MINIMAL MODELS OF RIGID ANALYTIC SURFACES

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Contents

1. Introduction

The main purpose of the present paper is to give the existence theorem of relatively minimal regular models of proper regular rigid analytic surfaces. We study bimeromorphic morphisms between quasi-compact regular rigid analytic surfaces. Then we calculate certain cohomology groups to show the existence of relatively minimal regular models. We also study minimal models of fibered surfaces.

We show that any bimeromorphic morphism between quasi-compact regular rigid analytic surfaces is a finite succession of blowing-downs. Hopf proved the corresponding theorem in the case of compact smooth complex analytic surfaces in [10]. Ueno stated the theorem in the case of proper smooth rigid analytic surfaces without proof in [23].

We give two proofs of the theorem. The first proof is similar to Hopf's. The second proof is based on the fact that we may algebraize bimeromorphic morphisms locally.

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We explain the details of the second proof. First, since any proper rigid analytic space of dimension one is projective (see Appendix), extending a relatively ample divisor, we may algebraize any bimeromorphic morphism over a sufficiently small neighborhood of any fundamental point. Then, to conclude the proof of the theorem, we use Köpf's relative GAGA theorems in [15] and Shafarevich's study of birational transformations between two-dimensional Noetherian schemes in [21].

Notations and Conventions. We fix a complete non-Archimedean valuation field K with a non-trivial valuation and assume that rigid analytic spaces are defined over K. We mainly use the terminologies and notations of [4]. However, we make a modification of the definition of smoothness. Let X be a rigid analytic space. When the local ring $\mathcal{O}_{X,x}$ is regular for all $x \in X$, we say that the rigid analytic space X is *regular*. When the base change $X \times_K K'$ for any extension K'/K of complete valuation fields is regular, we say that the rigid analytic space X is *smooth* (or *geometrically regular*). If the base field K is perfect, then regularity is equivalent to smoothness (see [12]).

The *dimension* of a rigid analytic space X is the supremum of the Krull dimension of the local ring $\mathcal{O}_{X,x}$ for all $x \in X$. When the Krull dimension is constant for all $x \in X$, we say that the rigid analytic space X is *of pure dimension*.

Let us denote the residue field at a point x on a rigid analytic space by $K(x)$, which is a finite extension of K. We set a complete valuation on $K(x)$ which is the unique extension of that on K.

We call the absolute value of a non-zero element of K raised to a rational number power a *special real number*. We denote the set of all special real numbers by $\sqrt{|K^\times|}$.

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2. Bimeromorphic Morphisms

2.1. **Blowing-ups and Blowing-downs.**

A proper surjective morphism $\phi\colon X \to Y$ of rigid analytic spaces is called a *bimeromorphic morphism* if there exist analytic subsets S and T of X and Y of codimension at least one respectively such that the restriction $\phi| X - S \to Y - T$ is an isomorphism. Note that Corollary 7 in [4, 9.1.4] implies that the subsets $X - S$ and $Y - T$ are admissible open subsets.

A *surface* is a reduced separated rigid analytic space of pure dimension two. Let $\phi\colon X \to Y$ be a bimeromorphic morphism between quasi-compact surfaces with the analytic subsets S and T in the above definition. Then the analytic subset S is the union of a finite number of points and proper curves.

By Proposition 4.6 in [3], the fiber $\phi^{-1}(y)$ is a point or a proper connected curve for each $y \in Y$. By the same proposition, we may assume that the analytic subset T is a finite number of points and that the analytic subset S is equal to the preimage $\phi^{-1}(T)$. We call an element y of T a *fundamental point* for ϕ and the curve $\phi^{-1}(y)$ an *exceptional curve* for ϕ.

Conrad defined and studied blowing-ups of rigid analytic spaces in [7, 4.1] (see also [20] for algebraically closed base field cases). The blowing-ups provide examples of bimeromorphic morphisms since the blowing-ups are proper (Corollary 2.3.9 in [7]). Any blowing-up on an affinoid space is isomorphic to the analytification of the algebraic one. We only treat regular surfaces and their blowing-ups at a point. Let $\pi\colon \widetilde{X} \to X$ be the blowing-up of a regular surface X at a point p. Then the resulting surface is also regular. In this case, we say that the morphism π is the *blowing-down* of $\widetilde{X}$ along the divisor $\pi^{-1}(p)$. The following extension theorem shows the uniqueness of such contractions of divisors.

Theorem 2.1 (extension theorem)**.** *Let T be a nowhere dense analytic subset of a normal rigid analytic space Y. Assume that a morphism $f\colon Y-T \to Z$ of rigid analytic spaces satisfies the following condition: There exists an admissible covering $\{U_i\}_{i\in I}$ of Y such that each image $f(U_i-T)$ is contained in an admissible affinoid open subset of Z. Then there exists a unique morphism $g\colon Y \to Z$ such that the restriction $g| Y - T \to Z$ is equal to f.*

Proof. First, we assume that the rigid analytic spaces Y and Z are affinoid spaces $\mathrm{Sp}\, A$ and $\mathrm{Sp}\, B$ respectively. Applying the Riemann extension theorem to the pull-backs of the analytic functions on $\mathrm{Sp}\, B$ under f ([2, §3]), we obtain the unique K-algebra homomorphism $B \to A$. The uniqueness follows from the uniqueness in the Riemann extension theorem. Therefore, we obtain the desired morphism $g\colon \mathrm{Sp}\, A \to \mathrm{Sp}\, B$. The construction of g is compatible with the restriction of $\mathrm{Sp}\, A$ to any admissible affinoid open subset. Thus, by assumption, the general case follows from Proposition 1 in [4, 9.3.3]. □

In the following subsections, we prove the following theorem.

Theorem 2.2 (Hopf's theorem)**.** *Any bimeromorphic morphism between quasi-compact regular surfaces is a finite succession of blowing-downs.*

2.2. **Analytic Approach.** In this subsection, we prove Hopf's theorem via an analytic approach.

We prove Hopf's theorem by induction on the number of the irreducible components of the union of all the exceptional curves. Let y be one of the fundamental points for a bimeromorphic morphism $\phi\colon X \to Y$. We take an admissible affinoid open subset V and two analytic functions f' and

g' on V that generates the maximal ideal of the local ring $O_{Y,y}$. We may assume that the subset V does not contain the other fundamental points. Put $U := \phi^{-1}(V)$. Since any proper morphism is quasi-compact, the admissible open subset U is quasi-compact. Put $f := \phi^* f'$ and $g := \phi^* g'$.

Claim. The principal divisors (f) and (g) are given by $C + E$ and $D + F$ respectively where the divisors C, D, E, and F satisfy the following conditions.

(1) The divisors C and D are prime divisors that are not contained in $\phi^{-1}(y)$.
(2) The supports of the divisors E and F equal $\phi^{-1}(y)$.

Proof. Since the restriction $\phi|\, U - \phi^{-1}(y) \to V - \{y\}$ is an isomorphism, the claim follows from Corollary 2.2.9 in [5]. □

Claim. Choosing suitable analytic functions f' and g', we may assume that the divisor E equals F in the above claim.

Proof. Since the admissible open subset U is quasi-compact, by Theorem 5.9, we have a finite locally principal covering of U for all prime divisors that appear in the finite sum (f) or (g). Then the above claim follows from the following lemma. □

Lemma 2.3. *Let I be an ideal of an affinoid algebra A. Let f and g be two elements of A. Assume that at least one of the two elements is not contained in I. Then there exists a positive real number ϵ satisfying the following condition. For all two non-zero elements α and β of K whose absolute values are less than ϵ, neither $f + \alpha g$ nor $\beta f + g$ is contained in I.*

Proof. Let $\psi\colon B(1)^2 \to A^2$ be the continuous map defined by $\psi(s,t) = (f + sg, tf + g)$. If the ideal I contains f or g, then the condition is fulfilled on an open neighborhood of $(0,0)$ minus $\{s = 0\}$ or $\{t = 0\}$ respectively. Therefore, there exists a desired positive real number ϵ in this case. Assume that the ideal I contains neither f nor g. By Proposition 2 in [4, 3.7.2], the subset $I \times A \cup A \times I$ of A^2 is closed. Therefore, the complement of $\psi^{-1}(I \times A \cup A \times I)$ is an open neighborhood of $(0,0)$. This proves the second case. □

Claim. The ratio $(f : g)$ defines a morphism from U to $\mathbb{P}^1_V$ over V.

Proof. We have only to show that the two divisors C and D do not intersect. Suppose that these two divisors intersected. Let p be one of the intersections. Then the point p is contained in $\phi^{-1}(y)$.

Let $\pi\colon D^n \to D$ be the normalization of D. The restriction $\phi \circ \pi|\, D^n - (\phi \circ \pi)^{-1}(y) \to \{g' = 0\} - \{y\}$ is an isomorphism. Since the normalization D^n is separated, there exists a finite family $\{W_q\}_{q \in \pi^{-1}(p)}$ of disjoint admissible affinoid open subsets of D^n such that each element W_q contains q. Put $W := \bigcup_{q\in\pi^{-1}(p)} W_q$ and $h := \pi^* f$. Taking an admissible affinoid covering of D^n and applying Lemma 2.3 in [14], we obtain a special real number ϵ such that the admissible affinoid open subset $\{z \in D^n \mid |h| \leq \epsilon\}$ is contained in W. Put $G := \{x \in X \mid |f'| \leq \epsilon, g' = 0\}$. Then the preimage of G under ϕ is an admissible affinoid open subset of D^n. Thus, Theorem 2.1 shows that the rigid space G is isomorphic to an admissible affinoid open subset of D^n. Therefore, the pull-back h is a parameter at the point $\pi^{-1}(p)$ while the analytic function $\pi^* g$ vanishes.

We may make the same argument with respect to C. The two results show that the two analytic functions f and g generate the maximal ideal of the local ring $\mathcal{O}_{X,p}$ This contradicts the assumption that the point y is a fundamental point for ϕ. □

Let $\pi\colon \widetilde{Y} \to Y$ be the blowing-up of Y at the point y. The above claim enables us to define the morphism $\widetilde{\phi}\colon X \to \widetilde{Y}$ of rigid analytic spaces such that $\phi = \pi \circ \widetilde{\phi}$. Then the number of the irreducible components of the union of all the exceptional curves for $\widetilde{\phi}$ is less than that for ϕ. Therefore, Hopf's theorem follows by induction on this number.

2.3. **Algebraic Approach.** In this subsection, we prove Hopf's theorem by an algebraic approach. To algebraize bimeromorphic morphisms locally, we prepare the following lemma.

Lemma 2.4. *Let $\pi\colon X \to Y$ be a proper morphism of rigid analytic spaces. Assume that the fiber at a point p on Y is of dimension at most one. Then there exists an admissible open subset U of Y that contains p such that the base change $\pi \times_Y U\colon X \times_Y U \to U$ is projective.*

Proof. Put $X_p := \pi^{-1}(p)$. By Theorem 5.12, the fiber X_p is a projective curve over $K(p)$. By Proposition 7.1.32 in [17], we have an effective ample Cartier divisor $\sum_{i\in I} a_i D_i$ on X_p.

We fix $i \in I$. Since the morphism π is proper, there exists two admissible affinoid open subsets P and Q of the preimage of an admissible affinoid open subset V of Y under π such that the relations $D_i \in P$ and $P \Subset_V Q$ hold. Since the affinoid open subset Q does not contain the whole fiber X_p, there exists an effective Cartier divisor E on X_p whose support is not contained in Q. Then, by Corollary 7.3.23 in [17], there exist a meromorphic function f on X_p and a positive integer b such that the inequality $(f) \geq D_i - bE$ holds.

We may assume that the admissible affinoid open subset P is given by $\{x \in Q \mid |g_1(x)| \leq \epsilon, \ldots, |g_n(x)| \leq \epsilon\}$ where the family $\{g_1, \ldots, g_n\}$ is an affinoid generating family of Q over V and ϵ is a special real number that is less than one. Put $R := \bigcup_{j=1,\ldots,n} Q\langle g_1/\epsilon, \ldots, g_n/\epsilon, \epsilon/g_j\rangle$. Replacing ϵ with a larger special real number, we may assume that the restriction of R to X_p dose not contain the support of (f).

We choose an analytic function h on Q whose restriction to the analytic subset $Q \cap X_p$ is equal to the restriction of f to it. Then the relations $X_p \cap R \subset R - \{h = 0\} \subset R$ hold. Applying Lemma 1.1 in [14], we obtain a connected admissible affinoid open subset W_i of V that contains p such that the restriction $\{h = 0\} \cap \pi^{-1}(W_i)$ is contained in $(Q - R) \cap \pi^{-1}(W)$. Then we obtain the Cartier divisor $\pi^{-1}(W_i) \cap P \cap \{h = 0\}$ on Q. Thus, by Lemma 1.1 in [24], we obtain the Cartier divisor F_i on $\pi^{-1}(W_i)$, whose restriction to X_p is greater than D_i.

Repeating the same procedure for each $i \in I$, we obtain the intersection W' of the admissible open subsets W_i and the sum F' of the restriction of the Cartier divisors F_i to $\pi^{-1}(W')$. By Proposition 7.5.5 in [17], the restriction of the Cartier divisor F' to X_p is ample. Applying Theorem 3.2.9 in [7] to the line bundle defined by the Cartier divisor F', we conclude the proof of the lemma. □

Proof of Hopf's theorem. Let $\phi\colon X \to Y$ be a bimeromorphic morphism between quasi-compact regular rigid analytic spaces of pure dimension two. Since the statement is local with respect to Y, we may assume that the morphism ϕ has a single fundamental point y.

By the above lemma and Chow's theorem (Theorem 5.3), we may algebraize the bimeromorphic morphism ϕ over an admissible affinoid open subset which contains y. Therefore, the theorem follows from the Hopf's theorem for two-dimensional Noetherian regular schemes ([21, 4, p.55]). □

3. Minimal Models

A *relatively minimal regular surface* (resp. *relatively minimal smooth surface*) is a proper regular surface X (resp. proper smooth surface) such that any bimeromorphic morphism from X to a proper regular surface (resp. proper smooth surface) is an isomorphism. A *relatively minimal regular model* (resp. *relatively minimal model*) of a proper regular surface X (resp. proper smooth surface) is a relatively minimal regular surface Y (resp. relatively minimal surface) such that there exists a bimeromorphic morphism from X to Y.

To show the existence of relatively minimal regular models and relatively minimal models, we prove the following proposition.

Proposition 3.1. *Let $\pi\colon \widetilde{X} \to X$ be the blowing-up of a proper regular surface X at a point p. Then the inequality*

$$\dim_K H^1(\widetilde{X}, \Omega^1_{\widetilde{X}}) > \dim_K H^1(X, \Omega^1_X)$$

holds. For any line bundle $\mathcal{L}$, the equality

$$\dim_K H^q(\widetilde{X}, \pi^*\mathcal{L}) = \dim_K H^q(X, \mathcal{L})$$

holds for all q.

Proof. We take an admissible affinoid open subset U of Y such that two analytic functions on U generate the maximal ideal of the local ring $O_{Y,p}$. Then the assertion follows from the Mayer-Vietoris sequence (Proposition 5.6) and the local calculation in [21, 5, pp.59–65]. □

Proposition 3.1 and Hopf's theorem (Theorem 2.2) show the existence of relatively minimal regular models and relatively minimal models.

Theorem 3.2 (existence of relatively minimal regular models)**.** *Any proper regular surface admits a relatively minimal regular model.*

Theorem 3.3 (existence of relatively minimal models)**.** *Any proper smooth surface admits a relatively minimal model.*

4. Minimal Models of Fibered Surfaces

A (regular) *fibered surface with fibers of arithmetic genus g* (over a regular curve) is a triple (X, S, π) where X is a regular surface, S is a regular curve, and π is a proper flat morphism from X to S satisfying the following condition. There exists a nowhere dense analytic subset S_0 of S such that for all $p \in S - S_0$, the fiber $\pi^{-1}(p)$ is an irreducible curve of arithmetic genus g over $K(p)$.

Remark. The flatness of the morphism π is equivalent to the surjectivity of the morphism π.

We define algebraic fibered surfaces over algebraic regular curves in the same way. An (algebraic) fibered surface (X, S, π) is said to be *smoothly fibered* if the projection π is smooth. An (algebraic) fibered surface (X, S, π) is said to be *proper* if the total space X and the base space S are proper.

A *relatively minimal fibered surface* is a fibered surface (X, S, π) satisfying the following condition. For any fibered surface (Y, S, ρ), any bimeromorphic morphism from X to Y over S is an isomorphism. A *relatively minimal model* of a fibered surface (X, S, π) is a relatively minimal fibered surface (Y, ρ, S) with a bimeromorphic morphism from X to Y over S. If all the relatively minimal models of X are isomorphic to Y over S, we call each relatively minimal model the *minimal model* of X over S or the *minimal fibered surface*. A prime divisor E on a fibered surface (X, S, π) is said to be *exceptional* if there exists a fibered surface (Y, S, ρ) and a bimeromorphic morphism $\beta\colon X \to Y$ over S satisfying the following conditions.

(1) The restriction $\beta|_{X-E}$ is an isomorphism.

(2) The image $\beta(E)$ is a point on Y.

We also define algebraic (relatively) minimal fibered surfaces in the same way except for replacing bimeromorphic morphisms by proper birational morphisms.

Theorem 4.1 (Castelnuovo's criterion). *A prime divisor E on a fibered surface (X, S, π) is exceptional if and only if the following conditions are satisfied.*

(1) *The divisor E is contained in a fiber at a point on S.*
(2) *The cohomology group $H^1(E, O_E)$ vanishes.*
(3) *The degree of the normal bundle $O_X(E)|_E$ of E is equal to $-\dim_K C(E)$.*

In this case, the divisor E is isomorphic to the projective line over $C(E)$.

Proof. For any bimeromorphic morphism $\phi\colon X \to Y$ over S between fibered surfaces, the image of exceptional curves of ϕ under π is a nowhere dense analytic subset of S. Therefore, Lemma 2.4, Chow's theorem (Theorem 5.3), and the GAGA theorems (see [15]) enable us to use algebraic results. Thus, the theorem follows from the algebraic Castelnuovo's criterion (Theorem 9.3.8 in [17], [21, 6, p.102], Theorem 3.10 in [16]). □

Lemma 4.2. *An algebraic fibered surface over an affinoid algebra is minimal if and only if its analytification is minimal.*

Proof. We have only to show the only if part. We assume that the analytification is not minimal. Then Hopf's theorem (Theorem 2.2) and Castelnuovo's criterion (Theorem 4.1) show that the analytification contains an exceptional divisor. Thus, the lemma follows from the GAGA theorems and the algebraic Castelnuovo's criterion. □

The following lemma shows that the analytification commutes with base change.

Lemma 4.3. *Let $\mathcal{Y}$ be a locally of finite type scheme over an affinoid algebra A. Then, for a K-algebra homomorphism $A \to B$ of affinoid algebras, there exists a canonical isomorphism*

$$\mathcal{Y}^{\mathrm{an}} \times_{\mathrm{Sp}\,A} \mathrm{Sp}\,B \xrightarrow{\sim} (\mathcal{Y} \times_{\mathrm{Spec}\,A} \mathrm{Spec}\,B)^{\mathrm{an}}$$

where $\mathcal{Y}^{\mathrm{an}}$ is the analytification of A-scheme $\mathcal{Y}$ and $(\mathcal{Y} \times_{\mathrm{Spec}\,A} \mathrm{Spec}\,B)^{\mathrm{an}}$ is the analytification of B-scheme $\mathcal{Y} \times_{\mathrm{Spec}\,A} \mathrm{Spec}\,B$.

Proof. We can prove this lemma by the same method as in the proof of Satz 1.9 in [15]. □

Lemma 4.4. *The image of any flat morphism of affinoid spaces is a finite union of affinoid subdomains.*

Proof. This lemma is shown in [19, 3.4.8] (see also Proposition 3.1.7 1 in [8]). □

A morphism $\rho\colon Y \to T$ is said to be *locally projective* if there exists an admissible covering $\{U_i\}_{i\in I}$ of T such that each base change $\rho \times_T U_i \colon Y \times_T U_i \to U_i$ is projective.

Lemma 4.5. *For any proper flat surjective morphism $\rho\colon Y \to T$ to a regular curve with fibers of dimension at most one, there exist two morphisms $\alpha_1 \colon T_1 \to T$ and $\alpha_{12} \colon T_2 \to T_1$ satisfying the following conditions.*

(1) *The morphism α_1 is an admissible affinoid covering.*
(2) *The morphism α_{12} is étale quasi-compact, and surjective.*
(3) *The base change $\rho \times_T T_2$ is locally projective.*

Proof. This lemma is a special case of Theorem 2.1.4 in [6]. □

Lemma 4.6. *For any fibered surface $\pi\colon X \to S$ and any étale morphism $\alpha\colon T \to S$, the base change $\pi \times_S T \colon X \times_S T \to T$ is also a fibered surface. Moreover, if the arithmetic genus of the fibers of π is at least one and the morphism α is surjective, the fibered surface X is minimal if and only if the base change $\pi \times_S T \colon X \times_S T \to T$ is minimal.*

Proof. Since the morphism α is étale, by (21.D) Theorem 51 in [18], the base space T and the fiber product $X \times_S T$ are regular. This implies the first statement.

We show the second statement. By Castelnuovo's criterion (Theorem 4.1) and Lemma 4.4, it suffices to show the case when the base spaces S and T are affinoid spaces. Moreover, by Lemma 2.4, we may assume that the projection π is projective. By Chow's theorem (Theorem 5.3), Lemma 4.3, and Lemma 4.2, the second statement follows from Proposition 9.3.28 in [17]. □

Theorem 4.7 (existence of minimal models)**.** *Any fibered surface with fibers of arithmetic genus at least one over a quasi-compact regular curve admits a minimal model.*

Proof. The last two lemmas and Theorem 9.3.21 in [17] show that the image of the exceptional divisors is a finite number of points. Thus, Castelnuovo's criterion (Theorem 4.1) implies that there exists a relatively minimal model of the fiberd surface. Lemma 2.4 and Corollary 9.3.24 in [17] show that the relatively minimal model is a minimal model of the fiberd surface. □

5. Appendix

5.1. **Coherent Algebras and Finite Rigid Analytic Spaces.** In this subsection, we study coherent O_X-algebras on a rigid analytic space X and finite rigid analytic spaces over X. By $\mathbf{CohAlg}(X)$ we denote the category of coherent O_X-algebras on a rigid analytic space X. By $\mathbf{Fin}(X)$ we denote the category of finite rigid analytic spaces over X. We prove that the category $\mathbf{CohAlg}(X)$ is equivalent to the category $\mathbf{Fin}(X)$. Then we prove that we can algebraize finite rigid analytic spaces over a projective rigid analytic space over an affinoid space.

First, recall that for any K-homomorphism $A \to B$ between affinoid algebras and any finite A-module M, the completion tensor product $M \widehat{\otimes}_A B$ is isomorphic to a finite B-module $M \otimes_A B$ (Proposition 6 in [4, 3.7.3]). Therefore, for an arbitrary morphism $f\colon X \to Y$ of rigid analytic spaces and an arbitrary coherent O_Y-module $\mathcal{F}$, we may define the pull-back $f^*\mathcal{F}$ of $\mathcal{F}$ under f, which is a coherent O_X-module. In particular, we obtain a functor $\mu_{XY}\colon \mathbf{CohAlg}(Y) \to \mathbf{CohAlg}(X)$. We also obtain a functor $\nu_{XY}\colon \mathbf{Fin}(Y) \to \mathbf{Fin}(X)$ by the base change of finite rigid analytic spaces over X via f.

By local calculation, we obtain the projection formula.

Proposition 5.1 (projection formula)**.** *Let $\pi\colon X \to Y$ be a finite morphism of rigid analytic spaces. Then, for any coherent O_X-module $\mathcal{F}$ and any coherent O_Y-module $\mathcal{G}$, there exists a canonical isomorphism $\pi_*(\mathcal{F} \otimes_{O_X} \pi^*\mathcal{G}) \cong \pi_*\mathcal{F} \otimes_{O_Y} \mathcal{G}$.*

For a finite rigid analytic space X' over X, the push-forward of the structure sheaf of X' is a coherent O_X-algebra (Proposition 5 in [4, 9.4.2]). This gives a functor $\phi_X\colon \mathbf{Fin}(X) \to \mathbf{CohAlg}(X)$. Since any finite algebra over an affinoid algebra is an affinoid algebra (Proposition 5 in [4, 6.1.1]), from a coherent O_X-algebra we obtain a finite rigid analytic space over X by pasting spaces and morphisms (Proposition 1 in [4, 9.3.2] and Proposition 1 in [4, 9.3.3]). This gives a functor $\psi_X\colon \mathbf{CohAlg}(X) \to \mathbf{Fin}(X)$. Then we have a following theorem.

Theorem 5.2. *For an arbitrary rigid analytic space X, the functors ϕ_X and ψ_X give an equivalence of the category of coherent O_X-algebras and the category of finite rigid analytic spaces over X. The equivalence commutes with base change in the following sense. For an arbitrary morphism $f\colon X \to Y$ of rigid analytic spaces, the diagram*

$$\begin{tikzcd}
\mathbf{CohAlg}(Y) \arrow[r, equal, "\sim"] \arrow[d, "\mu_{XY}"'] & \mathbf{Fin}(Y) \arrow[d, "\nu_{XY}"'] \\
\mathbf{CohAlg}(X) \arrow[r, equal, "\sim"] & \mathbf{Fin}(X)
\end{tikzcd}$$

is commutative.

In the rest of this subsection, we prove that we can algebraize finite rigid spaces over a projective rigid analytic space over an affinoid space. We fix an affinoid algebra A. Köpf gave the *analytification functor* from the category of locally of finite type schemes over an affine scheme $\operatorname{Spec} A$ to the category of rigid analytic spaces over an affinoid space $\operatorname{Sp} A$ in [15, §1]. Let us denote each analytification of a locally of finite type scheme $\mathcal{X}$ over $\operatorname{Spec} A$, a morphism f over $\operatorname{Spec} A$ between such schemes, and a coherent $\mathcal{O}_{\mathcal{X}}$-module $\mathcal{F}$ by $\mathcal{X}^{\mathrm{an}}$, f^{an}, and $\mathcal{F}^{\mathrm{an}}$ respectively. When the morphism $f\colon \mathcal{X} \to \mathcal{Y}$ is proper, we have the isomorphism

$$(R^q f_* \mathcal{F})^{\mathrm{an}} \cong R^q f_*^{\mathrm{an}} \mathcal{F}^{\mathrm{an}} \tag{q}$$

for any coherent $\mathcal{O}_{\mathcal{X}}$-module $\mathcal{F}$ and all q (Folgerung 3.13 in [15]). First, note that the following analogue of Chow's theorem follows from the GAGA theorems (Satz 5.1 in [15], Satz 4.11 in [15]).

Theorem 5.3 (Chow's theorem)**.** *The analytification functor gives an equivalence between the category of projective schemes over an affine scheme* $\operatorname{Spec} A$*, where A is an affinoid algebra, and the category of projective rigid analytic spaces over an affinoid space* $\operatorname{Sp} A$*.*

By $\mathbf{Fin}(\mathcal{X})$ we denote the category of finite schemes over a scheme $\mathcal{X}$.

Theorem 5.4. *For any projective scheme $\mathcal{X}$ over an affinoid algebra, the analytification functor yields an equivalence between the category* $\mathbf{Fin}(\mathcal{X})$ *and the category* $\mathbf{Fin}(\mathcal{X}^{\mathrm{an}})$*. In particular, any finite rigid analytic space over a projective rigid analytic space over an affinoid space is projective.*

Proof. Let F be the composite of functors

$$\mathbf{Fin}(\mathcal{X}) \xrightarrow{\sim} \mathbf{CohAlg}(\mathcal{X}) \xrightarrow{\sim} \mathbf{CohAlg}(\mathcal{X}^{\mathrm{an}}) \xrightarrow{\sim} \mathbf{Fin}(\mathcal{X}^{\mathrm{an}}),$$

where the second analytification functor gives an equivalence by the GAGA theorems. We have only to show that there exists a natural transformation from the functor F to the analytification functor of finite schemes over $\mathcal{X}$. This follows from Theorem 5.2 and the above isomorphism (0). The last statement follows from Corollaire 6.1.11 in [9]. □

5.2. **Cohomology Groups.** We review the theory of cohomology groups and give Mayer-Vietoris sequences of cohomology groups of abelian sheaves.

First, we review some facts of cohomology groups of abelian sheaves on rigid analytic spaces. The category of abelian sheaves on a rigid analytic space is enough injective (see [25]). For a quasi-separated paracompact rigid analytic space, Čech cohomology agrees with cohomology (Lemma 2.5.7 in [8] and Remark 2.5.5 in [8]). If a quasi-separated paracompact rigid analytic space is of pure dimension d, then the q-th cohomology group of any abelian sheaf vanishes for $q > d$ (Corollary 2.5.10 in [8]). The q-th cohomology group of any coherent module on an affinoid space vanishes for $q > 0$ (Theorem 8,7 in [22], Satz 2.4 in [13]). Therefore, using Leray-Cartan spectral sequence (Corollaire 3.3 [1]) with respect to an admissible affinoid covering, we can calculate cohomology groups of any coherent module on a separated paracompact rigid analytic space. The q-th cohomology groups of any $\mathcal{O}_X$-coherent module $\mathcal{F}$ on a proper rigid analytic space X is a finite dimensional K-vector space (Theorem 3.3 in [11]). Let us denote the dimension of this K-vector space by $h^q(\mathcal{F})$. In this case, we define the *Euler characteristic* $\chi(\mathcal{F})$ of $\mathcal{F}$ in the following way:

$$\chi(\mathcal{F}) := \sum_{q=0}^{\infty}(-1)^q \dim_K H^q(X, \mathcal{F}).$$

Since the pull-back of an admissible affinoid covering by a finite morphism is again an admissible affinoid covering (Proposition 1 in [4, 9.4.4]), we have the following proposition.

Proposition 5.5. *Let $\pi\colon X \to Y$ be a finite morphism between separated paracompact rigid analytic spaces X and Y. Then, for a coherent $\mathcal{O}_X$-module $\mathcal{F}$, there exists an isomorphism*

$$H^q(X, \mathcal{F}) \cong H^q(Y, \pi_*\mathcal{F})$$

for all $q \geq 0$. In particular, if the rigid analytic spaces X and Y are proper, then the equality $\chi(\mathcal{F}) = \chi(\pi_*\mathcal{F})$ holds.

Proposition 5.6 (Mayer-Vietoris sequence)**.** *Let $\{U_1, U_2\}$ be an admissible covering of an admissible open subset of a rigid analytic space X. Then, for any abelian sheaf $\mathcal{F}$, there exists a canonical exact sequence:*

$$0 \longrightarrow H^0(U_1 \cup U_2, \mathcal{F}|_{U_1\cup U_2}) \longrightarrow H^0(U_1, \mathcal{F}|_{U_1}) \oplus H^0(U_2, \mathcal{F}|_{U_2}) \longrightarrow$$
$$\longrightarrow H^0(U_1 \cap U_2, \mathcal{F}|_{U_1\cap U_2}) \longrightarrow H^1(U_1 \cup U_2, \mathcal{F}|_{U_1\cup U_2}) \longrightarrow \cdots .$$

Proof. We use geometric points on rigid analytic spaces (see [25]). Let $\iota_0\colon U_1 \cup U_2 \to X$, $\iota_1\colon U_1 \to X$, $\iota_2\colon U_2 \to X$, and $\iota_3\colon U_1 \cap U_2 \to X$ be the inclusion morphisms. Theorem 1 in [25, 4] implies that the sheaf sequence

$$0 \longrightarrow \iota_{0!}(\mathcal{F}|_{U_1\cup U_2}) \longrightarrow \iota_{1!}(\mathcal{F}|_{U_1}) \oplus \iota_{2!}(\mathcal{F}|_{U_2}) \longrightarrow \iota_{3!}(\mathcal{F}|_{U_1\cap U_2}) \longrightarrow 0.$$

is exact. Therefore, we obtain the long exact sequence:

$$0 \longrightarrow H^0(X, \iota_{0!}(\mathcal{F}|_{U_1\cup U_2})) \longrightarrow H^0(X, \iota_{1!}(\mathcal{F}|_{U_1})) \oplus H^0(X, \iota_{2!}(\mathcal{F}|_{U_2})) \longrightarrow$$
$$\longrightarrow H^0(X, \iota_{3!}(\mathcal{F}|_{U_1\cap U_2})) \longrightarrow H^1(X, \iota_{0!}(\mathcal{F}|_{U_1\cup U_2})) \longrightarrow \cdots .$$

Let $\mathcal{G}$ be an abelian sheaf on an admissible open subset U of X. Let $\iota\colon U \to X$ be the inclusion morphism. For any geometric point x on X, we have the isomorphism:

$$\iota_!\mathcal{G}_x \cong \begin{cases} \mathcal{G}_x, & x \in U, \\ 0, & \text{otherwise.} \end{cases}$$

Therefore, if a sheaf sequence $\mathcal{G} \to \mathcal{I}^\bullet$ is the canonical injective resolution, so is the extension $\iota_!\mathcal{G} \to \iota_!\mathcal{I}^\bullet$ of the sheaf sequence. Thus, we obtain the isomorphism

$$H^q(U, \mathcal{G}) \cong H^q(X, \iota_!\mathcal{G})$$

for all q. This proves the proposition. □

For a rigid analytic space X, by $\mathcal{O}_X^\times$ we denote the abelian sheaf of units in the sheaf of the ring $\mathcal{O}_X$. Then there exists a canonical isomorphism $\operatorname{Pic} X \cong H^1(X, \mathcal{O}_X^\times)$.

Lemma 5.7. *Let $\mathcal{I}$ be a coherent $\mathcal{O}_X$-ideal on a paracompact quasi-separated rigid analytic space X. Assume that $\mathcal{I}^2 = 0$. Put $X_0 := (X, \mathcal{O}_X/\mathcal{I})$. Let $\pi\colon X_0 \to X$ be the natural closed immersion. Then the sheaf sequence*

$$0 \longrightarrow \mathcal{I} \longrightarrow \mathcal{O}_X^\times \longrightarrow \pi^{-1}\mathcal{O}_{X_0}^\times \longrightarrow 0$$

is exact where, on an admissible affinoid open subset U of X, the morphism $\mathcal{I} \to \mathcal{O}_X^\times$ is given by $a \mapsto 1 + a$ and the the morphism $\mathcal{O}_X^\times \to \pi^{-1}\mathcal{O}_{X_0}^\times$ is induced by the natural projection $\mathcal{O}_X(U) \to \mathcal{O}_X(U)/\mathcal{I}(U)$. Therefore, we obtain the long exact sequence:

$$\cdots \longrightarrow H^1(X, \mathcal{I}) \longrightarrow \operatorname{Pic} X \longrightarrow \operatorname{Pic} X_0 \longrightarrow H^2(X, \mathcal{I}) \longrightarrow \cdots .$$

In particular, if the rigid analytic space X is of dimension one, the morphism $\pi^*\colon \operatorname{Pic} X \to \operatorname{Pic} X_0$ is surjective.

Proof. Since the sheaf sequence is exact on any affinoid open subset, it is exact at any geometric point on X. Therefore, the sheaf sequence is exact. □

5.3. **Weil Divisors and Cartier Divisors.** We define Weil divisors and Cartier divisors on normal rigid analytic spaces. Then we prove that these are the same notion on regular rigid analytic spaces.

We refer to [3] and [5] for the definition and fundamental results of Weil divisors on normal rigid analytic spaces. We refer to [3] for these of meromorphic functions on rigid analytic spaces. Let us denote the sheaf of meromorphic functions on a rigid space X by $\mathcal{M}_X$. A *Cartier divisor* on X is a global section of the abelian sheaf $\mathcal{M}_X^\times/\mathcal{O}_X^\times$. The *Cartier divisor class group* of X is the quotient group $\Gamma(X, \mathcal{M}_X^\times/\mathcal{O}_X^\times)/\Gamma(X, \mathcal{M}_X^\times)$. Let us denote this abelian group by $\operatorname{CaCl}(X)$. The long exact sequence induced by the sheaf exact sequence

$$0 \longrightarrow \mathcal{O}_X^\times \longrightarrow \mathcal{M}_X^\times \longrightarrow \mathcal{M}_X^\times/\mathcal{O}_X^\times \longrightarrow 0$$

gives the injective group homomorphism $\operatorname{CaCl}(X) \to \operatorname{Pic}(X), D \mapsto \mathcal{O}_X(D)$.

We may represent a Cartier divisor by the family $\{(U_i, f_i)\}_{i\in I}$ where the family $\{U_i\}_{i\in I}$ is an admissible covering of X and f_i is an element of $\mathcal{M}_X^\times(U_i)$.

The meromorphic function f_i is called a *defining function* of D on U_i. The Cartier divisor D is said to be *effective* if the Cartier divisor D is represented by the family $\{(U_i, f_i)\}_{i \in I}$ where each meromorphic function f_i is an analytic function on U_i. An effective Cartier divisor defines the closed subspace. We sometimes identify the closed subspace with the Carier divisor.

When the rigid space X is normal, we may describe Cartier divisors in the following way. Note that we may restrict Weil divisors to an arbitrary admissible open subset. A *locally principal covering* for a Weil divisor D on a normal rigid analytic space X is an admissible covering $\{U_i\}_{i \in I}$ of X such that the restriction of $D|_{U_i}$ is a principal divisor (f_i) on U_i. Then a Cartier divisor on X is a Weil divisor on X that admits a locally principal covering. Note that we may assume that a locally principal covering is an admissible affinoid covering if exists.

Lemma 5.8. *A prime Weil divisor D on a regular affinoid space* $\operatorname{Sp} A$ *admits a finite locally principal Zariski covering.*

Proof. Let I be the ideal of A corresponding to D. We take a point x on $\operatorname{Sp} A$. Let $\mathfrak{m}$ be the corresponding maximal ideal. Then the localization $A_{\mathfrak{m}}$ is a unique factorization domain since it is a regular local ring. Therefore, we may write $I_{\mathfrak{m}} = f_x A_m$ where $f_x \in A_m$. Let U_x be the subset of $\operatorname{Sp} A$ that is the complement of the support of the divisor $D - (f_x)$. By Corollary 7 in [4, 9.1.4], the subset U_x is an admissible open subset of $\operatorname{Sp} A$. We take such an admissible open subset for each point x on $\operatorname{Sp} A$. Since the admissible open subset U_x contains x, the family $\{U_x\}_{x \in \operatorname{Sp} A}$ is a Zariski covering of $\operatorname{Sp} A$. We take a finite subcovering $\mathcal{U}$ of this covering. By Corollary 7 in [4, 9.1.4], the covering $\mathcal{U}$ is an admissible covering of $\operatorname{Sp} A$. Since the equality $D|_{U_x} = (f_x)$ holds, the covering $\mathcal{U}$ is locally principal for D. □

The above lemma implies the global case.

Theorem 5.9. *All Weil divisors on a regular rigid analytic space X are Cartier divisors on X.*

5.4. **Proper Rigid Analytic Spaces of Dimension One.** A *curve* is a reduced separated rigid analytic space of pure dimension one. In this section, we assume that C is a proper curve. We prove the Riemann-Roch theorem for proper curves. Using this theorem, we show that any proper rigid analytic space of dimension one is projective.

Since any affinoid algebra is excellent (see [5, 1.1]), the singular locus of quasi-compact curve is the union of a finite number of points. A *regular divisor* on a proper curve is a formal finite sum of regular points. A *prime regular divisor* is a divisor which is defined by a single regular point. The *degree* of a regular divisor $\sum_P a_P P$ is the sum $\sum_P a_P \dim_K K(P)$. Let us denote this integer by $\deg_C D$. Since we may regard a regular divisor as a Cartier divisor, a regualr divisor D on C defines the line bundle on C. Let us denote this line bundle by $\mathcal{O}_C(D)$.

To calculate the dimensions of cohomology groups of line bundles, let us show the following lemma.

Lemma 5.10. *Let P be a prime regular divisor on a proper curve C. Then, for any line bundle $\mathcal{L}$, the following inequalities and equality hold:*

(1) $0 \leq h^0(\mathcal{L} \otimes \mathcal{O}_C(P)) - h^0(\mathcal{L}) \leq \dim_K K(P)$;
(2) $0 \leq h^1(\mathcal{L}) - h^1(\mathcal{L} \otimes \mathcal{O}_C(P)) \leq \dim_K K(P)$;
(3) $\chi(\mathcal{L} \otimes \mathcal{O}_C(P)) = \chi(\mathcal{L}) + \dim_K K(P)$.

Proof. We define the coherent $\mathcal{O}_C$-module $\mathcal{F}$ by the following sheaf exact sequence:

$$0 \longrightarrow \mathcal{O}_C(-P) \longrightarrow \mathcal{O}_C \longrightarrow \mathcal{F} \longrightarrow 0.$$

Tensoring the line bundle $\mathcal{L}$, since the support of cokernel $\mathcal{F}$ is the point P, we obtain the sheaf exact sequence:

$$0 \longrightarrow \mathcal{L} \longrightarrow \mathcal{L} \otimes \mathcal{O}_C(P) \longrightarrow \mathcal{F} \longrightarrow 0.$$

Since, by Proposition 5.5, the equality

$$h^i(C, \mathcal{F}) = \begin{cases} \dim_K K(P), & i = 0, \\ 0, & i \geq 1 \end{cases}$$

holds, the lemma follows from the long exact sequence induced by the above exact sequence. □

Theorem 5.11 (Riemann-Roch theorem for proper curves (the first form))**.** *For any regular divisor D on a proper curve C, the equality*

$$\chi(\mathcal{O}_C(D)) = \chi(\mathcal{O}_C) + \deg_C D$$

holds.

Proof. We write $D = \sum_P a_P P$. By induction on $|\sum_P a_P|$, the theorem follows from the equality of Lemma 5.10. □

Theorem 5.12. *Any proper rigid analytic space of dimension one is projective.*

Remark. If the base field K is separably closed, the theorem follows from Theorem 2.1.4 in [6].

Proof. It suffices to show that any proper rigid analytic space of pure dimension one is projective. Moreover, by Lemma 5.7 and Corollary 3.1.6 [7], we have only to show that any proper curve is projective. Let $\bigcup_{i \in I} C_i$ be the irreducible decomposition of a proper curve C. Choose a regular point

P_i on each C_i. By Lemma 5.10 and Theorem 5.11, there exists a positive integer n_i such that the line bundle $\mathcal{O}_C(n_i P_i)$ admits a section that is not a section of $\mathcal{O}_C((n_i - 1)P_i)$. The section gives the meromorphic function f_i, which is analytic except for the pole at the point P_i. Let f be the summation of all the meromorphic functions f_i. Then the restriction of f to each C_i is non-constant. The meromorphic function f gives the morphism $\phi\colon C \to \mathbb{P}^1_K$ of rigid analytic spaces.

Proposition 4 in [4, 9.6.2] and the following sentences show that the morphism ϕ is proper. Therefore, the proper mapping theorem (Proposition 3 in [4, 9.6.3]) shows that the image $\phi(C)$ is an analytic subset of $\mathbb{P}^1_K$. We put the reduced structure on $\phi(C)$. Since the morphism ϕ is proper, we have the Stein factorization $\mu \circ \lambda\colon C \to S \to \phi(C)$ (Proposition 5 in [4, 9.6.3]). By Lemma 4 in [4, 9.6.3], the morphism λ is surjective and for any point s on S, the preimage $\lambda^{-1}(s)$ is a connected analytic subset of C. If the preimage $\lambda^{-1}(s)$ is not a point, then the meromorphic function f is constant on a irreducible component of C. This is absurd. Therefore, the preimage $\lambda^{-1}(s)$ is a point on C. Thus, Lemma 4 in [4, 9.6.4] shows that the morphism λ is an isomorphism. Since the morphism μ is finite, Theorem 5.4 implies that the rigid analytic space S is projective. Therefore, the proper curve C is projective. □

Theorem 5.13 (Riemann-Roch theorem for proper curves (the second form))**.** *Let $\pi\colon C^n \to C$ be the normalization of a proper curve C. For any line bundle $\mathcal{L}$ on C, the equality*

$$\chi(\mathcal{L}) = \chi(\mathcal{O}_C) + \deg_{C^n} \pi^*\mathcal{L}$$

holds.

Proof. We define a coherent $\mathcal{O}_C$-module $\mathcal{F}$ by the following sheaf exact sequence:

$$0 \longrightarrow \mathcal{O}_C \longrightarrow \pi_*\mathcal{O}_{C^n} \longrightarrow \mathcal{F} \longrightarrow 0.$$

Since the singular locus of each U_i is a finite number of points, the support of $\mathcal{F}$ is a finite number of points. Then, tensoring $\mathcal{L}$, we obtain the sheaf exact sequence:

$$0 \longrightarrow \mathcal{L} \longrightarrow \pi_*\mathcal{O}_{C^n} \otimes \mathcal{L} \longrightarrow \mathcal{F} \longrightarrow 0.$$

These sequences give the equalities

$$\chi(\pi_*\mathcal{O}_{C^n}) - \chi(\mathcal{O}_C) = \chi(\mathcal{F})$$

and

$$\chi(\pi_*\mathcal{O}_{C^n} \otimes \mathcal{L}) - \chi(\mathcal{L}) = \chi(\mathcal{F}).$$

Eliminating $\chi(\mathcal{F})$, we obtain the equality:

$$\chi(\mathcal{L}) - \chi(\mathcal{O}_C) = \chi(\pi_*\mathcal{O}_{C^n} \otimes \mathcal{L}) - \chi(\pi_*\mathcal{O}_{C^n}).$$

Since, by the projection formula (Proposition 5.1), we have the isomorphism

$$\pi_* \mathcal{O}_{C^n} \otimes \mathcal{L} \cong \pi_* \pi^* \mathcal{L},$$

the right-hand side of the above equality is equal to $\chi(\pi_*\pi^*\mathcal{L}) - \chi(\pi_*\mathcal{O}_{C^n})$. Therefore, Proposition 5.5 gives the equality

$$\chi(\pi_*\pi^*\mathcal{L}) - \chi(\pi_*\mathcal{O}_{C^n}) = \chi(\pi^*\mathcal{L}) - \chi(\mathcal{O}_{C^n}),$$

which is equal to $\deg_{C^n} \pi^*\mathcal{L}$ by the Riemann-Roch theorem for proper regular curves (Theorem 5.11). □

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