ABSTRACTS

Harmonic Analysis and its Applications 2023

August 28 - September 1, 2023

at

Hokkaido University

Lecture A

Piero D'Ancona
(Sapienza Universitá di Roma)

Scattering theory for the NLS and the concentration–compactness method

Abstract

The main goal of the course is to describe the problem of scattering for the nonlinear Schrödinger equation (NLS) and some recent techniques from harmonic and real analysis used to solve it. In particular we shall consider the following topics:

- Pointwise and Strichartz estimates for free and perturbed Schrödinger equations
- Long time existence for NLS
- The problem of scattering in energy space
- The concentration–compactness method of P.L.Lions and its application by Kenig and Merle
- Scattering for the 1D NLS with variable coefficients.

Lecture B

Sebastian Herr
(Universität Bielefeld)

Bilinear Fourier restriction, critical function spaces, and applications to nonlinear Dirac and wave equations

Abstract

A characteristic feature of dispersive partial differential equations (such as Schrödinger or wave equations) is that solutions spread out in space and decay in time while preserving the spatial L^2 norm. This effect can be quantified by so-called Strichartz estimates and such estimates play a fundamental role in the perturbative analysis of nonlinear dispersive PDEs.

Strichartz estimates are dual to estimates for the restriction of the Fourier transform of the characteristic surface corresponding to the differential operator. From this perspective, the curvature of the characteristic surface determines the decay. Fourier restriction theory is a classical topic in harmonic analysis which started in the 1960s with first results and conjectures of Elias Stein and his students.

Both in the analysis of nonlinear dispersive PDEs and in Fourier restriction theory more information can be obtained by passing to a bilinear setting. More precisely, the product of two wave packets traveling in transversal directions enjoys better space-time decay.

In these three lectures I will introduce atomic function spaces and describe recent progress on bilinear Fourier restriction estimates in such spaces. Then, I will outline how these can be used to prove small data global well-posedness and scattering for cubic Dirac equations and the wave maps equation in scaling-critical spaces. This is based on joint work with Timothy Candy.

Lecture C

Isroil A. Ikromov
(Institute of Mathematics named after V.I. Romanovsky AS of Uzb., Samarkand, Uzbekistan)

On the sharp estimates for convolution operators with oscillatory kernel

Abstract

In this talk, we discuss the $L^p \mapsto L^{p'}$-boundedness problem for the convolution operator M_k with oscillatory kernel and a smooth amplitude function of order $-k$ for large values of the argument. The convolution operator M_k is related to solutions to the Cauchy problem for the strictly hyperbolic equations.

The operator M_k is associated to the characteristic hypersurfaces $\Sigma \subset \mathbb{R}^3$ of the equation. We study the convolution operators assuming that Σ is contained in a sufficiently small neighborhood of a given point $v \in \Sigma$ at which exactly one of the principal curvatures of Σ does not vanish. Such surfaces exhibit singularities of type A in the sense of Arnol'd's classification. Denoting by k_p the minimal exponent such that M_k is $L^p \mapsto L^{p'}$-bounded for $k > k_p$, we show that the number k_p depends on some discrete characteristics of the surface given as the graph of a function having singularities of type A.

Lecture D

Olli Saari
(Universitat Politecnica de Catalunya)

Phase space localizing operators and applications

Abstract

In these talks, I will discuss aspects of uniform bounds for one and two dimensional bilinear Fourier multipliers. The focus is on multipliers that have a singularity along a linear subspace, such as the bilinear Hilbert transforms, and uniformity refers to independence of the operator norm of the subspace containing the singularity. Such problems were first introduced for the bilinear Hilbert transform by Calderón and later studied by a number of authors, in particular Thiele, Grafakos-Li and Uraltsev-Warchalski. I will discuss the basic problem statements, the modern update of the method of phase plane projections and finally some advances in planar problems. This is based on a joint project with Marco Fraccaroli and Christoph Thiele.

Talk A

Tohru Ozawa
(Department of Applied Physics, Waseda University)

Proof of the Gagliardo-Nirenberg Sobolev inequalities via heat semigroup

Abstract

We give a simple and direct proof of the Gagliardo-Nirenberg Sobolev inequalities on the basis of the heat semigroup. This talk is based my recent joint-work with Dr. Taiki Takeuchi.

Talk B

Haruya Mizutani
(Osaka University)

Strichartz estimates for Schrödinger equations with long-range potentials

Abstract

The Strichartz estimate is one of fundamental tools in the study of nonlinear dispersive equations. This talk deals with (global-in-time) Strichartz estimates for Schrödinger equations with potentials decaying at infinity. The case when the potential decays sufficiently fast has been extensively studied in the last three decades. However, it has remained mostly unknown for slowly decaying potentials in which case the standard perturbation argument does not work. We instead employ several techniques from long-range scattering theory and microlocal/semiclassical analysis, and prove Strichartz estimates for a class of positive potentials decaying arbitrarily slowly. A typical example is the positive Coulomb potential in three space dimensions. As an application, we also obtain a modified scattering type result for the final state problem of a nonlinear Schrödinger equation with long-range potentials. This is partly joint work with Masaki Kawamoto (Ehime University).

Talk C

Toru Nogayama
(Chuo University)

Maximal regularity in Besov-Morrey spaces

Abstract

In this talk, we consider the maximal regularity for heat equations in Besov-Morrey spaces. The maximal regularity is one of the important estimates to investigate some problems for PDE. To lead this estimate, there is the general theory. However, Besov-Morrey spaces fall out of this framework. We directly show this estimate in Besov-Morrey spaces by using the Hardy-Littlewood maximal operator and duality argument. This is the joint work with Yoshihiro Sawano at Chuo University.

Talk D

Shinya Kinoshita
(Tokyo Institute of Technology)

Decoupling inequality under thin annulus constraint and its application to the periodic Zakharov system

Abstract

In this talk, we consider the Cauchy problem of the Zakharov system on the multidimensional torus $\mathbb{T}^d$. We establish the well-posedness of the Zakharov system in low regularity Sobolev spaces when $d \geq 3$. The results for $d = 3$ and $d \geq 5$ are optimal up to ε loss. The key tool is the ℓ^2 decoupling inequality for paraboloid under thin annulus constraint. This talk is based on the joint work with Shohei Nakamura (Osaka) and Akansha Sanwal (Innsbruck).

Talk E

Hitoshi Tanaka
(Tsukuba University of Technology)

The dyadic analysis with cubes and rectangles

Abstract

With rectangular doubling weight, a generalized Hardy-Littlewood-Sobolev inequality for rectangular fractional integral operators is verified. The result is a nice application of M-linear embedding theorem for dyadic rectangles. Some topics for the dyadic analysis with cubes are also introduced.