

Introduce the students to the basic theory of elliptic curves with emphasis on the arithmetic properties, assuming the minimum amount of prerequisites. The subject of elliptic curves sits at the intersection of analysis, topology and number theory, i.e., almost all the areas of mathematics. As such, it has been the center of intensive studies classically and recently, ranging from the old problem of the congruent numbers, of computing the elliptic integral, to the proof of the Fermat's Last Theorem, to name a few. We give a series of five lectures, aimed at the undergraduate students, which introduces them to this fascinating subject at an elementary level with little background material required.

The students will learn the Mordell-Weil theorem stating that the rational points on an elliptic curve form a finitely generated Abelian group, the topological and analytic properties of an elliptic curve via the Weierstrass p-function, the classification of the elliptic curves by the j-invariant, culminating on the Weil conjecture counting the number of points on an elliptic curve over finite fields.

Lecture 1 (2 hours): Introduction

Lecture 2 (2 hours): Mordell-Weil theorem

Lecture 3 (2 hours): Elliptic curves over  $\mathbb{C}$  (the analytic theory)

Lecture 4 (2 hours): The *j*-invariant

Lecture 5 (2 hours): Weil conjecture for elliptic curves

It would be desirable if the student has the basic knowledge of complex analysis of one variable and the basic knowledge of algebra (groups and fields). What is required for this series of lectures as prerequisites is tried to be kept at the low level, even though the student is expected to fill in the gaps of his/her knowledge needed to understand the lectures by reading the textbooks and/or by coming to the office hours. The goal of this series of lectures is to expose the students to the basic theory of elliptic curves with the minimum amount of background knowledge at an early stage of his/her learning of mathematics.



## 主催:京都大学スーパーグローバル大学創成支援事業 数学系サブユニット

スーパーグローバル大学創成支援事業の数学系サブユニットでは、高い数学の研究能力と語学力を備え、国際的な舞台で活躍できる若手研究者の育成を目指し世界トップレベルの研究者による様々な国際教育プログラムを実施しています。