

Ill-Posedness of the Third Order NLS Equation with Raman Scattering Term

Yoshio Tsutsumi

Department of Mathematics, Kyoto University,
Kyoto 606-8502, JAPAN

We consider the ill-posedness of the Cauchy problem for the third order NLS equation with Raman scattering term on the one dimensional torus.

$$\partial_t u = \alpha_1 \partial_x^3 u + i\alpha_2 \partial_x^2 u + i\gamma_1 |u|^2 u + \gamma_2 \partial_x (|u|^2 u) - i\Gamma u \partial_x (|u|^2), \quad (1)$$
$$t \in [-T, T], \quad x \in \mathbf{T} = \mathbf{R}/2\pi\mathbf{Z},$$

$$u(0, x) = u_0(x), \quad x \in \mathbf{T}, \quad (2)$$

where α_j, γ_j ($j = 1, 2$) and Γ are real constants and T is a positive constant. We assume that $\Gamma, \alpha_1 \neq 0$ and $\frac{2\alpha_2}{3\alpha_1} \notin \mathbf{Z}$. It has been universally used among physicists as a mathematical model for the photonic crystal fiber oscillator (see, e.g., [1]). The study of nonlinear dispersive and wave equations has drastically been developing for the last two decades. Especially, the relations between the nonlinear structure and the smoothing type effect are getting clearer. I explain how to prove the ill-posedness of the Cauchy problem (1)-(2) by using the smoothing type estimate. I also talk about the local unique existence of solutions in the analytic function space.

This talk is based on the joint work [2] with Nobu Kishimoto, RIMS, Kyoto University.

References

- [1] G. Agrawal, *Nonlinear Fiber Optics*, 4th edition, Elsevier / Academic Press, Burlington, 2007.
- [2] N. Kishimoto and Y. Tsutsumi, Ill-posedness of the third order NLS equation with Raman scattering term, *preprint*, arXiv: 1706.09111v1 [math.AP].