HOLOMORPHIC FOLIATIONS WITH MANY CLOSED LEAVES

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Abstract. In this talk we discuss holomorphic foliations with isolated singularities on a complex compact surface. We assume that in the complement of some compact curve the leaves of the foliation are closed, i.e., closed off the singular set of the foliation. Then we prove that, under mild and natural conditions on the singularities of the foliation in this curve (sometimes also in the curve itself), there is a meromorphic first integral for the foliation in the compact surface. This result can be seen as a generalization of a classical result of Darboux-Jouanolou assuring the existence of a rational first integrals for codimension one algebraic foliations on the complex projective space provided that there are infinitely many invariant algebraic hypersurfaces.

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