Kyoto Dynamics Days 4

Mechanics and Dynamics

December 17-18, 2004

Department of Mathematics, Kyoto University

- Program -

December 17

2:00-3:00 Richard Montgomery (Univ. California, Santa Cruz)
New periodic orbits for the N-body problem

ABSTRACT: In December 1999 Alain Chenciner and the speaker ``rediscovered'' a new periodic orbit for three equal masses moving in the plane according to Newton's laws of gravity. Chris Moore found the orbit numerically in the early 1990s, using some of the same ideas. The three equal masses chase each other around a fixed analytic figure eight shaped planar curve. We describe the method of proof and how it led to a whole host of new orbits for the N-body problem. The tools of the proof are a combination of calculus of variations, differential geometry and symmetries. The chief technical difficulty is avoiding collisions. After a sketch of the existence proof, we will survey some subsequently discovered orbits.

3:15-4:15 Toshiaki Fujiwara (Kitasato Univ.)
Synchronized Similar triangles for Planar Three-Body Orbit

ABSTRACT: If a three-body orbit has zero angular momentum and constant moment of inertia, the triangle whose vertexes are the positions of masses and the triangle whose perimeters are the momenta are always inversely similar. We call these triangles ``Synchronized Similar Triangles'' (SST), because they are always similar. Any planar three-body orbit can be transformed to an orbit with zero angular momentum and constant moment of inertia. Therefore, we can find SST in the transformed variables. Using SST, Fukuda, Kameyama, Ozaki, Yamada and the speaker proved that three-body orbit with zero angular momentum must have infinitely many syzygies --- this marvellous theorem was first formulated and proved by Montgomery. Recent project with Diacu, P¥'erez-Chavela and Santoprete to find a simple proof of the "Saari's conjecture" for planar three-body orbit will also be presented.

4:30-5:30 Konstantin Mischaikow (Georgia Tech.)
A topological approach to the study of patterns

ABSTRACT: Patterns are often the most striking features of spatially explicit dynamical systems. However, finding appropriate methods for characterizing and quantifying such patterns, especially when they are complicated, remains a challenge. I will describe a new approach based on computational homology where we are capable of distinguishing the dynamics at different parameters, quantifying spatial-temporal chaos and perhaps distinguishing between competing models. The specific applications involve data from numerical simulations of FitzHugh-Nagumo, Cahn-Hilliard, and experimental data of Rayleigh-Benard convection.

6:00- Dinner

<u>December 18</u>

10:00-11:00 Mitsuru Shibayama (Kyoto Univ.)
New periodic solutions to the 2n-body problem

ABSTRACT: Using a variational method, we prove the existence of periodic orbits for the 2n-body problem (n = 3,4,5) with equal masses in the plane. The orbit of masses for each solution consists of two symmetric closed curves. The most difficulty in the proof is the exclusion of collisions. We use a computer assisted proof to prove it, and then complete the proof for the cases of n = 3,4,5. We finally consider the extensions of the result, whose proof is done once possibility of collision is excluded.

11:15-12:15 Masaya Saito (National Astronomic Observatory)
Rectilinear three-body problem using symbolic dynamics

ABSTRACT: Rectilinear three-body problem with general masses and negative energy is studied using symbolic dynamics. We introduce the Poincare section, integrate orbits that start from it, and record collisional history of an orbit as a symbol sequence. In doing this, we associate the point on the Poincare section with symbol sequences. Each digit of the sequences contains the information about the configuration of collision. In the present paper, we regard the distribution of the symbol sequences as the structure of the Poincare section and study how the structure of the Poincare section changes as mass ratio of the particles changes. We then define the rule to classify the symbol sequences and divide the Poincare section according to the rule. It means that points on the Poincare section is classified according to the number of alternate collisions of the left and right and to which particle is ejected. The divided regions via the classification are stratified and construct blocks, whose shape is like a scallop. In most mass configurations, there are regions bifurcated into two (or more) branches. Only one of branches consists of the scallop-like blocks. This bifurcation makes the structure of the Poincare section complex. However, in special mass configurations, no bifurcations occur and regions are well stratified. We found that the special mass configuration is the one that the flow on the triple collision manifold is totally degenerate. The location of each region is related with kind of symbol sequences. This relation can be derived from the studies of the flow on the triple collision manifold and of the homothetic orbit.

2:00-3:00 Toshihiro Iwai (Kyoto Univ.)

Stratified dynamical systems and their boundary behavior for three bodies in space

ABSTRACT: The center-of-mass system for many particles are stratified into strata by the rotation group action. The principal stratum consists of nonlinear confirgurations. The colinear configurations form a lower dimensional stratum. Mechanics for many particles with nonlinear configurations and for those with collinear configurations are set up on the tangent and cotangent bundles of respective strata and can be reduced by the use of rotational symmetry. A question arises as to how a many-body system behave in the neighborhood of collinar configurations. The system may make a vibration to bend its collinear configuration. This motion takes place acorss the boundary of the principal stratum. It is of great interest to study such boundary behaviors of motion. This talk will start with the stratified dynamical systems and then deal with boundary behaviors in the case of three bodies in space.

3:15-4:15 Richard Montgomery (Univ. California, Santa Cruz)

A 3-body problem whose flow is conjugate to a hyperbolic geodesic flow

ABSTRACT: We investigate the three-body problem with an attractive $1/r^2$ potential. (The gravitational potential is $1/r$.) We show that modulo symmetries, the dynamics of the bounded zero-angular momentum solutions is equivalent to a geodesic flow on the thrice-punctured sphere, or ``pair of pants''. Our main result is that when the three masses are equal then the metric has negative curvature (hyperbolic) everywhere except at two points (the Lagrange points). The metric is also complete and has infinite area. Corollaries of hyperbolicity are the uniqueness of the $1/r^2$ figure eight, later proved by Fujiwara et al using a different method, a complete symbolic dynamics for encoding the collision-free solutions, and the fact that collision solutions are dense within the bound solutions.