

ON THE PARABOLIC BIFURCATION OF HOLOMORPHIC MAPS

MITSUHIRO SHISHIKURA

Tokyo Institute of Technology
Ohokayama, Meguro, Tokyo 152, Japan

ABSTRACT. A holomorphic map having a parabolic fixed point changes its dynamics drastically under a small perturbation. In order to study such a bifurcation, a special coordinate transformation is introduced. An invariant "residue" for fixed points is defined.

§0. INTRODUCTION

The dynamics of holomorphic maps of one complex variable was studied extensively during 80's. For quadratic polynomials, Douady and Hubbard have obtained the combinatorial description of the Mandelbrot set, in terms of external rays and external angles [DH]. One of their main tools is the analysis of the parabolic bifurcation (the bifurcation of rationally indifferent periodic points) using "Ecalles cylinders". This Ecalle cylinder is also used to study the structure of the Mandelbrot set near $c = 1/4$ the cusp of the main cardioid, the non local connectivity of the cubic connectedness locus, etc, [L],[DD].

We are concerned with fixed points of holomorphic maps. We call a fixed point *parabolic* (or *rationally indifferent*) if the multiplier (=the derivative at the fixed point) is a root of unity. In other word, if the origin 0 is a parabolic fixed point of f , f has the form:

$$f(z) = \lambda z + a_2 z^2 + \dots$$

where $\lambda = \exp(2\pi i p/q)$, $p, q \in \mathbb{Z}$, $q \geq 1$, $(p, q) = 1$. It is well known that under a small perturbation, a periodic cycle of period q can bifurcate from such a fixed point. Furthermore the behaviour of orbits depends sensitively on the perturbation, hence the global dynamics may change drastically. So our goal is to analyze the bifurcation of parabolic points in such a way that we can investigate the change of the global dynamics.

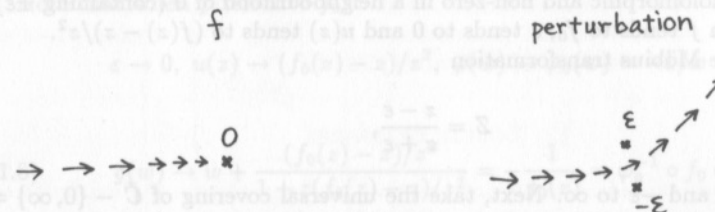


FIGURE 1. Dynamics of f and its perturbation

In the first part of this paper, we try to study the parabolic bifurcation using some special (and singular) coordinate transformation and reinterpret the method of Ecalle cylinders. In the second part, we introduce an invariant, the residue or the holomorphic index, for fixed points and study its properties specially for parabolic fixed points. In this paper we announce only the results, and the precise statements and the proofs should be published elsewhere. For simplicity, we treat only the case where the multiplier $\lambda = 1$.

Acknowledgement. The author would like to thank A. Douady, J.H. Hubbard and Tan Lei for having discussions on this subject.

§1. COORDINATE CHANGE

Let f_0 be a holomorphic map defined in a neighbourhood of 0 and suppose that 0 is a parabolic fixed point of f_0 with multiplier 1, i.e.

$$(1.1) \quad f_0(z) = z + a_2 z^2 + a_3 z^3 + \dots$$

We need to put :

$$(1.2) \text{ Non-degeneracy Assumption. } a_2 \neq 0.$$

In this case, by the linear scaling of the coordinate $z \rightarrow a_2 z$, we may assume that $a_2 = 1$. Now we perturb f_0 to f which is supposed to be defined in a neighbourhood of 0 and close to f_0 in the sense that $|f - f_0|$ is small in certain neighbourhood of 0. Since 0 is a fixed point of f_0 with multiplicity 2 (as a solution of $f_0(z) - z = 0$), f has two fixed points near 0 counted with multiplicity. Suppose f has two distinct fixed points near 0. By a coordinate change of the form $z \rightarrow z + c$ with c small, we may assume that these two fixed points are ε and $-\varepsilon$ (symmetric with respect to 0). Then we can write :

$$(1.3) \quad f(z) = z + (z^2 - \varepsilon^2)u(z)$$

where $u(z)$ is holomorphic and non-zero in a neighbourhood of 0 (containing $\pm\varepsilon$). Moreover when f tends to f_0 , ε tends to 0 and $u(z)$ tends to $(f(z) - z)/z^2$.

First, by the Möbius transformation

$$(1.4) \quad Z = \frac{z - \varepsilon}{z + \varepsilon},$$

we send ε to 0 and $-\varepsilon$ to ∞ . Next, take the universal covering of $\mathbb{C} - \{0, \infty\} = \mathbb{C} - \{0\}$ (in Z -coordinate), i.e. we introduce ζ by

$$(1.5) \quad Z = \exp(2\pi i \zeta).$$

Let us denote $z_1 = f(z)$ and use Z_1, ζ_1 to denote corresponding coordinates, i.e.

$$\exp(2\pi i \zeta_1) = Z_1 = \frac{z_1 - \varepsilon}{z_1 + \varepsilon}$$

where ζ_1 is determined up to an ambiguity by modulo \mathbf{Z} . In this ζ coordinate, f can be lifted to a map (depending on the choice of lift) as follows:

$$Z_1 = \frac{z_1 - \varepsilon}{z_1 + \varepsilon} = \frac{z - \varepsilon + (z^2 - \varepsilon^2)u(z)}{z + \varepsilon + (z^2 - \varepsilon^2)u(z)} = \frac{z - \varepsilon}{z + \varepsilon} \frac{1 + (z + \varepsilon)u(z)}{1 + (z - \varepsilon)u(z)} = Z \frac{1 + \varepsilon u(z)/(1 + zu(z))}{1 - \varepsilon u(z)/(1 + zu(z))}$$

Hence

$$\zeta \rightarrow \zeta_1 = \frac{1}{2\pi i} \log\left(\frac{z_1 - \varepsilon}{z_1 + \varepsilon}\right) = \zeta + \frac{1}{2\pi i} \log\left(\frac{1 + \varepsilon u(z)/(1 + zu(z))}{1 - \varepsilon u(z)/(1 + zu(z))}\right)$$

where we choose a branch of logarithm so that $\log 1 = 0$, and we keep the original coordinate function z in the expression. Then

$$\zeta_1 = \zeta + \frac{1}{\pi i} \cdot \frac{\varepsilon u(z)}{1 + zu(z)} + O(\varepsilon^2).$$

So we introduce a new coordinate by $w = \pi i \zeta / \varepsilon$, then the corresponding map $g(w)$ is:

$$(1.6) \quad w \rightarrow g(w) = w + \frac{1}{2\varepsilon} \log\left(\frac{1 + \varepsilon u(z)/(1 + zu(z))}{1 - \varepsilon u(z)/(1 + zu(z))}\right) = w + \frac{u(z)}{1 + zu(z)} + O(\varepsilon).$$

The original coordinate z can be expressed under the form:

$$(1.7) \quad z = \varphi(w) = \varepsilon \frac{1 + \exp(2\varepsilon w)}{1 - \exp(2\varepsilon w)}.$$

Since we took the universal covering, w and $w + \frac{\pi i}{\varepsilon}$ should be identified.

Note that when f tends to f_0 ,

$$\varepsilon \rightarrow 0, \quad u(z) \rightarrow (f_0(z) - z)/z^2, \quad \varphi(w) \rightarrow \varphi_0(w) = -1/w \text{ and}$$

$$(1.8) \quad g(w) \rightarrow w + \frac{(f_0(z) - z)/z^2}{1 + z(f_0(z) - z)/z^2} = -\frac{1}{f_0(z)} = \varphi_0^{-1} \circ f_0 \circ \varphi_0(w).$$

For this convergence, we should restrict w to a suitable region, for example

$$\{w \in \mathbb{C} \mid |\operatorname{Im}(\varepsilon w)| \leq \frac{\pi}{2}\} - \bigcup_{n \in \mathbf{Z}} \{w \mid |w - n \frac{\pi i}{\varepsilon}| < R\}$$

for some constant $R > 0$. Hence one can consider that $g(w)$ is a perturbation of $g_0(w) = \varphi_0^{-1} \circ f_0 \circ \varphi_0(w)$. The main difference is that for the perturbed system g , there is an additional identification:

$$(1.9) \quad w \sim w + \frac{\pi i}{\varepsilon}.$$

In fact, this identification gives the principal dependence on the perturbation (or on the parameter if f is parametrized). If we consider an orbit for f which comes into a certain neighbourhood of 0, it corresponds to an orbit for g which comes out of a set like $\{w \mid |w| < R\}$. Moreover if the f -orbit comes out of the neighbourhood, the g -orbit should arrive at another set $B_n = \{w \mid |w - n \frac{\pi i}{\varepsilon}| < R\}$, probably for $n=1$ or -1 . Whether this happens or not depends on the relative position of B_1 or B_{-1} with respect to B_0 , hence on the argument of $\frac{\pi i}{\varepsilon}$. So if we pretend that ε is the parameter of the system, (1.9) shows a sensitive dependence on the parameter.

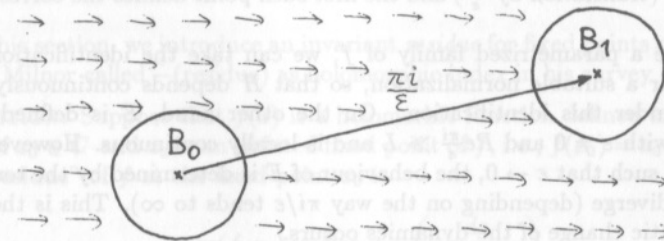


FIGURE 2. Dynamics of g

One can use this analysis, for example, to prove the landing property of external rays of rational angles with odd denominators (see [DH]). The original proof of this fact was given by the method of Ecalle cylinders.

§2. ECALLE CYLINDERS

Using the coordinate in the previous section, we can re-prove the existence of Ecalle cylinders and re-interpret their application. For unperturbed system f_0 , the Ecalle cylinders $C_{0,+}$ and $C_{0,-}$ are defined as follows. In the coordinate $w = -1/z = \varphi_0^{-1}(z)$, $g_0(w) = \varphi_0^{-1} \circ f_0 \circ \varphi_0(w)$ has the expansion:

$$g_0(w) = w + 1 + O\left(\frac{1}{w}\right).$$

Let $l_+ = \{w \mid \operatorname{Re} w = L\}$ and $l_- = \{w \mid \operatorname{Re} w = -L\}$ for $L > 0$. If L is large enough, g_0 is injective and $g_0(w) > w + \frac{1}{2}$ in $\{w \mid |w| \geq L\}$. Then l_+ (resp. l_-) is mapped to its right hand side, and l_+ and $g_0(l_+)$ (resp. l_- and $g_0(l_-)$) bounds a simply connected region S_+ (resp. S_-). The positive Ecalle cylinder $C_{0,+}$ is

$$C_{0,+} = \overline{S_+} / \sim$$

where $\overline{S_+} = S_+ \cup l_+ \cup g_0(l_+)$ is a closed region in \mathbb{C} and the equivalence relation \sim is defined by $w \sim g_0(w)$ for $w \in l_+$. It is easy to see that $C_{0,+}$ has naturally the structure of a Riemann surface and is conformally isomorphic to the bi-infinite cylinder $\mathbb{C} - \{0\} \simeq \mathbb{C}/\mathbb{Z}$. The negative Ecalle cylinder $C_{0,-}$ is defined similarly from $\overline{S_-}$.

The dynamics of g_0 induces a mapping H_0 from a neighbourhood of the ends of the negative cylinder C_- to a neighbourhood of the ends of the positive cylinder C_+ as follows. If $w \in \overline{S_-}$ and $|\operatorname{Im} w|$ is large enough, its orbit eventually arrives at $\overline{S_+}$. This correspondence defines the map.

For a perturbed system f , if the perturbation is small and if $\operatorname{Re} \frac{\pi i}{\varepsilon} \gg L$, we can define similarly the positive and negative cylinders C_+ , C_- and the mapping H near the ends of C_- . Since there is the identification (1.9), we can define an isomorphism E from C_+ to C_- as follows. If $w \in \overline{S_+}$, its orbit eventually arrives at $(\overline{S_-}) + \frac{\pi i}{\varepsilon}$ (translation by $\frac{\pi i}{\varepsilon}$) and the first such point defines the corresponding point in C_- .

If we have a parametrized family of f , we can take the identification of C_{\pm} with \mathbb{C}/\mathbb{Z} for a suitable normalization, so that H depends continuously on the parameter under this identification. On the other hand, E is defined for the parameters with $\varepsilon \neq 0$ and $\operatorname{Re} \frac{\pi i}{\varepsilon} \gg L$ and is locally continuous. However, if one takes a limit such that $\varepsilon \rightarrow 0$, the behaviour of E is determined by the term $\pi i/\varepsilon$, and it may diverge (depending on the way $\pi i/\varepsilon$ tends to ∞). This is the reason why the drastic change of the dynamics occurs.

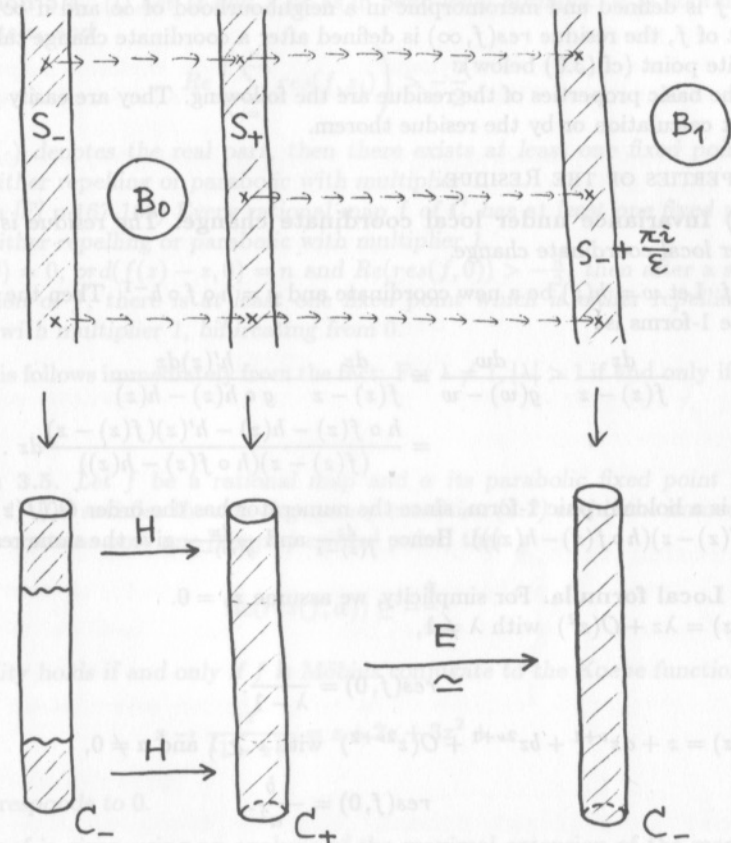


FIGURE 3. Ecalle cylinders and maps H and E

§3. RESIDUE FOR FIXED POINTS

In this section, we introduce an invariant residue for fixed points of holomorphic maps. Milnor called $-(\text{residue})$ as holomorphic index in his survey article [M], §9.

DEFINITION. Suppose that $f(z)$ is a holomorphic function defined in a neighbourhood of $z_0 \in \mathbb{C}$ and z_0 is an isolated fixed point of f , i.e., $f(z_0) = z_0$ and $f(z) \neq z$. The "residue" of f at the fixed point z_0 is

$$\operatorname{res}(f, z_0) = \operatorname{Res}_{z=z_0} \left(\frac{dz}{f(z) - z} \right),$$

where the right hand side is the residue of the 1-form at z_0 .

If f is defined and meromorphic in a neighbourhood of ∞ and if ∞ is a fixed point of f , the residue $res(f, \infty)$ is defined after a coordinate change taking ∞ to a finite point (cf (3.0) below).

The basic properties of the residue are the following. They are easily proved by direct calculation or by the residue theorem.

PROPERTIES OF THE RESIDUE.

(3.0) Invariance under local coordinate change. The residue is invariant under local coordinate change.

Proof. Let $w = h(z)$ be a new coordinate and $g = h \circ f \circ h^{-1}$. Then the difference of the 1-forms is

$$\begin{aligned} \frac{dz}{f(z) - z} - \frac{dw}{g(w) - w} &= \frac{dz}{f(z) - z} - \frac{h'(z)dz}{g \circ h(z) - h(z)} \\ &= \frac{h \circ f(z) - h(z) - h'(z)(f(z) - z)}{(f(z) - z)(h \circ f(z) - h(z))} dz. \end{aligned}$$

This is a holomorphic 1-form, since the numerator has the order $O((f(z) - z)^2) = O((f(z) - z)(h \circ f(z) - h(z)))$. Hence $\frac{dz}{f(z) - z}$ and $\frac{dw}{g(w) - w}$ give the same residue. \square

(3.1) Local formula. For simplicity, we assume $z_0 = 0$.

If $f(z) = \lambda z + O(z^2)$ with $\lambda \neq 1$,

$$res(f, 0) = \frac{1}{\lambda - 1}.$$

If $f(z) = z + az^{\nu+1} + bz^{2\nu+1} + O(z^{2\nu+2})$ with $\nu \geq 1$ and $a \neq 0$,

$$res(f, 0) = -\frac{b}{a^2}.$$

REMARK. In the above formulae, the multiplier λ and the ratio b/a^2 are well-known formal and analytic invariants. The residue unifies these two invariants.

(3.2) Semi-local formula. Let D be a bounded region in \mathbb{C} bounded by smooth Jordan curves and $f : \bar{D} \rightarrow \bar{D}$ a continuous map. Suppose f is holomorphic in D and has no fixed point on ∂D . Then

$$\sum_{\substack{z_i \in D \\ \text{fixed point of } f}} res(f, z_i) = \frac{1}{2\pi i} \int_{\partial D} \frac{dz}{f(z) - z}.$$

(3.3) Global formula. (Fatou [F] p.167-169) For a rational map $f : \bar{\mathbb{C}} \rightarrow \bar{\mathbb{C}}$,

$$\sum_{z_i : \text{fixed point of } f} res(f, z_i) = -1.$$

Proposition 3.4. (i) Let D and f be as in Semi-local Formula. If D contains n fixed points z_i and

$$Re \left(\sum_{i=1}^n res(f, z_i) \right) > -\frac{n}{2},$$

where $Re(\cdot)$ denotes the real part, then there exists at least one fixed point z_i which is either repelling or parabolic with multiplier 1.

(ii) (Fatou [F] p.167-169) Every rational map f of $\bar{\mathbb{C}}$ has at least one fixed point which is either repelling or parabolic with multiplier 1.

(iii) If $f(0) = 0$, $ord(f(z) - z, 0) = n$ and $Re(res(f, 0)) > -\frac{n}{2}$, then after a small perturbation of f , there is at least one fixed point which is either repelling or parabolic with multiplier 1, bifurcating from 0.

Proof. This follows immediately from the fact: For $\lambda \neq 1$, $|\lambda| > 1$ if and only if $Re(\frac{1}{\lambda-1}) > -\frac{1}{2}$.

Theorem 3.5. Let f be a rational map and α its parabolic fixed point with multiplier 1. If α satisfies the non-degeneracy condition (1.2) and if the immediate basin of α contains only one (simple) critical point, then

$$Re(res(f, \alpha)) \geq -\frac{3}{4}.$$

The equality holds if and only if f is Möbius conjugate to the Koebe function

$$z \rightarrow \frac{z}{(1-z)^2} = z + 2z + 3z^2 + \dots$$

and α corresponds to 0.

The proof is given using an analysis of the maximal extension of the map H_0 between Ecalle cylinders in §2.

Corollary 3.6. Under the hypothesis of Theorem 3.5, after a small perturbation of f , there is at least one fixed point which is either repelling or parabolic with multiplier 1, bifurcating from α .

REFERENCES

[DD] R.L. Devaney and A. Douady, Homoclinic bifurcations and infinitely many Mandelbrot sets, preprint 1988.

[DH] A. Douady et J.H. Hubbard, Etude dynamique des polynômes complexes, I et II, avec la collaboration de P. Lavaurs, Tan Lei et P. Sentenac, Publication d'Orsay 84-02, 85-04, 1984/1985.

[F]P. Fatou, Sur les équations fonctionnelles, Bull. Soc. Math. France, 47 (1919) pp.161-271.

[L] P. Lavaurs, Systèmes dynamiques holomorphes: explosion de points périodiques paraboliques, Thèse de doctorat de l'Université de Paris-Sud, Orsay, France, 1989.

[M]J. Milnor, Dynamics in one complex variable, Introductory lectures, preprint State Univ. of New York at Stony Brook, 1990.

PERIODIC ORBITS NEAR THE BOUNDARY OF A 3-DIMENSIONAL MANIFOLD

J. Sotomayor IMPA

Estrada Dona Castorina 110, Rio de Janeiro, RJ, 22460

BRAZIL

M. A. Teixeira

IMECC, UNICAMP

Cidade Universitaria, C.Px. 6065, 13081, Campinas, SP

BRAZIL

Abstract

In this work we describe the orbit structure of one parameter families of vector fields X_λ (for real parameter values near 0), defined on a 3-dimensional smooth manifold M , such that X_0 has a periodic orbit tangent to ∂M , the boundary of M .

1. INTRODUCTION

In this paper we study smooth families of vector fields X_λ depending on a real 1-dimensional parameter λ in \mathbb{R}^1 , tangent to a 3-dimensional smooth manifold M , with boundary ∂M . Our main interest is to describe the orbit structure of the family, for parameter values near $\lambda = 0$, on a small neighborhood of a periodic orbit γ_0 of X_0 , tangent to ∂M .

Without loss of generality, we assume that M is contained in a boundaryless compact manifold M' and that the boundary is implicitly defined by a smooth real valued function f , as follows:

$df_p \neq 0$ for p in ∂M , $M = \{ f \geq 0 \}$ and $\partial M = \{ f = 0 \}$.

Also we assume that the families X_λ of vector

ADVANCED SERIES IN DYNAMICAL SYSTEMS

Editor-in-Chief: K. Shiraiwa

- Vol. 1: Dynamical Systems and Nonlinear Oscillations
edited by G. Ikegami
- Vol. 2: Dynamical Systems and Singular Phenomena
edited by G. Ikegami
- Vol. 3: Invitation to C^* -algebras and Topological Dynamics
by J. Tomiyama
- Vol. 5: Dynamical Systems and Applications
edited by N. Aoki
- Vol. 6: Stability Theory and Related Topics in Dynamical Systems
edited by G. Ikegami & K. Shiraiwa
- Vol. 7: The Study of Dynamical Systems
edited by N. Aoki
- Vol. 8: Bifurcation Phenomena in Nonlinear Systems and Theory of
Dynamical Systems
edited by H. Kawakami
- Vol. 9: Dynamical Systems and Related Topics
edited by K. Shiraiwa
- Vol. 10: Some Problems on the Theory of Dynamical Systems
in Applied Sciences
edited by H. Kawakami

Advanced Series in Dynamical Systems

Vol. 9

Proceedings of the International Conference
**Dynamical Systems and
Related Topics**

Editor

K. Shiraiwa

Department of Mathematics
Nagoya University

3 – 7 September 1990

Nagoya, Japan



World Scientific

Singapore • New Jersey • London • Hong Kong

Published by

World Scientific Publishing Co. Pte. Ltd.

P O Box 128, Farrer Road, Singapore 9128

USA office: Suite 1B, 1060 Main Street, River Edge, NJ 07661

UK office: 73 Lynton Mead, Totteridge, London N20 8DH

**Proceedings of the International Conference on
DYNAMICAL SYSTEMS AND RELATED TOPICS**

Copyright © 1991 by World Scientific Publishing Co. Pte. Ltd.

All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the Publisher.

ISBN 981-02-0551-1

Printed in Singapore by Utopia Press.

Scientific Committee

Lennart Carleson (Institute of Technology, Sweden)

Adrien Douady (Ecole Normale Supérieure, France)

Shantao Liao (Peking University, People's Republic of China)

Sheldon E. Newhouse (University of North Carolina, USA)

Jacob Palis Jr. (Instituto de Matemática Pura e Aplicada, Brazil)

David Ruelle (Institut des Hautes Études Scientifiques, France)

Kenichi Shiraiwa (Nagoya University, Japan)

Ya. G. Sinai (Landau Institute for Theoretical Physics, USSR)

Jean-Christophe Yoccoz (Université de Paris Sud, France)

Organizing Committee

Chairman Kenichi Shiraiwa (Nagoya University)

Nobuo Aoki (Tokyo Metropolitan University)

Gikō Ikegami (Nihon University)

Hidekazu Ito (Tōhoku University)

Ryōsuke Iwahashi (Nagoya City University)

Shigehiro Ushiki (Kyoto University)

Shigenori Matsumoto (Nihon University)

Yoshio Miyahara (Nagoya City University)

Koichi Yano (University of Tokyo)

Kazuo Yamato (Nagoya University)

June 1991

Kenichi Shiraiwa

Editor and the

Chairman of the Organizing Committee

Printed in Singapore by Utopia Press.

PREFACE

The papers in this volume are contributed by the lecturers of the *International Conference on Dynamical Systems and Related Topics* held at the Nagoya International Center in Nagoya, Japan on September 3-7, 1990. Most of them were presented at the conference. The number of participants from Japan is about 100, and that from outside of Japan is 41.

The conference was held under the auspices of Nagoya University, Nagoya City, and the Mathematical Society of Japan. And it was supported financially by Nagoya University, Nagoya City, Daiko Foundation, Inamori Foundation, Ishida Foundation, Japan Association for Mathematical Sciences, Kato Ryutaro Gakujutu Shinko Kikin (Kato Ryutaro Foundation for the Promotion of Science), and some private contributors participating in the conference. Without their kind help, we would not have been able to hold the conference. We express our sincere gratitude for their cooperation.

We should also mention that our conference had been planned as a satellite conference of the International Congress of Mathematicians held at Kyoto on August 21-29, 1990 (ICM 90). Our conference was made possible by the support of the organizing committee of ICM, and we also express our appreciation for their help.

For the preparation of the conference, the Scientific Committee had been organized. Their efforts toward the success of the conference are much appreciated. The collaboration of staffs of the College of General Education (especially the Department of Mathematics) of Nagoya University was also valuable.

Finally we would like to express our gratitude to all the lecturers and participants of the conference without whom there would be no conference at all. We hope the proceedings of this conference would contribute to the development of dynamical systems and related topics in the world.

June 1991
 Kenichi Shiraiwa
 Editor and the
 Chairman of the Organizing Committee

Nonlinear Stokes Phenomena 158
 by S. Hayashino

Organizing Committee

- Chairman Kenichi Shiraiwa (Nagoya University)
- Nobuo Aoki (Tokyo Metropolitan University)
- Giko Ikegami (Nihon University)
- Hidetaka Ito (Tohoku University)
- Ryōzaburō Jō (Nagoya City University)
- Shigeru Matsuzono (Nihon University)
- Yoshio Miyahara (Nagoya City University)
- Kōichi Yano (University of Tokyo)
- Kazuo Yamato (Nagoya University)

Scientific Committee

- Jan-Christophe Yoccoz (Université de Paris VII, France)
- Ya. G. Sinai (Landau Institute for Theoretical Physics, USSR)
- Kenichi Shiraiwa (Nagoya University, Japan)
- David Ruelle (Institut des Hautes Études Scientifiques, France)
- Jacob Palis Jr. (Institut de Matemática Pura e Aplicada, Brazil)
- Sheldon E. Newhouse (University of North Carolina, USA)
- Shantanu Das (Kangri University, People's Republic of China)
- André Gambaudo (Ecole Normale Supérieure, France)
- Robert Carmona (Institut de Technologie, Sweden)

CONTENTS

Preface	v
On Stability in Singularly Perturbed Nonlinear Systems	1
<i>O. V. Anashkin</i>	
The Set of Axiom A Diffeomorphisms with No Cycle	20
<i>N. Aoki</i>	
Optimal Control of a Dynamical System with Trajectory Retention	36
<i>N. V. Chan</i>	
Bifurcations of Vector Fields with Z_4 Symmetry	61
<i>C.-Q. Cheng</i>	
Topological Transitivity & Metric Transitivity on T^2	65
<i>T. Ding</i>	
Smooth Weakly Chaotic Interval Maps with Zero Topological Entropy	72
<i>B.-S. Du</i>	
Tracking Limit Cycles Escaping from Rescaling Domains	80
<i>F. Dumortier & R. Roussarie</i>	
The Bridge Equation and its Applications to Local Geometry	100
<i>J. Ecalle</i>	
Rotation Numbers for Area Preserving Homeomorphisms of the Open Annulus	123
<i>J. Franks</i>	
Entropy of Rational Selfmaps of Projective Varieties	128
<i>S. Friedland</i>	
Boundary of Morse-Smale Surface Diffeomorphisms: An Obstruction to Smoothness	141
<i>J. M. Gambaudo</i>	
Constrained Stability for Nonlinear Electrical Circuits	153
<i>G. Ikegami</i>	
Nonlinear Stokes Phenomena	158
<i>Ju. S. Il'yashenko</i>	

PREFACE

The papers in this volume are contributed by the lecturers of the International Conference on Dynamical Systems and Related Topics held at the Nagoya International Center in Nagoya, Japan on September 3-7, 1990. Most of them were presented at the conference. The number of participants from Japan is about 100, and that from outside of Japan is 41.

The conference was held under the auspices of Nagoya University, Nagoya City, and the Mathematical Society of Japan. And it was supported financially by Nagoya University, Nagoya City, Daiko Foundation, Inamori Foundation, Ishida Foundation, Japan Association for Mathematical Sciences, Kato Ryuzo Gakujutsu Shinko Kikin (Kato Ryuzo Foundation for the Promotion of Science), and some private contributors participating in the conference. With out their kind help, we would not have been able to hold the conference. We express our sincere gratitude for their cooperation.

We should also mention that our conference had been planned as a satellite conference of the International Congress of Mathematicians held at Kyoto on August 21-29, 1990 (ICM 90). Our conference was made possible by the support of the organizing committee of ICM, and we also express our appreciation for their help.

For the preparation of the conference, the Scientific Committee had been organized. Their efforts toward the success of the conference are much appreciated. The collaboration of staffs of the College of General Education (especially the department of Mathematics) of Nagoya University was also valuable.

Finally we would like to express our gratitude to all the lecturers and participants of the conference without whom there would be no conference at all. We hope the proceedings of this conference would contribute to the development of dynamical systems and related topics in the world.

Ken-ichi Shuburo

Editor and

Chairman of the Organizing Committee

Jacobi Vector Fields along Geodesic Flows <i>I. Nobuhiro</i>	166
Ergodic Properties of Nonsingular Transformations with Asymptotically Periodic Densities <i>T. Inoue & H. Ishitani</i>	175
On Riemann Surfaces of Solutions for Analytic Integrable Systems and Their Holonomy Representations <i>H. Ito</i>	183
On a Dynamical Systems Related to the Sequences $[nx + y] - [(n-1)x + y] \quad n = 1, 2, \dots$ <i>S. Ito</i>	192
Asymptotic Behavior of Dynamical Systems and Processes on Banach Infinite Dimensional Spaces <i>A. F. Izé</i>	198
Global Stability for N -dimensional Lotka-Volterra Competition Simple Chain Systems <i>X.-H. Ji</i>	211
Bowen-Margulis and Patterson Measures on Negatively Curved Compact Manifolds <i>V. A. Kaimanovich</i>	223
Topological Dynamical Systems and Corresponding C^* -algebras <i>S. Kawamura</i>	233
Heteroclinic Bifurcations Associated with Different Saddle Indices <i>H. Kokubu</i>	236
Periodic Bifurcation Equations of Continuous Piecewise-Linear Vector Fields <i>M. Komuro</i>	261
Fatou-Julia Theory for Meromorphic Functions <i>J. Kotus</i>	289
Families of Locally Invariant Manifolds for Measure-Preserving Diffeomorphisms <i>M. Kurata</i>	304
Dynamics of Expanding Maps of the Interval <i>R. Labarca & M. J. Pacifico</i>	307

The "Blue Sky Catastrophe" on Closed Surfaces <i>W.-G. Li & Z.-F. Zhang</i>	316
Maslov-Type Index Theory and Asymptotically Linear Hamiltonian Systems <i>Y.-M. Long</i>	333
Several Kinds of Limit Cycle Bifurcations <i>D.-J. Luo</i>	342
A Method of Characteristic Functions in the Stability Theory of Dynamical Systems <i>N. Magnitsky</i>	353
On Measurable Field Compatible with Rational Functions <i>P. M. Makienko</i>	373
Fredholm Matrices and Zeta Function for Piecewise Monotonic Transformation <i>M. Mori</i>	388
The Topological Stability of Diffeomorphisms <i>K. Moriyasu</i>	401
On Some Results of Hofbauer on Maps of the Interval <i>S. Newhouse</i>	407
Location of Algebraic Integers and Related Topics <i>K. Nishizawa, K. Sekiguchi & K. Yoshino</i>	422
On a Characterization of C^1 Structurally Stable Endomorphisms on the Circle <i>K. Odani</i>	451
Homoclinic Bifurcations, Sensitive-Chaotic Dynamics and Strange Attractors <i>J. Palis</i>	466
Diffeomorphisms with Pseudo Orbit Tracing Property <i>K. Sakai</i>	473
Some Remarks on the Hénon Map <i>A. Sannami</i>	476
On the Parabolic Bifurcation of Holomorphic Maps <i>M. Shishikura</i>	478

Periodic Orbits near the Boundary of a 3-dimensional Manifold <i>J. Sotomayor & M. A. Teixeira</i>	487
Continuous Ergodicity of Anosov Maps <i>W.-X. Sun</i>	495
Definition of Partition and its Production for Many Types of Diffeomorphisms <i>M. Takase</i>	503
Weak Regularity of Lyapunov Exponents on One Dimensional Dynamical Systems <i>M. Tsujii</i>	510
Julia Sets with Polyhedral Symmetries <i>S. Ushiki</i>	515
Persistence of Strange Attractors when Unfolding Homoclinic Tangencies <i>M. Viana</i>	539
Chaos Caused by a Topologically Mixing Map <i>J.-C. Xiong & Z.-G. Yang</i>	550
Eventually Nonnegative Realization of Difference Control Systems <i>B. G. Zaslavsky</i>	573
The Levels of the Set of Recurrent Points, Topological Entropy and Chaos for Self-Maps of the Interval <i>Z.-L. Zhou & X.-C. Huang</i>	603
PROBLEMS	
Show the Existence of Higher Dimensional Ducks <i>G. Ikegami</i>	613
Open Problems <i>T. Ito</i>	615
Problems <i>Ju. S. Il'yashenko</i>	617
List of Participants	623

Dynamical Systems and Related Topics

ABSTRACT

The paper deals with singularly perturbed nonlinear dynamical systems described by the ordinary differential equations. The asymptotic behaviour of solutions is studied. The main result is a theorem in which the well-known Tikhonov's condition of the asymptotic stability is replaced by the condition of the simple stability in Lyapunov's sense. Under some additional assumptions we prove the convergence of the original system solution to the reduced system solution as a small parameter tends to zero.

INTRODUCTION

In a great number of technical, biological, sociological systems the variables considered change with very different rates. Mathematical modelling these processes leads to the singularly perturbed differential equations. We consider the autonomous system in the form