スーパーグローバルコース数学オンライン集中講義３

Spaces of Metrics of Positive Scalar Curvature

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Online lecture by Zoom

January 7 - 19

Friday	Jan. 7	9:00-11:00
Wednesday	Jan. 12	9:00-11:00
Thursday	Jan. 13	9:00-11:00
Tuesday	Jan. 18	9:00-11:00
Wednesday	Jan. 19	9:00-11:00

Studing Riemannian manifolds with positive scalar curvature (psc) is so interesting since this subject intertwines beautifully several areas of mathematics, such as Riemannian geometry, geometric analysis (including Ricci flow, conformal geometry and minimal surfaces theory), index theory, surgery theory, homotopy theory as well as recent study of moduli spaces of manifolds and cobordism categories.

In these five lectures, I would like to discuss in some detail several fundamental issues concerning manifolds with psc-metrics:

- Obstructions to the existence of pcs-metrics: index theory, surgery theorems and related bordism theory.
- Existence of psc-metrics (simply-connected manifolds of dimension at least five).
- Topology of spaces and moduli spaces of psc-metrics.

It turns out that the theory of positive scalar curvature on high-dimensional spin manifolds is intimately tied to algebraic topology. This relation can be traced to two basic facts. The first is the surgery principle of Gromov-Lawson and Schoen-Yau, which allows the import of methods from cobordism theory into the theory. The second is the existence of the spin Dirac operator and the Lichnerowicz formula, which can be used to construct K-theoretic invariants of positive scalar curvature metrics.

I will start with some basics on scalar curvature: Einstein-Hilbert functional, conformal metrics, Yamabe problem. My main goal is describe recent advances of the theory which concern the homotopy theory of the space $\mathcal{R}^{psc}(M)$ of metrics of positive scalar curvature. Our journey starts with the surgery theorem. Recent refinements and ramifications of the surgery theorem led to insights about the homotopy type of $\mathcal{R}^{psc}(M)$ and the action of the diffeomorphism group $\mathrm{Diff}(M)$ on $\mathcal{R}^{psc}(M)$, among other things. In particular, I will present some new results on the moduli spaces of psc-metrics based on recent work by Tadayuki Watanabe.

I will review of the Dirac operator and the secondary index invariant inddiff. At the end, I plan to present the proof of a result by the author and Johannes Ebert and Oscar Randal-Williams which shows that the space of psc-metrics $\mathcal{R}^{psc}(M)$ is at least as complicated as the real K-theory.

要申込：受講希望者は、Googleフォームにて申込みを行って下さい。
下記URL からアクセスしてください。聴講のみの希望者も申込みが必要です。
URL：https://forms.gle/s5x76WdsbUMEAVoz5
締切日：１月6日（木）17:00　締切厳守！

本講義はスーパーグローバルコース登録学生のコース修了要件の１単位となります。
ただし、大学院科目として通常の単位に認定されるわけではありませんので、注意してください。

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