Workshop on the arithmetic geometry of Shimura varieties and Rapoport-Zink spaces

Date	July 4 (Mon) – 8 (Fri), 2011
Place	Department of Mathematics, Kyoto University
	Lecture Room 110
Organizer	Tetsushi Ito (Kyoto)

Program

July 4 (Mon)

10:30–10:45 Registration

11:00–12:00 Tetsushi Ito (Kyoto, Math Dept)
Shimura varieties and Rapoport-Zink spaces — an introduction

14:00–15:00 Kentaro Nakamura (Keio)
Zariski density of crystalline representations for any p-adic field

15:30–17:00 Chung Pang Mok (McMaster)
Galois representations associated to automorphic forms of GL_2 over CM fields and local-global compatibility up to semi-simplification

July 5 (Tue)

10:00–11:00 Noriyuki Abe (Hokkaido)
On a classification of irreducible admissible modulo p representations of a split p-adic group

11:15–12:15 Yoichi Mieda (Kyushu)
Lefschetz trace formula and ℓ-adic cohomology of the Rapoport-Zink spaces for $\mathrm{GSp}(4)$

14:00–15:00 Ana Caraiani (Harvard)
Local-global compatibility and monodromy

15:30–16:30 Dong Uk Lee (POSTECH)
Decomposition of Shimura datum of Hodge type

19:00– Banquet

July 6 (Wed)

10:00–11:00 Yuya Matsumoto (Tokyo)
On good reduction of some $K3$ surfaces

11:15–12:15 Sug Woo Shin (MIT/KIAS)
Supercuspidal part of the mod ℓ cohomology of compact
$\mathrm{GU}(1, n-1)$ Shimura varieties
(Free Afternoon)

July 7 (Thu)

10:00–11:00 Naoki Imai (Kyoto, RIMS)
Cohomology of rigid curves with semi-stable coverings

11:15–12:15 Takahiro Tsushima (Kyushu)
On the elementary computation of the stable reduction of $\mathscr{X}(\pi^2)$
and its cohomology

14:00–15:00 Przemyslaw Chojecki (Ecole Polytechnique)
Density of crystalline points and local-global compatibility of Emerton

15:30–16:30 Tetsushi Ito (Kyoto, Math Dept)
ℓ-adic cohomology of Rapoport-Zink spaces and supercuspidal packets

July 8 (Fri)

10:00–11:00 Miaofen Chen (Paris-Sud)
The determinant morphism for Rapoport-Zink spaces

11:15–12:15 Shushi Harashita (Yokohama National Univ)
Deligne-Lusztig varieties associated to G_2

14:00–15:00 Zhiwei Yun (MIT)
Arithmetic Fundamental Lemma for function fields

15:30–16:30 Kai-Wen Lan (Princeton)
Kodaira-type vanishing on good reduction fibers of Shimura varieties

Abstract

Speaker: Noriyuki Abe (Hokkaido)

Title: On a classification of irreducible admissible modulo p representations of a split p-adic group

Abstract: In the theory of modulo p representations of a split p-adic group, the notion of a supersingular representation is defined using the Satake transform. I give a classification of irreducible admissible modulo p representations in terms of supersingular representations. In particular, it is proved that an irreducible admissible representation is supersingular if and only if it is supercuspidal. These are generalizations of results of Barthel-Livne and Herzig.

Speaker: Ana Caraiani (Harvard)

Title: Local-global compatibility and monodromy

Abstract: Given a cuspidal automorphic representation of $\mathrm{GL}(n)$ over a CM field which is regular algebraic and conjugate self-dual, one can associate to it a Galois representation. This Galois representation is known in most cases to be compatible with local Langlands. When n is even, the compatibility is known up to semisimplification or when the representation satisfies an additional regularity condition. I will extend the compatibility to Frobenius semisimplification by identifying the monodromy operators on either side.

Speaker: Miaofen Chen (Paris-Sud)

Title: The determinant morphism for Rapoport-Zink spaces

Abstract: We will prove a local analogue on the nonarchimedean places of the determinant morphism for Shimura varieties defined by Deligne. Let $\mathscr{M}$ be an unramified Rapoport-Zink space of EL type or PEL type which is unitary or symplectic. Let $\mathscr{M}_{\mathrm{rig}}$ be the generic fiber of Berthelot of $\mathscr{M}$ in the sense of rigid analytic spaces over which there exists a tower of rigid analytic spaces $(\mathscr{M}_K)_K$ classifying the level structures. The objective of the talk is to study the geometrically connected components of this tower. In order to do this, we define a determinant morphism $\det_K$ from the tower $(\mathscr{M}_K)_K$ to a tower of rigid analytic spaces of dimension 0 associated to the cocenter of the reductive group related to the space $\mathscr{M}$. We prove that the geometric fibers of the determinant morphism $\det_K$ are the geometrically connected components of $\mathscr{M}_K$.

Speaker: Przemyslaw Chojecki (Ecole Polytechnique)

Title: Density of crystalline points and local-global compatibility of Emerton

Abstract: We will prove that (almost) crystalline points are dense inside the completed cohomology of unitary Shimura varieties of Harris-Taylor. Then, we will explain how this result leads to the weak version of local-global compatibility of Emerton for Shimura curves over totally real fields where p is totally decomposed. Assuming certain modularity conjecture, this will permit us to deduce the Fontaine-Mazur conjecture in this case.

Speaker: Shushi Harashita (Yokohama National Univ)
Title: Deligne-Lusztig varieties associated to G_2
Abstract: In some Shimura varieties, Deligne-Lusztig varieties appear as natural subvarieties. It would be important to understand the geometry of Deligne-Lusztig varieties. In this lecture, I am going to talk about defining equations and the affineness of (fine generalized) Deligne-Lusztig varieties associated to split G_2.

Speaker: Naoki Imai (Kyoto, RIMS)
Title: Cohomology of rigid curves with semi-stable coverings
Abstract: In this talk, we will explain construction of a semi-stable formal model of a rigid curve with a semi-stable covering. Then we consider a finite flat morphism between two rigid curves with semi-stable coverings. We discuss the induced pullback and pushforward between the ℓ-adic cohomology of the rigid curves. This is joint work with Takahiro Tsushima.

Speaker: Tetsushi Ito (Kyoto)
Title: Shimura varieties and Rapoport-Zink spaces — an introduction
Abstract: The aim of this talk is to motivate the participants of the workshop to work on Shimura varieties and Rapoport-Zink spaces. I will give a brief introduction to Shimura varieties and Rapoport-Zink spaces with a special emphasis on geometry and cohomology. I will also discuss several applications to the local/global Langlands correspondences and Galois representations. Many important results have been obtained so far, but so many challenging problems are still open.

Title: Cohomology of Rapoport-Zink spaces and supercuspidal packets
Abstract: The ℓ-adic cohomology of Rapoport-Zink spaces is expected to realize local Langlands and local Jacquet-Langlands correspondences. Supercuspidal representations are the "most interesting" ones, and they tend to concentrate in the middle degree cohomology. However, for the Rapoport-Zink space of $\mathrm{GSp}(4, \mathbb{Q}_p)$, in a recent joint work with Yoichi Mieda, we discovered certain supercuspidal representations do appear outside the middle degree. This phenomenon is related to A-packets of Saito-Kurokawa representations of $\mathrm{GSp}(4, \mathbb{A})$. In this talk, I try to discuss

a conjectural description of the degree of cohomology of supercuspidal representations in terms of the structure of local L & A-packets and Zelevinsky duality. This might be considered as a local analogue of Arthur's conjecture on the classification of L^2 automorphic representations.

Speaker: Kai-Wen Lan (Princeton)
Title: Kodaira-type vanishing on good reduction fibers of Shimura varieties

Abstract: While the classical Kodaira vanishing theorem is false in general in positive characteristics, Deligne, Illusie and Raynaud showed that it remains true under some mild (liftability and dimension) conditions. Since then it has been generalized in two directions: Esnault and Viehweg allowed the line bundle to be somewhat less than ample, and Illusie allowed certain nontrivial coefficients of geometric origin. I will explain a common generalization of these two, which implies interesting vanishing results for the cohomology of (noncompact) PEL-type Shimura varieties with torsion automorphic coefficients. (This is joint work with Junecue Suh.)

Speaker: Dong Uk Lee (POSTECH)
Title: Decomposition of Shimura datum of Hodge type

Abstract: We prove that arbitrary Shimura datum of Hodge type admits a covering into a product of certain abelian type Shimura data associated with its adjoint datum.

Speaker: Yuya Matsumoto (Tokyo)
Title: On good reduction of some $K3$ surfaces

Abstract: Let X be a variety over a local field K. If X is an abelian variety, a theorem of Serre-Tate shows that X has good reduction if and only if its ℓ-adic étale cohomology is unramified (a Galois representation of K is unramified if the action of the inertia group is trivial). In this talk, I prove that similar results hold if X belongs to certain classes of $K3$ surfaces.

Speaker: Yoichi Mieda (Kyushu)
Title: Lefschetz trace formula and ℓ-adic cohomology of the Rapoport-Zink spaces for $\mathrm{GSp}(4)$

Abstract: Rapoport-Zink spaces are certain moduli spaces of quasi-isogenies of p-divisible groups with additional structures and can be regarded as local analogues of Shimura varieties. Kottwitz' conjecture predicts that their ℓ-adic cohomology partially realize the local Langlands correspondence and the local Jacquet-Langlands correspondence for rather general reductive groups. In this talk, we will investigate the ℓ-adic cohomology of the Rapoport-Zink space for GSp(4) by using Lefschetz trace formula for open adic spaces, and give a partial result on the conjecture of Kottwitz.

Speaker: Chung Pang Mok (McMaster)

Title: Galois representations associated to automorphic forms of GL_2 over CM fields and local-global compatibility up to semi-simplification

Abstract: Let E be a CM extension over a totally real field F. Given a regular algebraic cuspidal automorphic representation π on $\mathrm{GL}_2(E)$, whose central character is invariant under Galois conjugation of E/F, we construct the corresponding compatible system of 2-dimensional p-adic Galois representations attached to π. The basic idea, originally due to Harris-Soudry-Taylor, is to consider lifting to $\mathrm{GSp}_4(F)$. We present two constructions: one originally due to Taylor, by using p-power congruences; another one depends on using eigenvariety for $\mathrm{GSp}_4(F)$ (recently constructed by Vincent Pilloni and others). Time allowed we will also indicate the proof of local-global compatibility up to semi-simplification. This whole work was made possible by Arthur's classification of discrete spectrum of GSp_4.

Speaker: Kentaro Nakamura (Keio)

Title: Zariski density of crystalline representations for any p-adic field

Abstract: I would like to talk about a theorem concerning Zariski density of n-dimensional crystalline representations for any n and for any p-adic field K. This theorem is the generalization of Colmez and Kisin's theorem in the case $n = 2$ and $K = \mathbb{Q}_p$, which played many crucial roles in their studies of p-adic local Langlands for $\mathrm{GL}_2(\mathbb{Q}_p)$. The key for the proof is to construct a p-adic family of trianguline representations. We construct this family by generalizing Kisin's theory of finite slope subspace X_{fs}. We note that Chenevier recently proved this theorem for any n and for $K = \mathbb{Q}_p$ by generalizing Colmez's proof.

Speaker: Sug Woo Shin (MIT/KIAS)

Title: Supercuspidal part of the mod ℓ cohomology of compact $\mathrm{GU}(1, n-1)$ Shimura varieties

Abstract: Let p be a prime where $\mathrm{GU}(1, n-1)$ becomes isomorphic to a general linear group. We describe the mod ℓ cohomology of the Shimura varieties in the part where $G(\mathbb{Q}_p)$ acts by a mod ℓ supercuspidal representation. Any prime ℓ and an arbitrarily deep level structure at ℓ are allowed. The method is different from that of Emerton and Gee on a similar problem, and the results seem to be complementing each other.

Speaker: Takahiro Tsushima (Kyushu)

Title: On the elementary computation of the stable reduction of $\mathscr{X}(\pi^2)$ and its cohomology

Abstract: Let F be a local field with integer ring $\mathscr{O}_F$, π a uniformizer and k the residue field.

The Lubin-Tate space $\mathscr{X}(\pi^n)$ is a deformation space of the unique (up to isomorphism) formal $\mathscr{O}_F$-module $\Sigma/\bar{k}$ equipped with Drinfeld level π^n-structure. Recently, Jared Weinstein finds a systematic way to find components in the stable reduction of the Lubin-Tate tower, using p-adic Hodge theory, etc. In my talk, on the basis of the work of R. Coleman and K. McMurdy on the stable reduction of the modular curve $X_0(p^3)$, we explicitly calculate the stable reduction of $\mathscr{X}(\pi^2)$. Our method is very elemetary only using blow-ups. As a result of the computation, we explicitly determine the action of GL_2, the division algebra with invariant $1/2$, and the inertia group on the stable reduction. As an application of these computations, we observe that the local Jacquet-Langlands correspondence and the local Langlands correspondence for GL_2, for the supercuspidal representations of level 1 or $1/2$, are realized in the étale cohomology H^1 of the Lubin-Tate space $\mathscr{X}(\pi^2)$. We also discuss a relationship with the cohomology H_c^2 of Lusztig surface (affine) for $\mathrm{GL}_2(\mathscr{O}_F/\pi^2)$. This work is a joint work with Yoichi Mieda and Tetsushi Ito.

Speaker: Zhiwei Yun (MIT)
Title: Arithmetic Fundamental Lemma for function fields
Abstract: Arithmetic Fundamental Lemma (AFL) is a conjecture of Wei Zhang relating orbital integrals of $\mathrm{GL}(n)$ and intersection numbers on the unitary Rapoport-Zink spaces. We will sketch a proof of this conjecture in the function field case. We will first establish a global analog of AFL on counting certain Shtukas, and then deduce the AFL from its global analog.