QUASI-LOG CANONICAL PAIRS ARE DU BOIS

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Abstract. We prove that every quasi-log canonical pair has only Du Bois singularities. Note that our arguments are free from the minimal model program.

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1. Introduction

The notion of quasi-log canonical pairs was first introduced by Florin Ambro (see [A]) and is now known to be ubiquitous in the theory of minimal models. The main purpose of this paper is to establish:

Theorem 1.1. *Every quasi-log canonical pair has only Du Bois singularities.*

Theorem 1.1 is a complete generalization of [KK, Theorem 1.4] and [Kl, Corollary 6.32]. This is because we get the following corollary by combining Theorem 1.1 with the main result of [F2].

Corollary 1.2 ([Kl, Corollary 6.32])**.** *Let (X, Δ) be a semi-log canonical pair. Then any union of slc strata of (X, Δ) is Du Bois. In particular, X has only Du Bois singularities.*

By considering the definition and basic properties of quasi-log canonical pairs, Theorem 1.1 can be seen as an ultimate generalization of [KK, Theorem 1.5] (see also Corollary 4.2 below).

Corollary 1.3 ([KK, Theorem 1.5])**.** *Let $g : X \to Y$ be a proper surjective morphism with connected fibers between normal varieties. Assume that there exists an effective $\mathbb{R}$-divisor Δ on X such that (X, Δ) is log canonical and $K_X + \Delta \sim_{\mathbb{R},g} 0$. Then Y has only Du Bois singularities.*

More generally, let $W \subset X$ be a reduced closed subscheme that is a union of log canonical centers of (X, Δ). Then $g(W)$ is Du Bois.

The proof of Theorem 1.1 in this paper is different from the arguments in [KK] and [Kl]. This is mainly because we can not apply the minimal model program to quasi-log canonical pairs. We will use some kind of canonical bundle formula for reducible varieties, which follows from the theory of variations of mixed Hodge structure on cohomology with compact support, for the study of normal irreducible quasi-log canonical pairs. We want to

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emphasize that we do not use the minimal model program in this paper. From the Hodge theoretic viewpoint, we think that Kollár and Kovács ([KK]) avoided the use of variations of mixed Hodge structure by taking dlt blow-ups, which need the minimal model program.

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Conventions. We will work over $\mathbb{C}$, the complex number field, throughout this paper. A *scheme* means a separated scheme of finite type over $\mathbb{C}$. A *variety* means a reduced scheme, that is, a reduced separated scheme of finite type over $\mathbb{C}$. We will freely use the standard notation of the minimal model program and the theory of quasi-log schemes as in [F3] (see also [F1]).

2. Preliminaries

In this section, let us recall some basic definitions and prove an easy lemma.

Let $D=\sum_i d_iD_i$ be an $\mathbb{R}$-divisor, where D_i is a prime divisor and $d_i\in\mathbb{R}$ for every i such that $D_i\neq D_j$ for $i\neq j$. We put

$$D^{<c}=\sum_{d_i<c} d_iD_i,\quad D^{\leq c}=\sum_{d_i\leq c} d_iD_i,\quad D^{=1}=\sum_{d_i=1} D_i,\quad \text{and}\quad \lceil D\rceil=\sum_i \lceil d_i\rceil D_i,$$

where c is any real number and $\lceil d_i\rceil$ is the integer defined by $d_i\leq\lceil d_i\rceil<d_i+1$. Moreover, we put $\lfloor D\rfloor=-\lceil -D\rceil$ and $\{D\}=D-\lfloor D\rfloor$.

Let Δ_1 and Δ_2 be $\mathbb{R}$-Cartier divisors on a scheme X. Then $\Delta_1\sim_{\mathbb{R}}\Delta_2$ means that Δ_1 is $\mathbb{R}$-linearly equivalent to Δ_2.

Let $f: X\to Y$ be a morphism between schemes and let B be an $\mathbb{R}$-Cartier divisor on X. Then $B\sim_{\mathbb{R},f}0$ means that there exists an $\mathbb{R}$-Cartier divisor B' on Y such that $B\sim_{\mathbb{R}} f^*B'$.

Let Z be a simple normal crossing divisor on a smooth variety M and let B be an $\mathbb{R}$-divisor on M such that Z and B have no common irreducible components and that the support of $Z+B$ is a simple normal crossing divisor on M. In this situation, $(Z, B|_Z)$ is called a *globally embedded simple normal crossing pair*.

Let us quickly look at the definition of qlc pairs.

Definition 2.1 (Quasi-log canonical pairs)**.** Let X be a scheme and let ω be an $\mathbb{R}$-Cartier divisor (or an $\mathbb{R}$-line bundle) on X. Let $f: Z\to X$ be a proper morphism from a globally embedded simple normal crossing pair (Z,Δ_Z). If the natural map $\mathcal{O}_X\to f_*\mathcal{O}_Z(\lceil -(\Delta_Z^{<1})\rceil)$ is an isomorphism, $\Delta_Z=\Delta_Z^{\leq 1}$, and $f^*\omega\sim_{\mathbb{R}}K_Z+\Delta_Z$, then $(X,\omega,f:(Z,\Delta_Z)\to X)$ is called a *quasi-log canonical pair* (*qlc pair*, for short). If there is no danger of confusion, we simply say that $[X,\omega]$ is a qlc pair.

Let $(X,\omega,f:(Z,\Delta_Z)\to X)$ be a quasi-log canonical pair as in Definition 2.1. Let $\nu: Z^\nu\to Z$ be the normalization. We put $K_{Z^\nu}+\Theta=\nu^*(K_Z+\Delta_Z)$, that is, Θ is the sum of the inverse images of Δ_Z and the singular locus of Z. Then (Z^ν,Θ) is sub log canonical in the usual sense. Let W be a log canonical center of (Z^ν,Θ) or an irreducible component of Z^ν. Then $f\circ\nu(W)$ is called a *qlc stratum* of $(X,\omega,f:(Z,\Delta_Z)\to X)$. If there is no danger of confusion, we simply call it a qlc stratum of $[X,\omega]$. If C is a qlc stratum of $[X,\omega]$ but is not an irreducible component of X, then C is called a *qlc center* of $[X,\omega]$. The union of all qlc centers of $[X,\omega]$ is denoted by $\mathrm{Nqklt}(X,\omega)$. It is important that $[\mathrm{Nqklt}(X,\omega),\omega|_{\mathrm{Nqklt}(X,\omega)}]$ is a quasi-log canonical pair by adjunction (see, for example, [F3, Theorem 6.3.5]).

Let us prepare an easy lemma for Corollary 1.3.

Lemma 2.2. *Let $[X,\omega]$ be a quasi-log canonical pair and let $g : X \to Y$ be a proper morphism between varieties with $g_*\mathcal{O}_X \simeq \mathcal{O}_Y$. Assume that $\omega \sim_{\mathbb{R}} g^*\omega'$ holds for some $\mathbb{R}$-Cartier divisor (or $\mathbb{R}$-line bundle) ω' on Y. Then $[Y,\omega']$ is a quasi-log canonical pair such that W' is a qlc stratum of $[Y,\omega']$ if and only if $W' = g(W)$ for some qlc stratum W of $[X,\omega]$.*

Proof. By definition, we can take a proper morphism $f : Z \to X$ from a globally embedded simple normal crossing pair (Z,Δ_Z) such that $f^*\omega \sim_{\mathbb{R}} K_Z+\Delta_Z$, $\Delta_Z = \Delta_Z^{\leq 1}$, and the natural map $\mathcal{O}_X \to f_*\mathcal{O}_Z(\lceil -(\Delta_Z^{<1})\rceil)$ is an isomorphism. Let us consider $h := g \circ f : Z \to Y$. Then we can easily see that $(Y, \omega', h : (Z,\Delta_Z) \to Y)$ is a quasi-log canonical pair with the desired properties. □

For the details of the theory of quasi-log schemes, see [F3, Chapter 6].

Let us recall the definition of semi-log canonical pairs for the reader's convenience.

Definition 2.3 (Semi-log canonical pairs)**.** Let X be an equidimensional scheme which satisfies Serre's S_2 condition and is normal crossing in codimension one. Let Δ be an effective $\mathbb{R}$-divisor on X such that no irreducible component of SuppΔ is contained in the singular locus of X and that $K_X + \Delta$ is $\mathbb{R}$-Cartier. We say that (X,Δ) is a *semi-log canonical* pair if (X^ν, Δ_{X^ν}) is log canonical in the usual sense, where $\nu : X^\nu \to X$ is the normalization of X and $K_{X^\nu} + \Delta_{X^\nu} = \nu^*(K_X + \Delta)$, that is, Δ_{X^ν} is the sum of the inverse images of Δ and the conductor of X. An *slc center* of (X,Δ) is the ν-image of an lc center of (X^ν, Δ_{X^ν}). An *slc stratum* of (X,Δ) means either an slc center of (X,Δ) or an irreducible component of X.

For the details of semi-log canonical pairs, see [F2] and [Kl].

3. Du Bois criteria

A *reduced pair* is a pair (X,Σ) where X is a reduced scheme and Σ is a reduced closed subscheme of X. Then we can define the *Deligne–Du Bois complex* of (X,Σ), which is denoted by $\underline{\Omega}^\bullet_{X,\Sigma}$ (see [F3, 5.3.1], [Kl, Definition 6.4], [Kv1, Definition 3.9], [S, Section 3], and so on). We note that $\underline{\Omega}^\bullet_{X,\Sigma}$ is a filtered complex in a suitable derived category. We put

$$\underline{\Omega}^p_{X,\Sigma} := \mathrm{Gr}^p_{\mathrm{filt}}\underline{\Omega}^\bullet_{X,\Sigma}[p].$$

By taking $\Sigma = \emptyset$, we obtain the *Deligne–Du Bois complex* of X:

$$\underline{\Omega}^\bullet_X := \underline{\Omega}^\bullet_{X,\emptyset}$$

and similarly

$$\underline{\Omega}^p_X := \mathrm{Gr}^p_{\mathrm{filt}}\underline{\Omega}^\bullet_X[p].$$

By definition and construction, there exists a natural map

$$\mathcal{I}_\Sigma \to \underline{\Omega}^0_{X,\Sigma},$$

where $\mathcal{I}_\Sigma$ is the defining ideal sheaf of Σ on X.

Definition 3.1 (Du Bois pairs and Du Bois singularities)**.** A reduced pair (X,Σ) is called a *Du Bois pair* if the natural map $\mathcal{I}_\Sigma \to \underline{\Omega}^0_{X,\Sigma}$ is a quasi-isomorphism. When the natural map $\mathcal{O}_X \to \underline{\Omega}^0_X$ is a quasi-isomorphism, we say that X has only *Du Bois singularities* or simply say that X is *Du Bois*.

For the details, see, for example, [Kv1, Sections 2, 3, 4, and 5], [KK, Section 6.1], and [F3, Section 5.3].

The following lemma is a very minor modification of [Kv2, Theorem 3.3] (see also [Kl, Theorem 6.27]).

Lemma 3.2. *Let $f : Y \to X$ be a proper morphism between varieties. Let $V \subset X$ be a closed reduced subscheme with ideal sheaf $\mathcal{I}_V$ and $W \subset Y$ with ideal sheaf $\mathcal{I}_W$. Assume that $f(W) \subset V$ and that the natural map*

$$\rho : \mathcal{I}_V \to Rf_*\mathcal{I}_W$$

admits a left inverse ρ', that is, $\rho' \circ \rho$ is a quasi-isomorphism. In this situation, if (Y, W) is a Du Bois pair, then (X, V) is also a Du Bois pair. In particular, if (Y, W) is a Du Bois pair, then X is Du Bois if and only if V is Du Bois.

We give a proof of Lemma 3.2 for the reader's convenience although it is the same as that of [Kv2, Theorem 3.3].

Proof of Lemma 3.2. By functoriality, we have the following commutative diagram

$$\begin{array}{ccc} \mathcal{I}_V & \xrightarrow{\ \rho\ } & Rf_*\mathcal{I}_W \\ {\scriptstyle\alpha}\downarrow & & \downarrow{\scriptstyle\gamma} \\ \underline{\Omega}^0_{X,V} & \xrightarrow[\ \beta\ ]{} & Rf_*\underline{\Omega}^0_{Y,W}. \end{array}$$

Since (Y, W) is assumed to be a Du Bois pair, we see that γ is a quasi-isomorphism. Therefore, $\rho' \circ \gamma^{-1} \circ \beta$ is a left inverse to α. Then (X, V) is a Du Bois pair by [Kv2, Theorem 3.2] (see also [Kv1, Theorem 5.5] and [Kl, Corollary 6.24]). When (X, V) is a Du Bois pair, it is easy to see that X is Du Bois if and only if V is Du Bois by definition (see [Kl, Proposition 6.15] and [Kv1, Proposition 5.1]). □

We prepare one more easy lemma.

Lemma 3.3. *Let $f : Y \to X$ be a proper birational morphism from a smooth irreducible variety Y onto a normal irreducible variety X. Let B_Y be a subboundary $\mathbb{R}$-divisor on Y, that is, $B_Y = B_Y^{\leq 1}$, such that $\mathrm{Supp} B_Y$ is a simple normal crossing divisor on Y and let M_Y be an f-nef $\mathbb{R}$-divisor on Y. Assume that $K_Y + B_Y + M_Y \sim_{\mathbb{R},f} 0$, $B_Y^{<0}$ is f-exceptional, and $f(B_Y^{=1})$ has only Du Bois singularities. Then X has only Du Bois singularities.*

Proof. Since $B_Y^{=1}$ is a simple normal crossing divisor on a smooth variety Y, $B_Y^{=1}$ is Du Bois (see, for example, [F3, Proposition 5.3.10]). In particular, $(Y, B_Y^{=1})$ is a Du Bois pair. We put $Z = f(B_Y^{=1})$. We note that

$$-\lfloor B_Y \rfloor - (K_Y + \{B_Y\}) \sim_{\mathbb{R},f} M_Y$$

is nef and big over X. Therefore, $R^i f_*\mathcal{O}_Y(-\lfloor B_Y \rfloor) = 0$ holds for every $i > 0$ by the relative Kawamata–Viehweg vanishing theorem. Then we have

$$\mathcal{I}_Z = f_*\mathcal{O}_Y(-B_Y^{=1}) \to Rf_*\mathcal{O}_Y(-B_Y^{=1}) \to Rf_*\mathcal{O}_Y(-\lfloor B_Y \rfloor) \simeq f_*\mathcal{O}_Y(-\lfloor B_Y \rfloor) = \mathcal{I}_Z,$$

where $\mathcal{I}_Z$ is the defining ideal sheaf of Z on X. This means that the natural map $\rho : \mathcal{I}_Z \to Rf_*\mathcal{O}_Y(-B_Y^{=1})$ has a left inverse. By Lemma 3.2, (X, Z) is a Du Bois pair. By assumption, $Z = f(B_Y^{=1})$ is Du Bois. Therefore, we obtain that X has only Du Bois singularities (see [Kl, Proposition 6.15] and [Kv1, Proposition 5.1]). □

4. Proof of Theorem 1.1

In this section, we prove Theorem 1.1 and Corollaries 1.2 and 1.3.

First let us prove Theorem 1.1. We note that the reduction step to normal irreducible varieties in the proof of Theorem 1.1 below is nothing but [FL, Proposition 1.4]. We include it for the benefit of the reader.

Proof of Theorem 1.1. Let $[X, \omega]$ be a quasi-log canonical pair. We prove Theorem 1.1 by induction on $\dim X$. If $\dim X = 0$, then the statement is obvious. Let X_1 be an irreducible component of X and let X_2 be the union of the irreducible components of X other than X_1. Then $[X_1, \omega|_{X_1}]$, $[X_2, \omega|_{X_2}]$, and $[X_1 \cap X_2, \omega|_{X_1\cap X_2}]$ are quasi-log canonical pairs by adjunction (see, for example, [F3, Theorem 6.3.5]). In particular, X_1, X_2, and $X_1 \cap X_2$ are seminormal (see [F3, Remark 6.2.11]). Then we have the following short exact sequence

$$0 \to \mathcal{O}_X \to \mathcal{O}_{X_1} \oplus \mathcal{O}_{X_2} \to \mathcal{O}_{X_1\cap X_2} \to 0 \tag{4.1}$$

(see, for example, [Kl, Lemma 10.21]). We note that $X_1 \cap X_2$ is Du Bois since $[X_1 \cap X_2, \omega|_{X_1\cap X_2}]$ is a quasi-log canonical pair with $\dim X_1 \cap X_2 < \dim X$. By (4.1) and [F3, Lemma 5.3.9], it is sufficient to prove Theorem 1.1 under the extra assumption that X is irreducible by induction on the number of the irreducible components of X. Therefore, from now on, we assume that X is irreducible. Let $\nu : Z \to X$ be the normalization. Then, by [FL, Theorem 1.1], $[Z, \nu^*\omega]$ naturally becomes a quasi-log canonical pair with

$$R\nu_*\mathcal{I}_{\mathrm{Nqklt}(Z,\nu^*\omega)} = \mathcal{I}_{\mathrm{Nqklt}(X,\omega)}. \tag{4.2}$$

By induction on dimension, $\mathrm{Nqklt}(Z, \nu^*\omega)$ and $\mathrm{Nqklt}(X, \omega)$ are Du Bois. This is because $[\mathrm{Nqklt}(Z, \nu^*\omega), \nu^*\omega|_{\mathrm{Nqklt}(Z,\nu^*\omega)}]$ and $[\mathrm{Nqklt}(X, \omega), \omega|_{\mathrm{Nqklt}(X,\omega)}]$ are quasi-log canonical pairs by adjunction (see, for example, [F3, Theorem 6.3.5]). If Z is Du Bois, then $(Z, \mathrm{Nqklt}(Z, \nu^*\omega))$ is a Du Bois pair. In this case, we can easily see that X is Du Bois by Lemma 3.2 and (4.2) since $\mathrm{Nqklt}(X, \omega)$ is Du Bois. Therefore, it is sufficient to prove that Z is Du Bois. This means that we may further assume that X is a normal irreducible variety for the proof of Theorem 1.1. By Theorem 4.1 below and Lemma 3.3, we obtain that X has only Du Bois singularities. We note that $\mathrm{Nqklt}(X, \omega)$ is Du Bois since $[\mathrm{Nqklt}(X, \omega), \omega|_{\mathrm{Nqklt}(X,\omega)}]$ is a quasi-log canonical pair with $\dim \mathrm{Nqklt}(X, \omega) < \dim X$ by adjunction (see, for example, [F3, Theorem 6.3.5]). Therefore, X is always Du Bois when $[X, \omega]$ is a quasi-log canonical pair. □

The following theorem is a special case of [F4, Theorem 1.5].

Theorem 4.1 (see [F4, Theorem 1.5])**.** *Let $[X, \omega]$ be a quasi-log canonical pair such that X is a normal irreducible variety. Then there exists a projective birational morphism $p : X' \to X$ from a smooth quasi-projective variety X' such that*

$$K_{X'} + B_{X'} + M_{X'} = p^*\omega,$$

where $B_{X'}$ is a subboundary $\mathbb{R}$-divisor, that is, $B_{X'} = B_{X'}^{\leq 1}$, such that $\mathrm{Supp}B_{X'}$ *is a simple normal crossing divisor and that $B_{X'}^{<0}$ is p-exceptional, and $M_{X'}$ is an $\mathbb{R}$-divisor which is nef over X. Furthermore, we can make $B_{X'}$ satisfy $p(B_{X'}^{=1}) = \mathrm{Nqklt}(X, \omega)$.*

The proof of Theorem 4.1 uses the notion of basic slc-trivial fibrations, which is some kind of canonical bundle formula for reducible varieties (see [F4]). Note that [F4] depends on some deep results on the theory of variations of mixed Hodge structure on cohomology with compact support (see [FF]).

Finally we prove Corollaries 1.2 and 1.3.

Proof of Corollary 1.2. Without loss of generality, by shrinking X suitably, we may assume that X is quasi-projective since the problem is Zariski local. Then, by [F2, Theorem 1.2], $[X, K_X + \Delta]$ has a natural qlc structure, which is compatible with the original semi-log canonical structure. For the details, see [F2]. Therefore, any union of slc strata of (X, Δ) is a quasi-log canonical pair. Thus, by Theorem 1.1, it is Du Bois. □

Proof of Corollary 1.3. Let $f : Z \to X$ be a resolution of singularities such that $f^*(K_X + \Delta) = K_Z + \Delta_Z$, $\Delta_Z = \Delta_Z^{\leq 1}$, and $\mathrm{Supp}\Delta_Z$ is a simple normal crossing divisor on Z. Since

Z is irreducible, we can see that (Z, Δ_Z) is a globally embedded simple normal crossing pair. Since $\lceil -(\Delta_Z^{<1}) \rceil$ is effective and f-exceptional, $f_*\mathcal{O}_Z(\lceil -(\Delta_Z^{<1}) \rceil) \simeq \mathcal{O}_X$. Therefore,

$$(X, \omega, f : (Z, \Delta_Z) \to X),$$

where $\omega := K_X + \Delta$, is a quasi-log canonical pair such that C is a log canonical center of (X, Δ) if and only if C is a qlc center of $(X, \omega, f : (Z, \Delta_Z) \to X)$. We take an $\mathbb{R}$-Cartier divisor (or $\mathbb{R}$-line bundle) ω' on Y such that $\omega \sim_{\mathbb{R}} g^*\omega'$. Then, by Lemma 2.2,

$$(Y, \omega', g \circ f : (Z, \Delta_Z) \to Y)$$

is a quasi-log canonical pair. By Theorem 1.1, Y is Du Bois. We note that $g(W)$ is a union of qlc strata of $(Y, \omega', g \circ f : (Z, \Delta_Z) \to Y)$. Therefore, by adjunction (see, for example, [F3, Theorem 6.3.5]), $[g(W), \omega'|_{g(W)}]$ is also a quasi-log canonical pair. Then, by Theorem 1.1 again, $g(W)$ has only Du Bois singularities. □

By combining [F2, Theorem 1.2] with Lemma 2.2, we can generalize Corollary 1.3 as follows.

Corollary 4.2. *Let $g : X \to Y$ be a projective surjective morphism between varieties with $g_*\mathcal{O}_X \simeq \mathcal{O}_Y$. Assume that there exists an effective $\mathbb{R}$-divisor Δ on X such that (X, Δ) is semi-log canonical and $K_X + \Delta \sim_{\mathbb{R}, g} 0$. Then Y has only Du Bois singularities.*

More generally, let $W \subset X$ be a reduced closed subscheme that is a union of slc strata of (X, Δ). Then $g(W)$ is Du Bois.

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