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Speaker: Walter Bergweiler

Address: Mathematisches Seminar, Christian–Albrechts–Universität zu Kiel,
Ludewig–Meyn–Str. 4, D–24098 Kiel, Germany

Email: bergweiler@math.uni-kiel.de

Title: Hyperbolic entire functions with bounded Fatou components

Abstract: The Eremenko-Lyubich class B consists of all transcendental entire functions f for which the set $\text{sing}(f^{-1})$ of critical and (finite) asymptotic values is bounded. A function $f \in B$ is called hyperbolic if every point of the closure of $\text{sing}(f^{-1})$ is contained in an attracting periodic basin. We show that if a hyperbolic map $f \in B$ has no asymptotic value and every Fatou component of f contains at most finitely many critical points, then every Fatou component of f is bounded. Moreover, the Fatou components are quasidisks in this case. If, in addition, there exists N such that every Fatou component contains at most N critical points, then the Julia set of f is locally connected.

For hyperbolic maps in B with only two critical values and no asymptotic value we find that either all Fatou components are unbounded, or all Fatou components are bounded quasidisks.

We illustrate the results by a number of examples. In particular, we show that there exists a hyperbolic entire function $f \in B$ with only two critical values and no asymptotic value for which all Fatou components are bounded quasidisks, but the Julia set is not locally connected.

The results are joint work with Núria Fagella and Lasse Rempe-Gillen.