednesday) |                                               |                                             |                                           |                                              |
| 9:30–9:55                 | Reception (Room 109)                          |                                             |                                           |                                              |
| 9:55–10:00                | Opening                                       |                                             |                                           |                                              |
| 10:00–11:00               | Plenary Talk (Room 110): Kenta Ishimoto       |                                             |                                           |                                              |
| 11:15–12:15               |                                               |                                             |                                           | Kota Takeda<br>(Chair: Xin Tong)             |
| 12:15–13:30               | Lunch Break                                   |                                             |                                           |                                              |
| 13:30–14:30               | Iva Halacheva<br>(Chair: Sin-Myung Lee)       | Marc Kegel<br>(Chair: Hajime Kubota)        | Jonathan Taylor<br>(Chair: Ryoya Arimoto) | Xin Tong<br>(Chair: Yoshiki Hiruta)          |
| 14:45–15:45               | Sin-Myung Lee<br>(Chair: Iva Halacheva)       | Hajime Kubota<br>(Chair: Marc Kegel)        | Ryoya Arimoto<br>(Chair: Jonathan Taylor) | Yoshiki Hiruta<br>(Chair: Kota Takeda)       |
| 16:00–17:00               | Poster Session (Room 108)                     |                                             |                                           |                                              |
| 17:30–19:30               | Exchange Meeting (Banquet)                    |                                             |                                           |                                              |
| 21st November (Thursday)  |                                               |                                             |                                           |                                              |
| 10:00–11:00               | Thorsten Heidersdorf<br>(Chair: Naoya Hirmae) | Alexis Marchand<br>(Chair: Kouki Yamaguchi) | Se Jin Kim<br>(Chair: Hiroto Nishikawa)   | Amirhossein Taghvaei<br>(Chair: Masato Hara) |
| 11:15–12:15               | Naoya Hirmae<br>(Chair: Thorsten Heidersdorf) | Kouki Yamaguchi<br>(Chair: Alexis Marchand) | Hiroto Nishikawa<br>(Chair: Se Jin Kim)   | Masato Hara<br>(Chair: Amirhossein Taghvaei) |

# Abstracts

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## *Plenary Speaker*

**Kenta Ishimoto (Kyoto University)**

### *Bio-inspired fluids: A mathematical perspective*

In recent decades, many subfields of mathematics have been inspired by biology, and fluid mechanics is no exception. In this talk, I will give an overview of recent advancements in bio-inspired fluid mechanics from an applied mathematical perspective, particularly focusing on cellular locomotion in fluids governed by the Stokes equations. A notable feature of bio-inspired fluids is the presence of self-deforming boundaries, which give rise to diverse dynamical phenomena, even though the fluid equation is a linear PDE. I will discuss a geometrical formulation and a data-driven modeling approach for cellular swimming behaviors.

## *Parallel Sessions: Representation Theory*

**Iva Halacheva (Northeastern University)**

### *Bethe subalgebras of the Yangian of $gl(n)$ , cacti, and crystals*

A useful approach to decomposing a representation of an algebra into manageable pieces is through the action of maximal commutative subalgebras. The family of Bethe subalgebras of the Yangian  $Y(gl(n))$  is such a collection of maximal commutative subalgebras, indexed by points of the Deligne-Mumford compactification of the moduli space  $M(0, n + 2)$ . The Bethe subalgebra  $B(C)$  corresponding to a point  $C$  in the real locus of this parameter space acts with simple spectrum on a given tame representation of  $Y(gl(n))$ . This results in a covering, with fiber over  $C$  given by the set of eigenlines for the action of  $B(C)$ . I will describe how each fiber can be identified with a collection of Gelfand-Tsetlin keystone patterns carrying a  $gl(n)$ -crystal structure, as well as the corresponding monodromy action realized by the mirabolic cactus group. This is joint work with Anfisa Gurenkova and Leonid Rybnikov.

**Sin-Myung Lee (Korea Institute for Advanced Study)**

### *Super duality and representations of quantum affine superalgebras*

From the birth of quantum groups, finite-dimensional representations of quantum affine algebras have been extensively studied with various motivations ranging from integrable systems to cluster algebras. The super analogue, quantum affine superalgebras, possesses even more interesting but challenging representation theory. Some of difficulties already appear for classical Lie superalgebras, and super duality is an elegant way to understand and overcome them by establishing a connection to the well-known theory for usual Lie algebras. In this talk, we describe a similar connection for quantum affine (super)algebras, as a part of an ongoing project with Jae-Hoon Kwon and Masato Okado.

**Thorsten Heidersdorf (Newcastle University)**

*Decomposition of orthosymplectic tensor powers*

After a brief introduction to the representation theory of supergroups I will explain how to decompose tensor powers of the natural representation of the orthosymplectic supergroup into indecomposable summands. Along the way we will meet Deligne's interpolating category  $\text{Rep}(O_t)$  for a complex parameter  $t$  which interpolates representations of the orthogonal group, and modules of Khovanov's arc algebra of type B.

**Naoya Hiramae (Kyoto University)**

*When are group algebras  $\tau$ -tilting finite?*

It is fundamental in the representation theory of finite dimensional algebras to ask whether the given algebra is  $\tau$ -tilting finite since  $\tau$ -tilting finiteness relates to many other properties concerning certain finiteness: brick finiteness, functorially finiteness of all torsion classes, completeness of  $g$ -fans, and sifting discreteness. In this talk, we will explain that  $\tau$ -tilting finiteness of group algebras is determined by so-called  $p$ -hyperfocal subgroups in some cases. This talk is based on joint work with Yuta Kozakai.

*Parallel Sessions: Topology*

**Marc Kegel (Institut für Mathematik Humboldt-Universität zu Berlin)**

*Characterizing and non-characterizing knots by 3-manifolds*

From a knot  $K$ , we can build 3-manifolds by performing Dehn surgery on that knot. We will discuss some new results explaining in which sense the diffeomorphism types of these 3-manifolds characterize the isotopy class of the knot  $K$ . This talk is based on joint work with Baker-McCoy, Casals-Etnyre, and Piccirillo.

**Hajime Kubota (Kyoto University)**

*On the knot concordance invariant Upsilon using grid homology*

Grid homology is a combinatorial reconstruction of knot Floer homology developed by Manolescu, Ozsváth, Szabó, and Thurston. The Upsilon invariant is a concordance invariant in knot Floer homology. Földvári gave a combinatorial restructure of the Upsilon invariant using grid homology. In this talk, we will give an alternative formulation of the grid Upsilon invariant and prove that the grid Upsilon invariant is equivalent to the original Upsilon invariant.

**Alexis Marchand (Kyoto University)**

## *Stable commutator length, surfaces, and rationality*

Stable commutator length (scl) is a measure of homological complexity of group elements, which has attracted attention for its connections with notions of negative curvature in geometric group theory, such as Gromov-hyperbolicity. I will introduce scl and the relevant notions from geometric group theory. Then I will discuss algorithmic computations of scl and present some results aiming to make progress towards understanding scl in surface groups.

**Kouki Yamaguchi (Kyoto University)**

### *The 3-loop polynomial of knots and topics related to it*

The Kontsevich invariant of knots is a powerful invariant of knots, but it is so hard to calculate its image. In this talk, we review the loop expansion of the Kontsevich invariant of knots, which may be one approach to investigate its image, and we consider how to grasp the information of its image via the loop expansion. Also, we state some recent results about the loop expansion, especially, the calculation of the 3-loop polynomial. Further, we state some topics related to the loop expansion or the 3-loop polynomial, for example, the LMO invariant of the cyclic branched covers of knots.

## *Parallel Sessions: Operator Algebra*

**Jonathan Taylor (University of Potsdam)**

### *Reconstruction of groupoid actions from homomorphisms between Cartan pairs*

In 2008, Renault showed that any  $C^*$ -algebra with a Cartan subalgebra can be realised as the reduced twisted groupoid  $C^*$ -algebra of a twist over an effective étale groupoid. For a fixed Cartan subalgebra, the resulting groupoid and twist are unique up to isomorphism. We show that that a (non-degenerate)  $*$ -homomorphism between  $C^*$ -algebras with Cartan subalgebras that preserve the Cartan structure must be induced by combinatorial data between the underlying groupoids. These combinatorial data (called actors) form a class of morphisms between groupoids, and these morphisms lift functorially to  $*$ -homomorphisms between the groupoid  $C^*$ -algebras. We show that there is then an equivalence of categories between the category of Cartan pairs of  $C^*$ -algebras with Cartan structure preserving  $*$ -homomorphisms, and the category of twists over effective groupoids with actors.

**Ryoya Arimoto (Kyoto University)**

### *Simplicity of crossed products of the actions of totally disconnected locally compact groups on their boundaries*

Results of Archbold and Spielberg, and Kalantar and Kennedy assert that a discrete group admits a topologically free boundary if and only if the reduced crossed product of continuous functions on its Furstenberg boundary by the group is simple. In this talk, I will show a similar result for totally disconnected locally compact groups.

**Se Jin Kim (KU Leuven)**

*Measured groupoids and their von Neumann algebras*

Von Neumann algebras are a class of algebras due to Murray and von Neumann which originally arise as a way to model the relations between the operators arising from quantum systems. One natural way to construct such algebras is from the representation theory on measured groupoids. Groupoids are a class of algebraic objects that generalize equivalence relations, dynamical systems, and groups. Recently, the study of groupoids and their associated operator algebras have had significant interest due to their presence in many important areas in the subject. In this talk we present two contrasting properties of these groupoids in the measurable setting, called the icc property, and the Choquet--Deny property. The works presented in this talk are in joint work with Tey Berendschot, Soham Chakraborty, Milan Donvil, and Mario Klisse.

**Hiroto Nishikawa (Kyoto University)**

*On exactness and property A*

Property A is the amenable-type condition on metric spaces, which extends the group exactness. For an action of a discrete group on a set, we show that the Schreier graph has property A if and only if the permutation representation generates an exact  $C^*$ -algebra.

***Parallel Sessions: Applied Mathematics***

**Kota Takeda (Kyoto University)**

*Uniform error bounds of the ensemble square root filter for chaotic dynamics with multiplicative covariance inflation*

In this talk, we consider the sequential state estimation problems for a class of nonlinear dynamical systems on Hilbert spaces, including the two-dimensional Navier-Stokes equations and the Lorenz 63 and 96 equations. For such nonlinear dynamical systems, the ensemble Kalman filter (EnKF) is often used to approximate the mean and covariance of the probability distribution representing uncertainty in the state estimation. In particular, we focus on a deterministic version of the EnKF known as the ensemble square root filter (ESRF), performing well even with limited ensemble sizes compared to other stochastic implementations of the EnKF. We estimate the state estimation error for the ESRF when applied to the nonlinear dynamical systems.

**Xin Tong (National University of Singapore)**

*Ensemble Kalman inversion for high dimensional problems*

Ensemble Kalman inversion (EKI) is an ensemble-based method to solve inverse problems. Its gradient-free formulation makes it an attractive tool for problems with involved formulation. However, EKI suffers from the

"subspace property", i.e., the EKI solutions are confined in the subspace spanned by the initial ensemble. It implies that the ensemble size should be larger than the problem dimension to ensure EKI's convergence to the correct solution. Such scaling of ensemble size is impractical and prevents the use of EKI in high dimensional problems. To address this issue, we propose two novel approaches using localization and dropout regularization to mitigate the subspace problem. We prove that these methods converge in the small ensemble settings, and the computational cost of the algorithm scales linearly with dimension. We also show that they reach the optimal query complexity, up to a constant factor. Numerical examples demonstrate the effectiveness of our approach.

**Yoshiki Hiruta (Kyoto University)**

*Numerical studies for bistability in bioconvection and time-periodic dynamics in periodic flow*

We address an emergence of bistability in bioconvection system by increasing swimming speed. Linear stability and bifurcation analyses in model bioconvection system show swimming stabilizes small disturbance and it enhances localization of nonlinear convection solution. We numerically identified an unstable solution crucial for the formation/decaying convection. We will also talk about another topic dealing with a periodic motion in high dimensional space using a machine-learning technique.

**Amirhossein Taghvaei (University of Washington)**

*Toward data-driven nonlinear filtering algorithms*

In this talk, I present a new variational formulation of the Bayes' law, that will be used for construction of a new family of nonlinear filtering algorithms. The variational formulation is based on the optimal-transportation (OT) theory, and aims at approximating the Brenier Optimal transport map from the prior to the posterior distribution, as a solution to a stochastic optimization problem. Unlike sequential importance resampling (SIR) particle filters, the OT formulation does not require the analytical form of the likelihood. Moreover, it allows us to harness the approximation power of neural networks to model complex and multi-modal distributions and employ stochastic optimization algorithms to enhance scalability.

**Masato Hara (Kyoto University)**

*A scenario of learning dynamics by reservoir computing*

Reservoir computing is a kind of machine learning, which can learn and predict time series generated by a dynamical system. We think that a reservoir can predict time series because it becomes a (semi) conjugate system with a system behind the time series. In this talk, I will sketch a scenario of learning dynamics by reservoir computing from the viewpoint of dynamical system theory. The story is constructed from three steps, which exactly correspond to the three steps of the learning algorithm. This is a joint work with Professor Hiroshi Kokubu (Kyoto University).

**P01 : Masahisa Ebina (Kyoto University)**

*Limit theorems for spatial average of stochastic wave equations*

This poster considers stochastic wave equations driven by a centered Gaussian noise that is white in time and has some spatial correlation. We focus on the asymptotic behavior of specific spatial average of the solution over a Euclidean ball with radius  $R$  as  $R \rightarrow \infty$  and establish certain limit theorems, including the law of large numbers, the central limit theorem, and the large deviation principle. We will also present how the Malliavin calculus tool can be used to show these results.

**P02 : Florian Gruen (Kyoto University)**

*Regularity and structure of non-planar  $p$ -elasticae*

We prove regularity and structure results for  $p$ -elasticae in  $\mathbb{R}^n$ , with arbitrary  $p \in (1, \infty)$  and  $n \geq 2$ . Planar  $p$ -elasticae are already classified and known to lose regularity. In this paper, we show that every non-planar  $p$ -elastica is analytic and three-dimensional, with the only exception of flat-core solutions of arbitrary dimensions. As an application, we establish the Li–Yau type inequality for the  $p$ -bending energy of closed curves in  $\mathbb{R}^n$ , extending previous works for  $p = 2$  and  $n \geq 2$  as well as for  $p \in (1, \infty)$  and  $n = 2$ .

**P03 : Shunsuke Hirota (Kyoto University)**

*Rainbow Paths of Verma Supermodules*

Just as semisimple Lie algebras and Drinfeld–Jimbo quantum groups can be classified by their underlying root systems, which serve as their skeletons, basic Lie superalgebras and Nichols algebras of diagonal type can be classified by the Weyl groupoids that form their skeletons. The BGG category  $\mathcal{O}$  of semisimple Lie algebras is traditional in representation theory, and while many aspects concerning even roots can be generalized to its super analogue, it remains a highly nontrivial and intriguing subject. In this research, we investigated fundamental aspects related to odd roots for basic Lie superalgebras, for which there is no theory analogous to that of Lie algebras, and derived several results from the properties of the Weyl groupoid. Specifically, we characterized the shortest paths in graphs such as Young’s lattice in terms of Verma supermodules, extended some results concerning associated varieties and projective dimensions, and constructed a semibrick that realizes the module category of rank 1 Lie superalgebras.

**P04 : Donggeon Kim (Kyoto University)**

*Feedback control of the Kuramoto model defined on graphs*

This study presents the problem of feedback control for the Kuramoto model defined on graphs with natural

frequencies. We consider the state where all oscillators synchronize at the same frequency as the desired motion and the controller is designed using the continuum limit approach. Deterministic and random graphs cases are examined through theoretical analysis, with the theoretical results verified by numerical simulations. Notably, we theoretically demonstrate that when the feedback gain is sufficiently large, the response converges to the desired motion.

**P05 : Ryo Matsuda (Kyoto University)**

*The Bers boundary of infinite or finite Teichmüller spaces*

The Teichmüller space of a Riemann surface  $S$  is regarded as the deformations space of the complex structure of  $S$ . Therefore, understanding the boundary of the Teichmüller space is very important, as it leads to an understanding of the degeneration phenomenon of the complex structure. However, the structure of the Teichmüller space changes greatly depending on whether the Riemann surface is finite or infinite, and the structure of the boundary also changes completely. In this talk, we will discuss the results obtained from the perspective of self-similarity for Bers boundaries in Teichmüller space, both for finite and infinite types.

**P06 : Bingxue Tao (Kyoto University)**

*Hierarchically hyperbolic groups and quasi-trees*

A finitely generated group  $G$  has property (QT) if it acts isometrically on a finite product of quasi-trees so that orbit maps are quasi-isometric embeddings. Examples of groups with property (QT) include mapping class groups and fundamental groups of 3-dimensional graph manifolds. In this poster, we will explain how to unify and generalize these results to a broader class of groups called "hierarchically hyperbolic groups" using the projection complex machinery. In particular, we will show property (QT) of certain graphs of groups and how property (QT) behaves under taking graph products.

**P07 : Hajime Tateishi (Kyoto University)**

*Existence and nonexistence of invariant curves of billiard maps*

The billiard system consists of a billiard table (a two-dimensional domain) and a billiard ball (a point mass). In the system, we consider a billiard map, which is a map from the pair of a collision point on the boundary and the direction of movement to the next pair. This map is a well-known example of an area-preserving twist map. In this talk, I will show that billiard maps do not have an invariant curve under some assumptions.

**P08 : Takumi Yagi (Kyoto University)**

*Perturbations of Hénon maps for parameters near the boundary of the Mandelbrot set*

In one-dimensional complex dynamics, we consider complex polynomials  $z^2 + c$  for complex parameters  $c$ . The critical point 0 of  $z^2 + c$  plays an important role in determining the hyperbolicity and connectedness of the Julia set. In two-dimensional complex dynamics, we consider complex Hénon maps  $H(z, w) = (z^2 + c + aw, az)$ , with non-zero  $a$ , obtained by perturbations of  $z^2 + c$ . Hénon maps do not have critical points. As a result, it is challenging to investigate properties such as hyperbolicity and connectedness of the Julia sets of Hénon maps, especially for parameters  $c$  on the boundary of the Mandelbrot set. In this talk, we introduce some results concerning the properties of Julia sets of Hénon maps under such perturbations.

### **P09 : Yiyang Wang (Kyoto University)**

#### *Formal degrees and parabolic induction*

Formal degrees are important representation theoretic invariants of discrete series representations for reductive groups over local fields. Harish-Chandra classified all discrete series of reductive Lie groups and computed their formal degrees in 1970s, which has now become a fundamental building block in automorphic forms and the Langlands program. For p-adic groups, Hiraga-Ichino-Ikeda proposed their important conjecture (2008, JAMS), which expresses the formal degree as a product of arithmetic invariants of Langlands parameters. The poster will talk about the relation of this conjecture and the parabolic induction in Harish-Chandra's philosophy.

### **P10 : Zhirun Zhan (Kyoto University)**

#### *The uniqueness of the mild solutions to the Navier-Stokes equations in scale critical spaces*

Our work deals with the uniqueness of mild solutions to the Navier-Stokes equations in the whole space. It is known that the uniqueness of mild solutions to the unforced Navier-Stokes equations holds in  $L^\infty(0, T; L^d(\mathbb{R}^d))$  when  $d \geq 4$ , and in  $C([0, T]; L^d(\mathbb{R}^d))$  when  $d \geq 3$ . As for the forced Navier-Stokes equations, when  $d \geq 3$  the uniqueness of mild solutions in  $C([0, T]; L^{d, \infty}(\mathbb{R}^d))$  with force  $f$  and initial data  $u_0$  in appropriate Lorentz spaces is known. In this poster it is presented that for  $d \geq 3$ , the uniqueness of mild solutions to the forced Navier-Stokes equations holds in a wider function space than known scale-critical spaces.