Maximal Regularity for Time-Periodic Mixed-Order Boundary Systems

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In the 1960s, Agmon, Douglis, and Nirenberg developed an elliptic theory for mixed-order boundary systems [1, 2]. Given a matrix of differential operators $L = (L_{ij})$ and m different boundary operators B_h, they established ellipticity and complementing boundary conditions under which solutions to the system

$$Lu = f, \quad \text{on } \mathbb{R}^d_+, \qquad B_h u = g_h, \quad \text{on } \mathbb{R}^{d-1}, \quad h = 1, \dots, m,$$

admit optimal L^p a priori estimates. In particular, their ellipticity condition requires that $D := \det L$ is homogeneous of order $2m$ in the sense that its Fourier symbol satisfies $\hat{D}(\lambda\xi) = \lambda^{2m}\hat{D}(\xi)$ for all $\lambda > 0$. The choice of Dirichlet boundary conditions $B_h = \partial_\nu^h$ is always admissible in the elliptic case.

There are many examples for systems of equations for which the determinant is not homogeneous of any order. For example, the Cahn–Hilliard–Gurtin system

$$\begin{aligned} \partial_t u_1 - \Delta u_2 &= f_1 \quad \text{on } \mathbb{R} \times \mathbb{R}^d_+, \\ -(\partial_t - \Delta)u_1 + u_2 &= f_2 \quad \text{on } \mathbb{R} \times \mathbb{R}^d_+, \\ \partial_\nu u_1 &= g_1 \quad \text{on } \mathbb{R} \times \mathbb{R}^{d-1}, \\ \partial_\nu u_2 &= g_2 \quad \text{on } \mathbb{R} \times \mathbb{R}^{d-1}, \end{aligned}$$

has a determinant symbol $\hat{D}(\tau, \xi) = |\xi|^4 + i\tau|\xi'|^2 + i\tau$. Here, the last term exhibits a different scaling behavior than the first two terms, i.e. there are no $\alpha, \beta > 0$ such that $\hat{D}(\lambda^\alpha\tau, \lambda\xi) = \lambda^\beta\hat{D}(\tau, \xi)$ for all $\lambda > 0$. Consequently, the theory of Agmon, Douglis and Nirenberg is not applicable. For the whole space, an extensive method based on the notion of N-parabolicity has been developed in [3] in order to investigate such problems with a general inhomogeneous structure. I will show that the notion of N-parabolicity can be extended to boundary problems in order to obtain maximal L^p regularity results for mixed-order boundary systems such as the Cahn-Hilliard-Gurtin system in a time-periodic setting. Interestingly, we will find that unlike in the elliptic theory, Dirichlet boundary conditions (i.e., $B_h = \partial_\nu^h$) are generally not an admissible choice in this more general framework.

The talk is based on ongoing joint work with Guillaume Neuttiens.

References

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