2024 年　数学・数理科学グローバル講義 I （前期）

注）タイトルとアブストラクトが未定のものは分かり次第掲載します。

数学・数理科学グローバル特別講義 1
講師：Nigel Higson（Pennsylvania State University、微分幾何学分野）
講義日程：2024 年　5 月 10 日、13 日、17 日、20 日、27 日　15:00-17:00
タイトル：Spectral Theory By Example
概要：

I will present some of the mathematical foundations of spectral theory. Spectral theory began with Hilbert's treatment of integral equations, but it grew very rapidly as a mathematical theory, and also as an important tool across a wide variety of applications in science. My plan is to present some of the theory, and some of the applications—as many as I can, in fact, within a ten-hour lecture series.

1. Hilbert's Spectral Theorem
    * Self-adjoint matrices, integral equations and compact operators
    * Weyl's law
2. Further Applications of Hilbert's Spectral Theorem
    * Representations of compact groups
    * Sturm-Liouville theory
    * Complex functions and the Dirichlet principle
3. Von Neumann's Spectral Theorem
    * Self-adjoint and essentially self-adjoint operators
    * The Fourier transform
    * Weyl's boundary conditions for the Sturm-Liouville problem
4. Explicit Plancherel Formulas
    * Plancherel's theorem for the Fourier transform
    * Weyl's spectral theorem
    * SL(2,R) and Harish-Chandra's Plancherel formula for spherical functions
    * Levinson's theorem
5. The Prolate Spheroidal Wave Operator
    * Band-limited and time-limited signals in communication theory
    * Prolate spheroidal wave functions
    * The Riemann zeta function

数学・数理科学グローバル特別講義 2
講師：Igor Klep（University of Ljubljana、作用素環論分野）
講義日程：2024 年 5 月 20 日 10:00-12:00
2024 年 5 月 21 日～24 日 15:00-17:00
タイトル：Noncommutative analysis and real algebraic geometry
概要：Ever since Gauss it was known that a positive univariate real polynomial can be written as a sum of two squares of real polynomials. In a similar spirit, positive semidefinite quadratic forms (in any number of variables) are sums of squares of linear forms. During his 1885 PhD thesis defense Minkowski got into an argument with Hilbert about whether an extension to higher degree forms holds true (i.e., is every positive polynomial a sum of squares (sos) of polynomials?), thus providing a common generalization of the above two observations. A few years later Hilbert answered this in the negative; his proof was highly non-constructive, and the first explicit example of a positive polynomial that is not sos was given only 80 years later by Motzkin in 1967. Hilbert also posited that positive bivariate polynomials are sums of squares of rational functions, leading him to include the following problem (as #17) among the famous 23 problems for his address to the 1900 International Congress of Mathematicians:
Is every positive polynomial a sum of squares of rational functions?

A positive solution was presented in 1926 by Artin who developed the theory of formally real fields to solve this problem; we call this the beginning of real algebraic geometry (RAG). Nowadays real algebraic geometry is the branch of algebraic geometry studying real algebraic sets, i.e., real-number solutions to systems of polynomial equations. Pillars of RAG are generalizations of the above mentioned theorem of Artin, the so-called Positivstellensätze (=certificates of positivity):
Given polynomials p and q, is p positive where q is positive?

On the other hand, many problems in quantum physics or linear systems design in control theory have matrices as variables, and the formulas naturally contain noncommutative polynomials in matrices. Analyzing such problems has led to the development of a noncommutative (nc) real algebraic geometry. Often, the qualitative properties of the noncommutative case are much cleaner than those of their scalar counterparts. Indeed, the relaxation of scalar variables by matrix variables in several natural situations results in a beautiful structure.

This series of lectures will start by presenting classical results before moving on to newer modern results and techniques in noncommutative analysis and RAG. A selection of the following topics and their applications will be presented: noncommutative sums of squares (Helton's sum of squares theorem; McCullough's factorization theorem; etc.), convexity (linear matrix inequalities and their connection to complete positivity and dilation theory), noncommutative analytic maps (continuous implies analytic; Ax-Grothendieck theorem, nonlinear completely positive maps).

数学・数理科学グローバル特別講義 3
講師： Kai-Wen Lan(University of Minnesota、数論分野)
講義日程：2024 年 6 月 24 日～27 日 13:15-16:30
タイトル：Some vanishing results for the rational completed cohomology of Shimura varieties
概要：Shimura varieties are generalizations of modular curves, whose cohomology is useful for studying the relations between automorphic and Galois representations in the context of Langlands program. For each prime number p, the associated p-adic completed cohomology naturally captures lots of p-adic interpolations of such relations. In this lecture series, I will start with a general introduction to Shimura varieties and their compactifications, and explain how their classical and p-adic completed cohomology can be studied by using complex and p-adic Hodge theories. After these preparations, I will explain some vanishing results for the rational completed cohomology of Shimura varieties in my joint work in progress with Lue Pan. I will provide concrete examples when possible.

数学・数理科学グローバル特別講義 4
講師：Raphael Beuzart-Plessis （Aix-Marseille University、数論分野）
講義日程：2024 年 7月 2 日、9日、16日 15:00-17：00
2024年 7 月 4 日、11 日 9:30-11：30
タイトル：On the formal degree conjecture for classical groups
概要：The aim of these lectures will be to present a proof of a conjecture of Hiraga, Ichino and Ikeda expressing the formal degrees of discrete series representations of local reductive groups in terms of their Langlands parameterization (more precisely as adjoint gamma factors) in the case of classical p-adic groups. It will include a review of necessary material from

local harmonic analysis as well as the theory of twisted endoscopy. Time permitting, I will also discuss other applications of the same method such as to explicit Plancherel formulas for certain symmetric spaces.

数学・数理科学グローバル特別講義 5
講師：Chun Liu（Illinois Institute of Technology、流体数学・PDE 分野）
講義日程：2024 年　7 月 10 日、17 日　13:15-15:15
　　2024 年　7 月 11 日、12 日、16 日　15:00-17:00
タイトル：Energetic Variational Approaches for Active Materials and Reactive Fluids
概要：Active material and reactive fluids are those materials that can convert and transduce different types of energy, in particular between mechanical energy and various types of chemical energy. They are ubiquitous in modern applications and have become more and more important in areas like clean energy, environment sciences, as well as various medical and biology systems. In this lecture I will present a general framework to model a few specific types of active fluids. This is the extension of the classical energetic variational approaches for mechanical systems. These methods will also include a wide range of chemical reaction kinetics coupled with mechanical processes. This is a joint project with many collaborators, in particular, Bob Eisenberg, Yiwei Wang and Tengfei Zhang.
Lecture 1. Classical mechanics and energetic variational approaches
Lecture 2. General diffusion and chemical reaction kinetics
Lecture 3. Thermodynamics and temperature effects
Lecture 4. Wormlike micellar solutions: Micro-macro models and closures
Lecture 5. Temperature effects for phase field models

数学・数理科学グローバル特別講義 6
講師：Istvan Heckenberger（The Philipps University of Marburg、表現論分野）
講義日程：2024 年　8 月 5 日～9 日　時間未定
タイトル：Hopf and Nichols algebras
概要：Hopf algebras are a broad class of mathematical objects with an almost 100years of history and with applications in many areas in mathematics and physics. The structure of a rather general Hopf algebra is influenced to a large extent by its Nichols algebra and gives it a Lie theoretic flavor.In the course, the basic theory of Hopf and Nichols algebras as well as the relevant algebraic and combinatorial tools are discussed. It is explained, how braided

monoidal categories, generalized reflections, root systems and deformation techniques can be used to obtain structurally relevant information on Nichols algebras.