



FIGURE 1. The form of the set $g_y(x, y) = 0$

1. ABSTRACT

We consider the problem of finding heteroclinic solutions $x(s)$, $y(s)$, of the singularly perturbed system

$$(1.1) \quad \begin{aligned} x'' &= g_x(x, y) \\ \epsilon^2 y'' &= g_y(x, y) \end{aligned}$$

$$x = x(s), \quad y = y(s), \quad s \in \mathbb{R}, \quad ' = \frac{d}{ds}$$

which are approximated, when ϵ is small, by a non-smooth connection for the formal limiting system obtained by setting $\epsilon = 0$. The irregularity in the formal limit arises due to the branching nature of the set of solutions (x, y) of the degenerate relation $0 = g_y(x, y)$ (see Fig. 2). Thus the critical manifold will have a “corner” singularity. The salient features of the function g are

- (i) $g(x, y) = g(x, -y)$, $\forall (x, y) \in \mathbb{R}^2$. (This assumption is for convenience only.)
- (ii) g has three nondegenerate equal global minima at $(0, 0)$, (x_1, y_1) , and $(x_1, -y_1)$, where $x_1 > 0$, and $y_1 > 0$. Without loss of generality, we assume that $g(0, 0) = g(x_1, y_1) = g(x_1, -y_1) = 0$.
- (iii) There exists a continuous function, with finitely many points of nondifferentiability $Y : [-\delta, x_1 + \delta] \rightarrow [0, \infty)$, $\delta > 0$ small, such that $Y(0) = 0$, $Y(x_1) = y_1$, $g_y(x, Y(x)) = 0$, $\forall x \in [-\delta, x_1 + \delta]$ and $g(x, Y(x)) \neq 0$, $\forall x \in (0, x_1)$.

We show existence of solutions $(x, y) \in C^2(\mathbb{R}) \times C^2(\mathbb{R})$ of (1.1) which satisfy

$$(1.2) \quad (x(-\infty), y(-\infty)) = (0, 0) \quad \text{and} \quad (x(\infty), y(\infty)) = (x_1, y_1)$$

and their projection on the $x - y$ plane converges as $\epsilon \rightarrow 0$ to the set $\{(x, Y(x)), x \in (0, x_1)\}$.