Stochastic Analysis and Applications
German-Japanese symposium

RIMS Workshops

Organizer: Ichiro Shigekawa  (Kyoto University)

DATES: September 11, 2006 (Mon) 9:20–
September 15 (Fri) 15:50
PLACE: International Conference Hall II
Kyoto University Clock Tower Centennial Hall
Yoshida-Honmachi, Sakyo-ku, Kyoto 606-8501

Program

September 11 (Mon)
10:20—11:10  Lucian Beznea (Institute of Mathematics, Romania)
Markov processes associated with $L^p$-resolvents and applications
to stochastic differential equations on Hilbert space
11:30—12:20  Walter Hoh (Universität Bielefeld)
Some properties of solutions to the martingale problem
for a class of pseudo differential operators.
14:00—14:30  Masanori Hino (Kyoto University)
Reflecting Ornstein-Uhlenbeck processes on path spaces
14:40—15:10  Kazuhiro Kuwae (Kumamoto University)
On Stratonovich type integrals and Lyons-Zheng decompositions
over symmetric Markov processes
15:20—15:50  Masayoshi Takeda (Tohoku University)
Large Deviations for Additive Functionals
of Symmetric Stable Processes
16:10—16:40  Narn-Rueih Shieh (National Taiwan University)
Dilation-stable-like processes on fractals
16:50—17:20  Isamu Doku (Saitama University)
A limit theorem for rescaled immigration superprocesses
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September 14 (Thu)

9:20—10:10  Michael Röckner (Purdue University, Universität Bielefeld)
Elliptic and parabolic PDE for measures

10:20—11:10  Tobias Kuna (Universität Bielefeld, Universita de Roma La Sapienza)
Decay of correlation in simultaneously existing phases
via a modification of Dobrushin’s uniqueness method

11:30—12:20  Shigeki Aida (Osaka University)
Semi-classical limit of the bottom of spectrum of a Schrödinger
operator on a path space

13:00—15:00  Special Session (Centenial Hall: first floor)

15:20—16:10  S. Albeverio (Universität Bonn)
Some new developments in hydrodynamics and quantum field theory

16:20—17:10  N. Bouleau (ENPC ParisTech)
Arbitrary functions principle and Dirichlet forms

September 15 (Fri)

9:20—10:10  Patrik Ferrari (Technische Universität München)
Fluctuation properties of the TASEP with periodic initial configuration

10:20—11:10  Tadahisa Funaki (University of Tokyo)
Concentrations in (1+1)-dimensional interfaces with pinning

11:30—12:20  Hirofumi Osada (Kyushu University)
Tagged particle of Ginibre interacting Brownian motions

14:00—14:30  Minoru Yoshida (Kansai University)
Reflection positive random fields and Dirichlet spaces

14:40—15:10  Oleksandr Kutovyy (Universität Bielefeld)
On continuous contact model

15:20—15:50  Song Liang (Tohoku University)
A Mechanical Model of Markov Processes

This workshop is partially supported by JSPS Grant-in-Aid for Scientific Research A 17204009
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HI*: Head Investigator

Caution: In many parts of the abstract, the commas "," are missing due to computer
trouble.
Markov processes associated with $L^p$-resolvents and applications to stochastic differential equations on Hilbert space

Lucian BEZNEA

Institute of Mathematics "Simion Stoilow" of the Romanian Academy

The talk is based on joint works with Nicu Boboc and Michael Röckner. We give general conditions on a generator of a $C_0$-semigroup (resp. of a $C_0$-resolvent) on $L^p(E,\mu)$, $p \geq 1$, where $E$ is an arbitrary (Lusin) topological space and $\mu$ a $\sigma$-finite measure on its Borel $\sigma$-algebra, so that it generates a sufficiently regular Markov process on $E$. We present a general method how these conditions can be checked in many situations. Applications to solve stochastic differential equations on Hilbert space in the sense of a martingale problem are given.
Some properties of solutions to the martingale problem for a class of pseudo differential operators.

Walter Hoh
University of Bielefeld

Nonlocal generators of jump type Markov processes typically have a representation as a pseudo differential operator with negative definite symbol. We discuss the construction of such processes via the martingale problem. In order to prove uniqueness of the solutions it is of interest to obtain regularity results for corresponding pseudo differential equations.

For this purpose we first construct a strong Markov selection and then present a modification of the approach of Bass and Levin in order to prove Hölder continuity of associated harmonic functions. Hereby we give conditions in terms of the symbol of the operator that guarantee the necessary estimates for certain exit times.

These conditions allow to treat more general jumping mechanisms that the stable case, but permit nonisotropic situation which also are not covered by the extensions due to Song and Vondraček.

Reflecting Ornstein-Uhlenbeck processes on path spaces

Masanori Hino
Graduate School of Informatics, Kyoto University

Consider a set of continuous maps from $[0,1]$ to a domain in $\mathbb{R}^d$. Although its topological boundary in the path space is not smooth in general, by using the theory of BV functions on the Wiener space and the theory of Dirichlet forms, we can discuss the existence of the surface measure and the Skorohod representation of the reflecting Ornstein-Uhlenbeck process on the set. This talk is based on a joint work with H. Uchida.
On Stratonovich type integrals and Lyons-Zheng’s decompositions over symmetric Markov processes

Kazuhiro Kuwae

Department of Mathematics, Faculty of Education, Kumamoto University

I will talk about the Fisk-Stratonovich type integrals by Dirichlet processes appeared in Fukushima’s decomposition in the framework of Dirichlet forms associated with symmetric Markov processes. We also show that our Fisk-Stratonovich type integrals admit Lyons-Zheng’s type decompositions under the law for quasi-everywhere starting point which strengthen the relation between two definitions on the Fisk-Stratonovich integrals by Nakao and by Lyons-Zheng Lyons-Zhang for diffusions without killing inside.

Large Deviations for Additive Functionals of Symmetric Stable Processes

Masayoshi Takeda

Tohoku University, Mathematical Institute

We establish the large deviation principle for additive functionals of symmetric $\alpha$-stable processes employing the Gärtner-Ellis Theorem.
Dilation-stable-like processes on fractals

Narn-Rueih Shieh
National Taiwan University

In this talk we report some concerns on stable-like processes on fractals which may allow different stability index in different "directions". We report the simplest case, the process on product fractals with independent components. The main tools are multivariate subordinations and time-changes.

A limit theorem for rescaled immigration superprocesses.

Isamu Doku
Saitama University

We consider a class of immigration superprocesses with non-vanishing deterministic immigration rate. We can prove that under a suitable scaling the rescaled superprocesses converge to a new class of superprocesses associated with coalescing spatial motion.
Stochastic dynamics on spaces of monotone maps of $S^1$ and on the $L^2$-Wasserstein space

Karl-Theodor Sturm
University of Bonn

TBA

The Einstein Relation

Andras Telcs
Department of Computer Science and Information Theory, Technical University Budapest

The talk emphasizes the importance and beauty of the so called Einstein relation. This relation connects the mean exit time volume and resistance growth. It plays a surprisingly important role in the study of the diffusion in different spaces.
Application of Prüfer angle methods in spectral asymptotics

Uta Freiberg
Friedrich-Schiller-University Jena

Generalized second order differential operators of the form \( d/dmd/dx \) are considered. They act on the \( L^2([0, 1], m) \)-space where \( m \) is a self similar measure which is singular with respect to Lebesgue measure. In the particular case that \( m \) has a self-similar support, we explain how the method of Prüfer angles can be used in order to get spectral asymptotics.

Essential self-adjointness of Dirichlet operators on a path space with Gibbs measures via an SPDE approach

Hiroshi Kawabi
Faculty of Mathematics, Kyushu University

In this talk we show the essential self-adjointness of Dirichlet operators in \( L^2(\mu) \), where \( \mu \) is a Gibbs measure on an infinite volume path space \( C(\mathbb{R};\mathbb{R}^d) \). This operator can be regarded as a perturbation of the Ornstein-Uhlenbeck operator by a nonlinearity and corresponds to a parabolic SPDE on \( \mathbb{R} \). In view of quantum field theory the solution of SPDE is called a \( P(\phi)_1 \)-time evolution. This talk is based on a joint work with Michael Röckner.
Asymptotic Expansions for the Laplace Approximations for Itô Functionals of Brownian Rough Paths

Yuzuru Inahama
Tokyo Institute of Technology

(Jointwork with Hiroshi KAWABI). In this talk we establish asymptotic expansions up to any order for the Laplace approximations for Itô functionals of Brownian rough paths under the condition that the phase function has finitely many non-degenerate minima. Our main tool is the Banach space-valued rough path theory of T. Lyons. We use a large deviation principle and the stochastic Taylor expansion up to any order with respect to the topology of the space of geometric rough paths.
Mass transportation and rough curvature bounds for discrete spaces

Anca Bonciocat

Universität Bonn, Institut für Angewandte Mathematik and Institute of Mathematics
Simion Stoilow Institute of the Romanian Academy

We develop a notion of rough curvature bounds for discrete spaces, based on the concept of optimal mass transportation. These rough curvature bounds will depend on a real parameter $h \geq 0$, which should be considered as a natural length scale of the underlying discrete space or as the scale on which we have to look at the space. Mass transportation and convexity properties of the relative entropy will be studied along $h$-geodesics. Instead of midpoints of a given pair of points $x_0, x_1$ we look at $h$-midpoints which are points $y$ with $d(x_0, y) \leq \frac{1}{2} d(x_0, x_1) + h$ and $d(x_1, y) \leq \frac{1}{2} d(x_0, x_1) + \frac{h}{2}$.

We prove that an arbitrary metric measure space $(M, d, m)$ has curvature $\geq K$ (in the sense of K.-T. Sturm) provided it can be approximated by a sequence $(M_h, d_h, m_h)$ of ('discrete') metric measure spaces with $h$-curvature $\geq K_h$ with $K_h \to K$ as $h \to 0$. The curvature bounds will also be preserved under the converse procedure: Given any metric space $(M, d, m)$ with curvature $\geq K$ and any $h; 0$ we define approximating standard discretizations $(M_h, d, m_h)$ of $(M, d, m)$ with $h$-curvature $\geq K$.

Finally, we apply our results to concrete examples of planar graphs.
Microscopic derivation of the three-dimensional Navier-Stokes equation from a stochastic interacting particle system

Robert Philipowski
University of Bonn

We study a system of stochastically interacting particles (vortices) and show that for a large number of vortices the weighted empirical measure of the system approximates the solution of the three-dimensional Navier-Stokes equation in vorticity form.

The scaling limit behaviour of periodic stable-like processes

Brice Franke
Ruhr-Universitaet Bochum, currently: Academia Sinica Taipei

A stable-like process is generated by a certain pseudo-differential operator of variable order where the order ranges between zero and two. In this talk we intend to investigate the behaviour of stable-like processes with a space-periodic order-function under scaling. Two different situations will occur. When the order function attains its minimum on a set of positive measure with respect to the invariant probability weak convergence toward a stable Levy-process holds. In the converse case one has convergence in probability toward the zero-function. The large deviation for this convergence can be described and turns out to be typical for processes having heavy-tailed increments. The results presented in this talk can be used to understand the recurrence and transience properties of stable-like diffusions with periodic generators.
Path behavior of Laplacian pinning models in (1+1)-dimension

Jean-Dominique Deuschel
Fachbereich Mathematik, Technische Universität Berlin

Joint work with Francesco Caravenna.

Consider a random field $\phi : \{0, \ldots, N\} \to \mathbb{R}$ with Laplacian interactions of the form $V(\Delta \phi_i)$, for a large class of potentials $V(\cdot)$, and with in addition a delta–pinning reward for the field to touch the $x$–axis, that plays the role of a defect line. The aim of this paper is to give a precise pathwise description of the field in the limit $N \to \infty$, as a function of the intensity $\varepsilon \geq 0$ of the pinning reward. We extract in particular the scaling limits of the model, showing that there is a phase transition at $\varepsilon = \varepsilon_c > 0$ between a delocalized regime ($\varepsilon < \varepsilon_c$), in which the field wanders away from the defect line, and a localized regime ($\varepsilon > \varepsilon_c$), in which the field sticks very close to it. A much subtler scenario shows up in the critical regime ($\varepsilon = \varepsilon_c$), where the rescaled field is shown to converge in distribution toward the derivative of a symmetric stable Lévy process of index $2/5$. Our approach is based on (Markov) renewal theory.
We provide an $N/V$-limit for the infinite volume, infinite particle gradient stochastic and Langevin dynamics associated with Gibbs states in continuous particle systems. Starting point in both cases is an $N$-particle dynamic with singular interaction in a finite volume with Lebesgue measure $V$. In the case of the $N$-particle gradient stochastic dynamic we consider reflecting boundary conditions. The corresponding generator is symmetric and the process can be constructed by symmetric Dirichlet forms. In the case of the $N$-particle Langevin dynamic we have to consider periodic boundary conditions in order to get diffusions. Here the corresponding generator has an antisymmetric part which can not be dominated by the symmetric part, i.e., the generator is non-sectorial. Hence we used the concepts of generalized Dirichlet forms to construct the corresponding process. In both cases we discuss the $N/V$-limit, i.e. the limit $N$ to infinity and $V$ to infinity such that $N/V$ converges to a density $> 0$, and compare the technical tools. In the case of gradient stochastic dynamics this gives the first construction of the infinite particle, infinite volume dynamic for one dimensional particles in the space of configurations with only one particle in one position. In the case of Langevin dynamics this gives the first construction of the infinite particle, infinite volume dynamic in the sense of the associated martingale problem.
The Skew Bessel process w.r.t. a curve

Gerald Trutnau
University of Bielefeld

Let \( \gamma : [0, \infty) \to [0, \infty) \) be continuous, locally of bounded variation, \( \delta > 0, \eta \in [-1, 1] \).

For \( z \geq 0 \) consider the following SDE

\[
Y_t = z + 2 \int_0^t \sqrt{|Y_s|} d(W_s + \eta K_s) + \delta t \quad (1),
\]

where \( K_t \) is an increasing process, \( K_0 \equiv 0 \), such that \( \int_0^t f(s, Y_s) dK_s = \int_0^t f(s, \gamma(s)^2) dK_s \)

for enough functions \( f \), \( W \) is a standard BM starting at zero, and \( \sqrt{\cdot} \) is the positive square root. It is easy to see that a solution to (1) stays positive. One can hence discard the absolute value in (1) as in the classical case \( \gamma \equiv 0 \), i.e. when \( Y_t \) is the squared Bessel process of dimension \( \delta \). Provided \( Y_t \) is unique in some appropriate sense the positive square root of \( Y_t \) is called \( \eta \)-skew Bessel process of dimension \( \delta \) w.r.t. \( \gamma \). Of special interest is the case where \( 2 \int_0^t \sqrt{Y_s} dK_s = \tilde{L}_t^0(Y - \gamma^2) \) is the symmetric local time of the semimartingale \( Y - \gamma^2 \).
Invariant measures and transition probabilities of continuous stochastic processes satisfy second order PDE of elliptic and parabolic type respectively, however, with coefficients of possibly low regularity. This motivates the study of such equations for measures from a purely analytic point of view. In the first part of the talk we shall start with reviewing existence, uniqueness and local regularity results in the elliptic case. In particular, the measures solving the PDE (essentially) always have densities with respect to Lebesgue measure. Subsequently, we shall present some recent global regularity results for these densities as well as conditions implying that they decay polynomially or exponentially at infinity. In the second part of the talk we shall pass to the parabolic case. Here the solutions will be measures on space time. Results on existence and local regularity are quite similar to the elliptic case. Results on uniqueness and global regularity have been established only very recently and are quite different from those in the elliptic case. Finally, it should be mentioned that though in this talk only the finite dimensional case is discussed, the same circle of problems is being analyzed in infinite dimensions and some of the above results have also been proved there.
Decay of correlation in simultaneously existing phases via a modification of Dobrushin’s uniqueness method

Tobias Kuna
University Bielfeld, Universit of Roma La Sapienza

In the work of A. Mazel J. Lebowitz and E. Presutti it was shown rigorously the existence of a phase transition with the density as order parameter for these systems. Although one has not a unique state we were nevertheless able to modify Drobushin’s uniqueness method in such a way that it becomes applicable. The Dobrushin condition cannot hold for all configurations because otherwise it would imply global uniqueness. However the configurations which violate the condition have small probability in the phase under consideration. This can be formalized using the notion of restricted ensemble. The decay of correlation for the restricted ensemble can be used to control the finite volume corrections of the pressure. The latter one are essential to obtain information about the initially considered systems from the restricted ensemble.
Semi-classical limit of the bottom of spectrum of a Schroedinger operator on a path space

Shigeki Aida
Osaka University

We discuss semi-classical limit of the bottom of spectrum of Schroedinger operators on path spaces. More precisely the path space is the space of continuous paths over a compact Riemannian manifold whose starting points are fixed and is endowed with a Brownian motion measure. Another path space is the pinned path space over a compact Lie group. That is the ending point is also fixed as well as the starting point. In this case we suppose that the ending point is outside the cut locus of the starting point. We show that under some assumptions the limit is written by the data of the Hessians at the zero points of the potential function of our Schroedinger operator.

Some New Developments in Functional integrals

Sergio Albeverio
University of Bonn, inst. Ang. Math

New developments concerning asymptotics of probabilistic and oscillatory functional integrals will be given. Particular attention will be given to summabibity and to phase functionals with superquadratic growth at infinity (of particular interest for applications).
When a random variable $Y$ is approximated by another random variable $Y_n$ defined on the same probability space and with values in the same measurable space, a Dirichlet form is most often provided by the approximation on the space $L^2(\mathbb{P}_Y)$ where $\mathbb{P}_Y$ is the law of $Y$. We give general results on such situations, especially by introducing four bias operators related by two linear relations, called the theoretical, the practical, the symmetric and the singular bias operator. The symmetric bias operator, when it exists, is the generator of the above mentioned Dirichlet form. This applies to many usual situations encountered in probability theory from Donsker theorem to solutions of SDE by discretization.

With the help of this general framework, we study some approximations related to the Arbitrary Functions Principle (AFP) introduced by H. Poincaré and extended by E. Hopf. This case is indeed particularly interesting from the point of view of the above method because it gives examples where the conditional law of $Y_n$ given $Y = y$ is a Dirac mass and where nevertheless the associated Dirichlet form doesn’t vanish. It is easily seen that the AFP occurs as soon as a quantity is measured by an instrument with regular graduation.

We give extensions of the AFP to the Wiener space and, to go further, we introduced the notion of Rajchman martingales which are continuous local martingales whose bracket is a.s. a Rajchman measure (i.e. a measure with characteristic function vanishing at infinity). Such martingales possess remarkable properties with respect to approximations by discretization.

Along this study we introduce and use a Girsanov theorem for Dirichlet forms which has its own interest.
Fluctuation properties of the TASEP with periodic initial configuration

Patrik Ferrari
Technical University Munich, Mathematics Department

We consider the totally asymmetric simple exclusion process (TASEP). Particles are on $\mathbb{Z}$ and jumps independently on the right-neighbor with unit rate provided the site is empty. On a macroscopic level the evolution of the density of particles follows the Burgers equation. Thus it is natural to focus on fluctuations and large deviations. Fluctuations of particle positions turns out to depend on initial conditions. Thus it is interesting to find out which class of initial condition leads to the same asymptotic behavior.

Concentrations in (1+1)-dimensional interfaces with pinning

Tadahisa Funaki
Graduate School of Mathematical Sciences, University of Tokyo

We study scaling limits for the (1+1)-dimensional interfaces with pinning especially under the critical situation that the rate function of the corresponding large deviation principle admits two minimizers. We obtain different type of limits depending on the boundary conditions under the presence (in a partial wetting regime) or absence of a wall.
Tagged particle of Ginibre interacting Brownian motions

Hirofumi Osada
Department of mathematics, Kyushu University

I prove the logarithmic growth of the trajectory of a tagged particle of Ginibre Interacting Brownian motions.

Reflection positive random fields and Dirichlet spaces

Minoru W. Yoshida
Dept. Math, Kansai Univ.

Some features of the reflection positive random fields having a correspondence with Euclidean quantum field theory are considered. We discuss how the reflection positive random fields are identified with some symmetric Markov process taking values in the space of distributions.

On continuous contact model

Oleksandr Kutovyy
Fakultat fur Mathematik, Universitaet Bielefeld

TBA
This is joint work with Shigeo Kusuoka.

We consider the motion of several massive particles (molecules) in an ideal gas of identical point particles (atoms) in a $d$-dimensional Euclidean space with certain interactions. It is well-believed that the motion of the molecules converges to a Markov process when the mass $m$ of atoms converges to 0, heuristically because the central limit theorem for “independent identically distributed” atoms. However, the atoms are actually not independent, especially not to the history, since they could be the ones interacted with some molecule(s) already.

This problem was first considered by Holley, in the case that the particles are moving in a one dimensional space. The corresponding question for general dimensional case has been considered by, e.g., Claderoni-Dürr-Kusuoka, Dürr-Goldstein-Lebowitz, but for the system of collision with only one molecule.

In this study, we consider the problem of multi-molecules without the independent assumption (which, as explained, actually does not hold). We prove the existence of the solution of the corresponding equation, and study the limit when $m$ converges to 0.