The purpose of these lectures is to study the tempered dual of a real reductive group as a noncommutative topological space.

The unitary dual of a locally compact group may be identified with the spectrum of its group C*-algebra. The C*-algebra point of view equips the unitary dual with a topology, and it also associates to every unitary representation of the group, irreducible or not, a closed subset of the dual. In the case of a real reductive group, the tempered dual is the closed set associated to the regular representation.

The tempered dual may also be thought of as the spectrum of the so-called reduced C*-algebra. Following standard practice in C*-algebra theory and noncommutative geometry, we shall interpret the problem of studying the tempered dual as a noncommutative topological space as the problem of studying the reduced C*-algebra up to Morita equivalence.

The extra effort that is required to study the tempered dual in this more elaborate way, and not just a set, is rewarded in spectacular fashion by a beautiful isomorphism statement in K-theory that was conjectured by Connes and Kasparov, and later proved by Wassermann and Lafforgue. I shall describe a proof of the Connes-Kasparov isomorphism for real reductive groups that mostly follows the approach outlined by Wassermann but also uses ideas introduced by Vincent Lafforgue, together with new index-theory calculations that extend Lafforgue’s ideas.

本講義は「スーパーグローバルコース数学特別講義3」として大学院の学生には1単位認定されます。