One dimensional diffusions conditioned to be non-explosive

Ichiro Shigekawa (Kyoto University)

We consider one dimensional diffusions conditioned to be non-explosive. Suppose we are given a minimal diffusion process $\{X_t, P_x\}$ on an interval (l_1, l_2). Let ζ be its explosion time. If $P_x[\zeta = \infty] > 0$, then the measure conditioned to be non-explosive is defied by

$$P_x[\,\cdot\,|\,\zeta = \infty] = P_x[\,\cdot\,\cap \zeta = \infty]/P_x[\zeta = \infty].$$

If $P_x[\zeta = \infty] = 0$, then the measure conditioned to be non-explosive is defined as the limit

$$\lim_{T\to\infty} P_x[\,\cdot\,|\,\zeta > T].$$

If the limit exists and the limit is a diffufin process, we call it a *surviving diffusion*. We are interested in the following problems:

1. When does the surviving diffusion?
2. Characterizasion of the surviving diffusion.

The the surviving diffusion is characterized as a h-transform of the original process by the λ-harmonic function φ, λ being the principal eigenvalue