On weakly reflective submanifolds in Hilbert spaces

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In infinite dimensional geometry, it is fundamental to consider differences or similarities between the finite and infinite dimensional cases. It is known ([2]) that the major part of the standard theorems in (Riemannian) manifold theory are still valid for differentiable manifolds modeled on a separable Hilbert space, namely Hilbert manifolds. On the other hand, an unusual example of Hilbert manifolds with a non-differentiable topological group structure ([5]) appears in connection with Kac-Moody algebras.

The study of submanifolds in Hilbert spaces is an important subject since it provides examples of Riemannian Hilbert manifolds. However it is difficult to deal with all such submanifolds since the spectral theory of the shaper operators is complicated in general. Thus it is natural to focus on proper Fredholm (PF) submanifolds ([4]) whose shape operators are compact. It is also noted that important examples of PF submanifolds can be obtained as orbits of a group action induced by the gauge transformations. These actions are called \( P(G, H) \)-actions ([5]).

In this talk, we first review basic facts on \( P(G, H) \)-actions. Then I will introduce infinite dimensional version of weakly reflective submanifolds, and explain their basic properties. Moreover I shall state results on the submanifold geometry of \( P(G, H) \)-orbits in terms of weakly reflective submanifolds. In particular, the following will be emphasized: (1) each fiber of a parallel transport map over a compact Riemannian homogeneous space \( G/K \) is a weakly reflective submanifold of a Hilbert space, (2) we can construct an infinite dimensional weakly reflective submanifold from each of known examples of weakly reflective submanifolds in \( G/K \).

参考文献


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