2025 年　数学・数理科学グローバル講義Ⅳ（後期）

注）タイトルとアブストラクトが未定のものは分かり次第掲載します。

数学･数理科学グローバル特別講義 5
講師：Song Sun（浙江大学 国際先端数学研究院(IASM)、複素幾何学･代数幾何学分野）
講義日程:10/9（木）10:00～12:30
10/10(金) 10:00～12:30
10/15(水) 13:00～15:30
10/20(月) 13:00～15:45

タイトル:Singularity formation in Kahler geometry
概要:The study of singularities is an important topic in various areas of geometry. In this short course I will explain the understanding of singularity formation of canonical geometric structures on Kahler manifolds, with an emphasis on the interaction between geometric analysis and algebraic geometry. In particular, we will discuss the bubbling phenomenon for Kahler-Einstein metrics and Hermitian-Yang-Mills connections, and describe a Yau-Tian-Donaldson type conjecture connecting Kahler-Ricci shrinkers and Fano fibrations. The latter uniformizes various special cases previously studied separately, and connects to the analysis of Ricci flow on Kahler manifolds.

数学･数理科学グローバル特別講義 6
講師:Arthur Charpentier（Université du Québec à Montréal、保険数学分野）
講義日程:11 月 4 日、5 日、10 日、11 日、13 日　時間未定

タイトル:Fairness and distribution in insurance, an actuarial perspective
概要:This course explores the interplay between biases, discrimination, and fairness in insurance, a field where the very logic of risk classification relies on differential treatment of policyholders. Insurance companies segment individuals into risk pools in order to assign premiums that reflect expected costs and incentivize risk reduction. While such segmentation is intrinsic to the industry, it raises critical questions: which forms of discrimination are permissible, where do biases in data and models intervene, and how do regulations around the world define acceptable practices? We will examine these questions by studying how discrimination may arise in predictive modeling for insurance, the measures used to detect it, and the methods available to mitigate unfairness—ranging from regulatory frameworks to

statistical corrections. Lecture notes will be provided, based on selected chapters from Charpentier (2024).

On the mathematical side, the course emphasizes the role of modern quantitative tools in addressing fairness. After reviewing statistical and machine learning approaches to insurance pricing, we will introduce frameworks for assessing group fairness (such as demographic parity and equalized odds) and for analyzing counterfactual fairness, grounded in causal reasoning. A central mathematical focus will be on optimal transport and related concepts, which provide a versatile toolkit for measuring and mitigating disparities, both at the group and individual level. We will explore their use in post-processing adjustments of models, as well as in constructing counterfactuals for fairness evaluation.

References

Charpentier, A. (2024). Insurance: Biases, Discrimination, and Fairness. Springer. (Main course reference; lecture notes will be based on selected parts.)

Barocas, S., Hardt, M., & Narayanan, A. (2019). Fairness and Machine Learning. fairmlbook.org. (A standard reference on fairness definitions and methods in ML.)

Villani, C. (2009). Optimal Transport: Old and New. Springer. (Foundational text for mathematical background on optimal transport.)

Pearl, J., & Mackenzie, D. (2018). The Book of Why: The New Science of Cause and Effect. Basic Books. (Reference on causal inference and counterfactual reasoning, central to counterfactual fairness.)

数学・数理科学グローバル特別講義 7

講師：Pedro Salomão（Université du Québec à Montréal 南方科技大学、幾何学分野）

講義日程：11/4（火）13:30～14:30、15:00～16:00
11/5（水）13:30～14:30、15:00～16:00
11/6（木）13:30～14:30、15:00～16:00
11/7（金）10:00～12:00、14:00～16:00

タイトル：Pseudo-holomorphic curves, Reeb dynamics, and applications to Celestial Mechanics

概要：This lecture series will explore the deep interplay between pseudo-holomorphic curve theory and Reeb dynamics in symplectic and contact topology, with a focus on recent applications to classical problems in Celestial Mechanics. We will develop the theory of finite-energy pseudo-holomorphic curves in symplectizations, discuss their relationship to periodic orbits of Reeb flows, and present finite energy foliations in various settings.

A central theme will be the role of convexity—both strong and weak—in shaping the qualitative dynamics of Hamiltonian systems. The lectures will cover classical and modern results, including the foundational works of Hofer, Wysocki, and Zehnder, as well as new constructions of finite energy foliations in non-convex energy levels.

We will then move to concrete applications. These include the Hénon–Heiles system, the Euler problem of two fixed centers, and, especially, the circular planar restricted three-body problem. In particular, we will discuss new results establishing the existence of global finite-energy foliations that organize the dynamics, and allow for a detailed qualitative analysis of the dynamics in these models.

The aim is to introduce a powerful toolkit for studying Hamiltonian dynamics via holomorphic curve techniques and to show its relevance in problems with rich physical and geometric content. We will conclude with a discussion of open problems and possible directions for further exploration.

Lecture 1 - Introduction to Pseudo-holomorphic Curves in Symplectizations

We begin with an introduction to pseudo-holomorphic curves in the context of symplectic and contact geometry. After reviewing the notion of contact structures and Reeb vector fields, we introduce symplectizations and the setup for finite-energy pseudo-holomorphic curves. The lecture will cover basic analytical properties, energy bounds, and bubbling-off phenomena, drawing from the foundational work of Hofer.

We also explain the link between pseudo-holomorphic curves and Reeb orbits, emphasizing the geometric intuition behind the use of holomorphic curves to study Hamiltonian dynamics.

Lecture 2 - Finite-energy Foliations and the Work of Hofer–Wysocki–Zehnder

This lecture will focus on finite-energy foliations in symplectizations. We present the main existence results by Hofer, Wysocki, and Zehnder under convexity assumptions, and explain the geometric meaning of these foliations in terms of Reeb flows. Topics include asymptotic behavior of curves near punctures, global surfaces of section, and the use of holomorphic curves to analyze the topology of energy levels.

We also introduce the notion of weakly convex energy levels, setting the stage for later applications in non-convex settings like the restricted three-body problem.

Lecture 3 - Construction of Finite-energy Foliations in Celestial Models

We move to the construction of finite-energy foliations in physically relevant Hamiltonian systems. After briefly reviewing the relevant dynamical systems (Hénon–Heiles, Euler's two-center problem), we describe how these models can be lifted to contact manifolds and how their Reeb dynamics reflect the original Hamiltonian dynamics.

We present joint work on constructing finite-energy foliations for these systems, even in the absence of strict convexity. These foliations provide a powerful tool for global analysis, yielding information about periodic orbits, homoclinic and heteroclinic orbits, escape dynamics, etc.

Lecture 4 - Finite-energy Foliations in the Restricted Three-Body Problem

This lecture will be devoted to recent results on the circular planar restricted three-body problem (CPR3BP). We explain how to regularize collision singularities using elliptic coordinates and how to lift the system to a contact-geometric setting where holomorphic curve techniques become applicable.

We construct a global finite-energy foliation for certain energy levels of the CR3BP slightly above the first Lagrange value, showing how it encodes the global dynamics and connects with classical families of periodic orbits. This work opens the door to a holomorphic curve approach to classical problems in celestial mechanics, including capture, escape, and transition dynamics.

Lecture 5 - Open Problems, Physical Implications, and Future Directions

In the final lecture, we reflect on the broader implications of the theory developed so far. We discuss physical and mathematical motivations for studying foliation structures in Hamiltonian systems, including connections with transition state theory in chemistry and black hole dynamics in general relativity.

We survey open problems related to the existence and classification of finite-energy foliations, especially in non-convex and degenerate settings. We also discuss potential generalizations: foliations in higher dimensions, relations with Floer homology and symplectic field theory, and the challenge of understanding chaotic regimes via holomorphic methods.

数学･数理科学グローバル特別講義８
講師：Gregory Miermont（Ecole normale superieure de Lyon、確率論分野）
講義日程：12/8（月）13:00～15:00
　　　　　12/9（火）14:00～16:00
　　　　　12/10（水）14:00～16:00
　　　　　12/11（木）9:30～11:30
　　　　　12/12（金）14:00～16:00
タイトル：Combinatorial, probabilistic and geometric aspects of planar maps
概要：A map is the combinatorial structure of a graph embedded in a surface. These objects arise naturally in many different branches of mathematics, starting with a very rich enumerative theory that lies at the boundary between enumerative combinatorics, algebraic combinatorics, random matrices and theoretical physics. Models of random maps also provide natural examples of discrete random surfaces, whose large-scale limit give rise to universal models of two-dimensional random geometry, with many unexpected connections to other models of random geometry, such as random trees, random hyperbolic surfaces, or conformal random geometry. In these lectures, I will first focus on various approaches to the enumeration of planar maps, based on Tutte's decomposition and the slice decompositions. I will then explain how these decompositions are connected to classical objects of probability theory, such as random walks and spatial branching processes. Finally, we will see how these connections can provide detailed information about the geometry of random maps and their local and scaling limits.

These lectures are intended to a wide audience, and the first three lectures require little knowledge in probability besides the very basic aspects of random walks and branching processes.